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(*b*. Vienenburg, near Hildesheim, Germany, 25 March 1798; *d*. Münster, Germany, 25 September 1852)

mathematics.

Gudermann's father was a teacher. After graduating from secondary school Gudermann was to have studied to become a priest, but in Göttingen he studied, among other things, mathematics. From 1823 he was a teacher at the secondary school in Kleve; and from 1832 until his death he taught at the Theological and Philosophical Academy in Münster, first as associate professor, and from 1839 as full professor, of mathematics.

Gudermann's scientific work forms part of German mathematics in the second quarter of the nineteenth century. The characteristic feature of this period is that the ideas of transforming mathematics had been expressed or indicated, but understanding and realizing them in results or comprehensive theories was still beyond the capabilities of the mathematicians; this was achieved only in the second half of the century. Much preparatory work had to be carried out for this transformation. As soon as comprehensive, sufficiently accurate and general theories had been established, the preparatory work was forgotten. This was also the fate of Gudermann's work; he is known as the teacher of Karl Weierstrass rather than as an original thinker.

The depth of Gudermann's understanding of the contemporary trends in mathematics is substantiated by the topic which he discussed in his own work. C. F. Gauss's influence on Gudermann is still unclear, but the topic he chose is close to the intellectual environment of Gauss and his followers. Basically, Gudermann considered only two groups of problems: spherical geometry and the theory of special functions.

His book *Grundriss der analytischen Sphärik* (1830) deals with the former. He considered the study of spherical geometry important for several reasons. In the introduction he pointed out that a plane was a special case of a spherical surface, that is, a sphere with an infinite radius. For this reason and because of its constant curvature there exist many similarities between spherical geometry and plane geometry; yet at the same time Gudermann considered scientifically more interesting the study of cases in which this similarity no longer holds. As part of this program he sought to establish an analytical system for spherical surfaces akin to that formed by the coordinate system in planimetry. But he had to admit the existence of insurmountable difficulties if the required simplicity of the analytical means was to be preserved. At some points in the book Gudermann came close to problems which were important for non-Euclidean geometry but did not stress them, nor did he explicitly mention this aspect.

Gudermann devoted much more attention to the theory of special functions. After the earlier works of Leonhard Euler, John Landen, and A. M. Legendre (Gauss's results were still in manuscript), Niels Abel's studies on elliptical functions, published mostly in A. L. Crelle's *Journal für reine und angewandte Mathematik*, represented an important divide in treating this area. In 1829 Carl Jacobi's book *Fundamenta nova theoriae functionum ellipticarum* was published. At the time Gudermann was one of the first mathematicians to expand on these results. Beginning with volume **6** (1830) of Crelle's *Journal*, he published a series of papers which he later summarized in two books: *Theorie der Potenzialoder cyklisch-hyperbolischen Functionen* (1833) and *Theorie der Modular-Functionen und der Modular-Integrale* (1844), which were to have had a sequel which was never written.

In these books Gudermann went back to the origin of the theory of special functions—the problems of integral calculus—and stressed the genetic connection between simply periodic functions and elliptical functions. Since he did not neglect the requirements of integral calculus, he saw the necessity of arranging the theory to allow for numerical calculations. The key appeared to be in the development of the functions into infinite series and infinite products and in the use of suitable transformations. This made it possible to present extensive numerical tables in his first book and to work through to the nucleus of the theory of special functions in his second book, indicating the way which subsequently proved to be exceptionally fruitful. Thus he also came close to Gauss's intentions. Gudermann also introduced a notation for elliptical functions—*sn, cn*, and *dn*— which was adopted. He himself called elliptical functions "Modularfunctionen." It was pointed out later that Gudermann's work had an excess of special cases which in time lost interest.

Gudermann's work drew deserved attention in Germany in the 1830's. Since he was one of the few university professors to treat the problems of elliptical functions systematically, Karl Weierstrass came from the University of Bonn in 1839–1840 to attend Gudermann's lectures and presented "Über die Entwicklung der Modularfunctionen" as a *Habilitationsschrift* in 1841.

Gudermann was one of the first to realize Weierstrass' mathematical talent and scientific ability. Weierstrass, using Gudermann's idea of the development of functions into series and products, formed the principal, mighty, and accurate tool of the theory of functions.

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