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(*b*. Breslau, Germany, 27 April 1837; *d*. Erlangen, Germany, 21 December 1912)

mathematics.

The son of David Gordan, a merchant, Paul Albert attended Gymnasium and business school, then worked for several years in banks. His early interest in mathematics was encouraged by the private tutoring he received from N. H. Schellbach, a professor at the Friedrich Wilhelm Gymnasium. He attended Ernst Kummer's lectures in number theory at the University of Berlin in 1855, then studied at the universities of Breslau, Königsberg, and Berlin. At Königsberg he came under the influence of Karl Jacobi's school, and at Berlin his interest in algebraic equations was aroused. His dissertation (1862), which concerned geodesics on spheroids, received a prize offered by the philosophy faculty of the University of Breslau. The techniques that Gordan employed in it were those of Lagrange and Jacobi.

Gordan's interest in function theory led him to visit F. B. Riemann in Göttingen in 1862, but Riemann was ailing and their association was brief. The following year, Gordan was invited to Giessen by A. Clebsch, with whom he worked on the theory of Abelian functions. Together they wrote an exposition of the theory. In 1874 Gordan became a professor at Erlangen, where he remained until his retirement in 1910. He married Sophie Deuer, the daughter of a Giessen professor of Roman law, in 1869.

In 1868 Clebsch introduced Gordan to the theory of invariants, which originated in an observation of George Boole's in 1841 and was further developed by Arthur Cayley in 1846. Following the work of these two Englishmen, a German branch of the theory was developed by S. H. Aronhold and Clebsch, the latter elaborating the former's symbolic methods of characterizing algebraic forms and their invariants. Invariant theory was Gordan's main interest for the rest of his mathematical career; he became known as the greatest expert in the field, developing many techniques for representing and generating forms and their invariants. Correcting an error made by Cayley in 1856, Gordan in 1868 proved by constructive methods that the invariants of systems of binary forms possess a finite base. Known as the Gordan finite basis theorem, this instigated a twenty-year search for a proof in case of higher-order systems of forms. Making use of the Aronhold-Clebsch symbolic calculus and other elaborate computational techniques, Gordan spent much of his time seeking a general proof of finiteness. The solution to the problem came in 1888, when David Hilbert proved the existence of finite bases for the invariants of systems of forms of arbitrary order. Hilbert's proof, however, provided no method for actually finding the basis in a given case. Although Gordan was said to have objected to Hilbert's existential procedures, in 1892 he wrote a paper simplifying them. His version of Hilbert's theorem is the one presented in many textbooks.

Apparently unaware of James J. Sylvester's attempts in 1878, Gordan and a student, G. Alexejeff, applied the theory of invariants to the problems of chemical valences in 1900. Alexejeff went so far as to write a textbook on invariant theory that was intended for chemists. After some very hostile criticism from the mathematician Eduard Study and an indifferent reception by chemists, the project of introducing invariants into chemistry was dropped. Gordan made a few more contributions to invariant theory, but in the thirty years following Hilbert's work, interest in the subject declined among mathematicians.

The second major area of Gordan's contributions to mathematics is in solutions of algebraic equations and their associated groups of substitutions. Working jointly with Felix Klein in 1874–1875 on the relationship of icosahedral groups to fifth-degree equations, Gordan went on to consider seventh-degree equations with the group of order 168; and toward the end of his career, equations of the sixth degree with the group of order 360. His work was algebraic and computational, and utilized the techniques of invariant theory. Typical of Gordan's many contributions to these subjects are papers in 1882 and 1885 in which, following Klein's exposition of the general problem, he carries out the explicit reduction of the seventh-degree equation to the setting of the substitution group of order 168.

Gordan made other contributions to algebra and gave simplified proofs of the transcendence of e and π. The overall style of Gordan's mathematical work was algorithmic. He shied away from presenting his ideas in informal literary forms. He derived his results computationally, working directly toward the desired goal without offering explanations of the concepts that motivated his work.

Gordan's only doctoral student, Emmy Noether, was one of the first women to receive a doctorate in Germany. She carried on his work in invariant theory for a while, but under the stimulus of Hilbert's school at Göttingen her interests shifted and she became one of the primary contributors to modern algebra.

BIBLIOGRAPHY

Further information on Gordan and his work may be found in Charles Fisher, “The Death of a Mathematical Theory,” in *Archive for History of Exact Sciences*, **3** , no. 2 (1966), 137–159; and “The Last Invariant Theorists,” in *European Journal of Sociology*, **8** (1967), 216–244. See also Felix Klein, *Lectures on the Icosahedron* (London, 1888), and *Lectures on Mathematics* (New York, 1911), lecture **9** ; Max Noether, “Paul Gordan,” in *Mathematische Annalen*, **75** (1914), 1–41, which contains a complete bibliography of Gordan’s works; and Hermann Weyl, “Emmy Noether,” in *Scripta mathematica*, **3** (l935), 201–220.

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