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(*b*. Glasgow, Scotland, 8 March 1788; *d*. Edinburgh, Scotland, 6 May 1856)

philosophy, logic.

Hamilton's father, William Hamilton, professor of astronomy at the University of Glasgow, died in 1790, leaving William to be raised by his mother, Elizabeth Hamilton. After receiving a degree from the University of Edinburgh in 1807, Hamilton went to Balliol College. Oxford, with a Snell Exhibition. He quickly acquired the reputation of being the most learned authority in Oxford on Aristotle, and the list of books that he submitted for his final examination in 1810 was unprecedented. He did not, however, receive a fellowship, primarily because of the unpopularity of Scots at Oxford. He returned therefore to Edinburgh to study and there became an advocate in 1813. Because he had little interest in his career in the law, he applied in 1821 for the chair of moral philosophy at Edinburgh vacated by the death of Thomas Brown. Hamilton was a Whig, and the Tory town council therefore chose his opponent, John Wilson. When in 1829 Macvey Napier became the editor of the *Edinburgh Review*, he persuaded Hamilton to write a series of articles for that journal. The articles, which appeared between 1829 and 1836, were the basis of his international reputation, a reputation that forced the town council to elect him in 1836 to the chair in logic and metaphysics, which he held until his death.

Hamilton's three most important articles for the *Edinburgh Review* were those on Cousin (1829), on perception (1830), and on logic (1833). In the first two, he revealed his unique philosophical position, a combination of the Kantian view that there is a limitation on all knowledge and the Scottish view that man has, in perception, a direct acquaintance with the external world.

The first paper deals with the possibility of human knowledge of the absolute. In it Hamilton argued against Cousin's view that man has immediate knowledge of the absolute and against Schelling's view that man can know the absolute by becoming identical with it. Hamilton tried to show that neither of these views is coherent and that there is something incoherent about the very notion of thought about the absolute. Hamilton's own position was close to Kant's, but he wanted to go further than Kant and say that the mind cannot use the absolute even as a regulative idea.

The second article is a defense of Reid's view, that the direct object of perception is external, against the attacks of Brown. Hamilton had little trouble in showing that Brown neither understood Reid's position nor could offer arguments that disproved either Reid's position or the position mistakenly attributed to him by Brown. Hamilton did, however, agree with Brown's claim that Reid was mistaken when he identified the direct object of acts of memory with some previously existing external object.

These metaphysical positions were developed further during the twenty years that Hamilton was professor at Edinburgh. Many of his mature opinions, as expressed in the appendixes to his edition of Reid's major works and to his own published lectures, modify what he wrote in these two articles; but he never really gave up these basic positions, which were extremely influential during his lifetime and still have some interest today. They are, however, far less important for the history of thought in general and for the history of science in particular than his work in logic.

Hamilton was one of the first in that series of British logicians—a series that included George Boole, Augustus De Morgan, and John Venn—who radically transformed logic and created the algebra of logic and mathematical logic. To be sure, Hamilton only helped begin this development, and given his dislike of mathematics, he probably would not have been very happy with its conclusion. Nevertheless, his place in it must be recognized.

The traditional, Aristotelian analysis of reasoning allowed for only four types of simple categorical propositions:

(A) All A are B.

(E) No A are B.

(I) Some A are B.

(O) Some A are not B.

Hamilton's first important insight was that logic would be more comprehensive and much simpler if it allowed for additional types of simple categorical propositions. In particular, Hamilton suggested that one treat the signs of quantity ("all," "some," "no") in the traditional propositions as modifiers of the subject term A and that one introduce additional signs of quantity as modifiers of the predicate term B. Hamilton called this innovation the quantification of the predicate. Other logicians before Hamilton had made the same suggestion, but Hamilton was the first to explore the implications of quantifying the predicate, of admitting eight simple categorical propositions:

(1) All A are all B.

(2) All A are some B (traditional A).

(3) Some A are all B.

(4) Some A are some B (traditional I).

(5) Any A is not any B (traditional E).

(6) Any A is not some B.

(7) Some A are not any B (traditional O).

(8) Some A are not some B.

The first important inference that Hamilton drew from this modification had to do with the analysis of simple categorical propositions. There were, according to the traditional, Aristotelian logic, two ways of analyzing a simple categorical proposition such as "All A are B": extensively, that is, as asserting that the extension of the term A is contained within the extension of the term B; or comprehensively, that is, as asserting that the comprehension of the term B is contained within the comprehension of the term A. In either case, the proposition expresses a whole-part relation. But the new Hamiltonian modification, because it distinguished (1) from (2), (3) from (4), (5) from (6), and (7) from (8), enables one to adopt a different analysis of these propositions. According to this new analysis, each of these propositions asserts or denies the existence of an identity-relation between the two classes denoted by the quantified terms. Thus, "All A are all B" asserts that the classes A and B are identical, while "Some A are not some B" asserts that there is a subset of the class A which is not identical with any subset of the class B. One result, therefore, of the quantification of the predicate is that simple categorical propositions become identity claims about classes. This is just the analysis of simple categorical propositions that Boole needed and used in formulating the algebra of logic.

Hamilton's work facilitated a considerably simplified analysis of the validity of reasoning. The traditional, Aristotelian analysis of mediate reasoning, for example, involved many concepts (such as the figure of a syllogism, major and minor terms) that were based on the distinction between the subject of a proposition and its predicate. This subject-predicate distinction had some significance when simple categorical propositions were understood as expressing asymmetrical whole-part relations. But given the new analysis, where these propositions are understood as expressing symmetrical identity relations, there is little point to a distinction between the subject and the predicate of a proposition. Further, if the subject-predicate distinction is dropped, then all of the traditional cumbersome machinery based on it should also be dropped. As a result, the complicated traditional rules for the validity of syllogistic reasoning disappear. One is then left, as Hamilton pointed out in his theory of the unfigured syllogism, with two simple rules for valid syllogisms: If $A = B$ and $B = C$, then $A = C$; and If $A = B$ and $B \neq C$, then $A \neq C$. Similarly, the traditional Aristotelian analysis of immediate reasoning, based upon the complicated distinctions between simple conversion, conversion *per accidens,* and contraposition, is replaceable by the simple rule that all eight propositions are simply convertible. This rule follows directly from the fact that all eight propositions are concerned with symmetrical identity relations.

New advances in a given science, besides simplifying the treatment of previously solved problems, usually enable one to solve problems that one could not previously handle. Hamilton's quantification of the predicate is no exception to this rule. The logician could now explain the validity of many inferences that resisted traditional analysis. The simplest example of this is the inference to the identity of classes A and C from premises asserting that they are both identical with some class B. Traditional analysis did not even recognize the existence of propositions asserting that two classes are identical; it could not, therefore, explain the validity of such an inference. Hamiltonian analysis, however, could do so by referring to the first of the two rules for the validity of all mediate reasoning.

Some of Hamilton's new ideas, such as his classidentity analysis of propositions, were incorporated into Boole's far more sophisticated system. This contribution to mathematical logic would in itself be sufficient to earn for Hamilton an important place in intellectual history, but his claim to recognition is strengthened by the significance of his innovations to the history of logic. As is well known, Kant and most other important eighteenth-century philosophers thought that nothing of importance had been done in formal logic since the time of Aristotle, primarily because of the completeness and perfection of the Aristotelian system. The only people who saw a future for logic were those who wanted to change logic from a formal analysis of the validity of reasoning to an epistemological and psychological analysis of the conditions for, and limits on, human knowledge. Hamilton, by showing that the Aristotelian analysis could be greatly improved and supplemented, changed the minds of many philosophers, logicians, and mathematicians and helped produce the interest in formal logic that was so necessary for the great advances of the nineteenth century.

Despite its great historical significance, Hamilton's quantification of the predicate has had little direct influence in more recent times. This is partly due to the fact that both it and the Boolean algebra of logic, which it so greatly influenced, have been superseded by Frege's far more powerful quantificational analysis—an analysis so different that Hamilton's theory has no relevance to it. It is, however, also due to a certain internal weakness in Hamilton's initial quantification of the predicate, which was pointed out by Hamilton's great adversary, Augustus De Morgan, during their long and acrimonious quarrels.

There really were two quarrels between Hamilton and De Morgan. The first had to do with Hamilton's charge that De Morgan had plagiarized some of Hamilton's basic ideas. In 1846 De Morgan sent a draft of one of his most important papers on logic to William Whewell, who was supposed to transmit it to the Cambridge Philosophical Society. De Morgan then received from Hamilton, in the form of a list of requirements for a prize essay set for Hamilton's students, a brief account of Hamilton's quantification of the predicate. At about this time, De Morgan asked Whewell to return the draft of his paper and then made some changes in it. Hamilton charged that the alterations were based on his communication to De Morgan. In his reply De Morgan claimed that he made the changes before he received the communication from Hamilton. Although it is not clear as to who was right about the date of the changes, it is clear that De Morgan did not plagiarize Hamilton's ideas. Even if De Morgan's ideas were suggested by Hamilton's communication, they are so different from Hamilton's that no one could consider them to be a plagiarism.

The second, far more important, quarrel was about the relative merits of their innovations in logic. This arose out of the first, since De Morgan was not content with pointing out the differences between the two systems. He also argued that Hamilton's innovations, unlike his own, were based on a defective list of basic propositions.

De Morgan first made this claim in an appendix to his book *Formal Logic* (1847). Although he offered several criticisms of Hamilton's list of eight basic categorical propositions, there was only one that was really serious—that Hamilton's first proposition is not a simple categorical proposition because it is equivalent to the joint assertion of the second and third propositions. Thus "All A are some B" and "Some A are all B" can both be true if, and only if, "All A are all B" is also true.

Hamilton was slow in responding to this argument, primarily because he had suffered an attack of paralysis in 1844 that made it very difficult for him to do any work. When, however, in 1852, he published a collection of his articles from the *Edinburgh Review*, he included in the book an appendix in which he argued that De Morgan's criticisms were based on a misunderstanding of the eight propositional forms. De Morgan thought that "some" meant "some, possibly all." If it did, then he would certainly be right in his claim that the first proposition is equivalent to the joint assertion of the second and third propositions. But Hamilton said that he had meant in his forms "some, and not all." Consequently, the conjunction of "All A are some B" and "Some A are all B" is inconsistent, and neither of these propositions can be true if "All A are all B" is true.

The controversy rested at this point until some years after Hamilton's death. Then, De Morgan renewed it in a series of letters in *Athenaeum* (1861–1862) and in his last article on the syllogism, in *Transactions of the Cambridge Philosophical Society* (1863). De Morgan began his new attack by casting doubt on the claim that Hamilton had meant "some, but not all." He did this by showing that much of what Hamilton had to say about the validity of particular inferences made sense only if we suppose that he meant "some, perhaps all." There is little doubt that De Morgan was right about this point. Yet De Morgan now had an even more crushing criticism of Hamilton's list of simple categorical propositions: Even if one grants, he said, that Hamilton meant "some, but not all," there is still something wrong with his list. After all, there are five, and only five, relations of the type discussed in categorical propositions that can hold between two classes: (1) the two classes are coextensive, (2) the first class is a proper subset of the second class, (3) the second class is a proper subset of the first class, (4) the two classes have some members in common but each has members that are not members of the other, and (5) the two classes have no members in common. Thus Hamilton's propositions 6–8 seem to be superfluous.

De Morgan's final critique clearly showed that Hamilton had not exercised sufficient care in laying the foundations for his new analysis of the validity of reasoning. This was quickly recognized by most logicians; Charles Sanders Peirce, the great American logician, described De Morgan's 1863 paper as unanswerable. While there is no doubt that De Morgan's critique helped lessen the eventual influence of Hamilton's work, it should not prevent the recognition of both the intrinsic merit of Hamilton's work and its role in the development of mathematical logic in Great Britain during the nineteenth century.

BIBLIOGRAPHY

I. Original Works. Hamilton's main writings are important essays supplementary to *The Works of Thomas Reid* (Edinburgh, 1846); *Discussions on Philosophy and Literature and Education and University Reform* (Edinburgh, 1852); *Lectures on Metaphysics*, H. Mansel and J. Veitch, eds. (Edinburgh, 1859); and *Lectures on Logic*, H. Mansel and J. Veitch, eds. (Edinburgh, 1861).

II. Secondary Literature. On Hamilton and his work, see T. S. Baynes, *An Essay on the New Analytic of Logical Forms* (Edinburgh, 1850); S. A. Grave, *The Scottish Philosophy of Common Sense* (Oxford, 1960); L. Liard, *Les logiciens anglais* *contemporains* (Paris, 1890); J. S. Mill, *An Examination of Sir William Hamilton's Philosophy* (London, 1889); S. V. Radmussen, *The Philosophy of Sir William Hamilton* (Copenhagen, 1925); and J. Veitch, *Sir William Hamilton* (Edinburgh, 1869).

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