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(*b*. Rouen, France, 30 October 1844; *d*. Versailles [?], France, 23 May 1889)

mathematics.

Halphen's mathematical reputation rests primarily on his work in analytic geometry. Specifically, his principal interests were the study of singular points of algebraic plane curves, the study of characteristics of systems of conics and second-order surfaces, the enumeration and classification of algebraic space curves, the theory of differential invariants and their applications, and the theory of elliptic functions and their applications. His papers are marked by brilliance combined with dogged perseverance.

Halphen was raised in Paris, where his mother moved shortly after she was widowed in 1848. His early schooling was at the Lycée Saint-Louis, and he was admitted to the École Polytechnique in 1862. He served with great distinction in the Franco-Prussian War, and in 1872 he married the daughter of Henri Aron; she eventually bore him four sons and three daughters. Also in 1872 Halphen returned to the École Polytechnique, where he was appointed *répétiteur* and rose to *examinateur* in 1884. His doctorate in mathematics was awarded in 1878 upon the presentation of his thesis, *Sur les invariants différentiels*. In 1880 Halphen won the Ormoy Prize (Grand Prix des Sciences Mathématiques) of the Academy of Sciences in Paris for advances he had made in the theory of linear differential equations, and in 1882 he received the Steiner Prize from the Royal Academy of Sciences in Berlin for his work on algebraic space curves. He was elected to membership in the French Academy in 1886, an honor which he enjoyed for only three years before he died of what was called "overwork."

Halphen first came to the attention of the mathematical community in 1873, when he resolved Michel Chasles's conjecture: Given a family of conics depending on a parameter, how many of them will satisfy a given side condition? Chasles had found a formula for this, but his proof was faulty. Halphen showed that Chasles was essentially correct, but that restrictions on the kinds of singularities were necessary. Halphen's solution was ingenious: he transformed the given system of conics into one algebraic plane curve, and the side condition into another; his results were then obtained from the study of the two curves.

After solving Chasles's problem, Halphen went on to make significant contributions in the theory of algebraic plane curves, especially in the study of their singular points. He was the first to classify singular points and extended earlier work of Bernhard Riemann by giving a general formula for the genus of an algebraic plane curve. Then, considering curves in the same genus, he extended a theorem of Max Noether which proved that in any class there always exist curves with only ordinary singularities.

This work led Halphen to the subject of differential invariants. He had noticed in his earlier work that under projective (i.e., linear and one-to-one) transformations certain differential equations remained unchanged. He was able to characterize all such equations and presented the results in 1878 as his thesis. Henri Poincaré was so impressed that he said: "... the theory of differential invariants is to the theory of curvature as projective geometry is to elementary geometry" ("Notice sur Halphen," p. 154; also *Oeuvres*, I, xxxv). Later Halphen applied these results to the integration of linear differential equations, greatly extending the classes of these equations which could be solved. For the latter work he was awarded its prize for 1880 by the French Academy of Sciences.

Halphen's most significant original work was the paper which won the Steiner Prize. In it he made a complete classification of all algebraic space curves up to the twentieth degree. This problem is much more difficult than the corresponding one for algebraic plane curves. A plane curve of degree *k* can be considered to be a special case of the most general curve of degree *k;* thus the class and genus of the curve are known if the degree is known, perhaps modified by singularities. But for space curves there is no such thing as a most general curve of degree *k* (a space curve requires at least two equations) and so, in Halphen's words, "...one never knows any geometric entity which includes, as special cases, all space curves of given degree. One cannot, therefore, assert *a priori* for any property of space curves, no matter how general, that it will depend only on the degree" ("Sur quelques propriétés des courbes gauches algébriques," p. 69; also *Oeuvres*, I, 203). For example, the genus has no algebraic relation to the degree but instead satisfies certain inequalities.

Halphen's last work was a monumental treatise on elliptic functions. He intended that it consist of three volumes, but he died before he could finish the last. The aim of the work was to simplify the theory of elliptic functions to the point where they could be put to use by the nonspecialist without losing any of the essential points. In the first volume he realized this aim, proving everything he needed without recourse to more general function theory. In the process Halphen not only simplified the theory but also eliminated much of the very cumbersome notation then in use. The second volume is concerned principally with applications from mechanics, geometry, and differential equations. The problems solved are all difficult and are either new or show new insights. The third volume was to contain material on the theory of transformation and applications to number theory.

The amount and quality of Halphen's work is impressive, especially considering that his mathematically creative life covered only seventeen years. Why, then, is his name so little known? The answer lies partly in the fact that some of his work, the theory of differential invariants, is now only a special case of the more general Lie group theory and thus has lost its identity. But part of the answer is related to a larger question: Why is so much mathematics of even the recent past lost? In Halphen's case, he worked in analytic and differential geometry, a subject so unfashionable today as to be almost extinct. Perhaps with its inevitable revival, analytic geometry will restore Halphen to the eminence he earned.

BIBLIOGRAPHY

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