Hardy, Godfrey Harold | Encyclopedia.com

(*b*. Cranleigh, England, 7 February 1877; *d*. Cambridge, England, 1 December 1947)

mathematics.

Hardy was the elder of two children of Isaac Hardy, a master at Cranleigh School, and Sophia Hall. The parents were intelligent and mathematically minded, but lack of money had precluded them from a university education. They provided an enlightened upbringing for Hardy and his sister.

The freedom to ask questions and to probe led Hardy to an early established disbelief in religious doctrine. (As a fellow of New College, Oxford, he refused to enter the chapel to take part in electing a warden.) Neither Hardy nor his sister married, and he owed much to her devoted care throughout his life, particularly in his later years.

As a boy Hardy showed all-around ability with a precocious interest in numbers. At the age of thirteen he moved from Cranleigh School with a scholarship to Winchester College, to this day a famous nursery of mathematicians. He went on to Trinity College, Cambridge, in 1896, was fourth wrangler in the mathematical tripos in 1898, was elected a fellow of Trinity in 1900, and won (with J. H. Jeans) a Smith's Prize in 1901. Success in the tripos depended on efficient drilling in solving problems quickly. Hardy, resenting the routine of the famous "coach" R. R. Webb, had the good fortune to be transferred to A. E. H. Love. No description of Hardy's development into a mathematician can be so vivid as his own:

My eyes were first opened by Professor Love, who taught me for a few terms and gave me my first serious conception of analysis. But the great debt which I owe to him was his advice to read Jordan's famous *Cours d'analyse;* and I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and learnt for the first time as I read it what mathematics really meant [*A Mathematician's Apology*, sec. 29].

Hardy flung himself eagerly into research and between 1900 and 1911 wrote many papers on the convergence of series and integrals and allied topics. Although this work established his reputation as an analyst, his greatest service to mathematics in this early period was *A Course of Pure Mathematics* (1908). This work was the first rigorous English exposition of number, function, limit, and so on, adapted to the undergraduate, and thus it transformed university teaching.

The quotation from the *Apology* continues, "The real crises of my life came ten or twelve years later, in 1911, when I began my long collaboration with Littlewood, and in 1913, when I discovered Ramanujan."

J. E. Littlewood, eight years younger than Hardy, proved in 1910 the Abel-Tauber theorem that, if na_n is bounded and $\Sigma a_n x^n \rightarrow s$ as $x \rightarrow 1$, then $\Sigma a_n = s$. The two then entered into a collaboration which was to last thirty-five years. They wrote nearly a hundred joint papers. Among the topics covered were Diophantine approximation (the distribution, modulo 1, of functions $f(n)$ of many types, such as θn^2 for irrational θ), additive and multiplicative theory of numbers and the Riemann zeta function, inequalities, series and integrals in general (for instance, summability and Tauberian theorems), and trigonometric series.

The partnership of Hardy and Littlewood has no parallel, and it is remarkable that, at its greatest intensity (1920–1931), Hardy lived in Oxford and Littlewood in Cambridge. They set up a body of axioms expressing the freedom of their collaboration, for example, "When one received a letter from the other he was under no obligation to read it, let alone to answer it." The final writing of the papers was done by Hardy.

Hardy called his discovery of Srinivasa Ramanujan the one romantic incident of his life. One morning early in 1913, he received a letter from this unknown Indian, containing a number of formulae without any proofs. Established mathematicians are exposed to manuscripts from amateurs, and Hardy could not at a glance assess it. A few hours' work convinced him that the writer was a man of genius. Ramanujan turned out to be a poor, self-taught clerk in Madras, born in 1887. Hardy brought him to England in April 1914 and set about the task of filling the gaps in his formal mathematical education. Ramanujan was ill from May 1917 onward; he returned to India in February 1919 and died in April 1920. In his three years of health and activity, he and Hardy had arrived at spectacular solutions of problems about the partition of numbers which called forth the full power of the Indian's natural insight and the Englishman's mastery of the theory of functions.

Denote by $p(n)$ the number of ways of writing n as the sum of positive integers (repetitions allowed), so that $p(5) = 7$. As n increases, $p(n)$ increases rapidly; for instance, $p(200)$ is a number of thirteen digits, a computation which in 1916 took a month. Hardy and Ramanujan established an asymptotic formula for $p(n)$, of which five terms sufficed to give the value of $p(200)$.

Hardy was a lecturer at Trinity College until 1919, when he became Savilian professor of geometry at Oxford; there he founded a flourishing school of research. For the year 1928–1929 he went to Princeton, exchanging places with Oswald Veblen. He returned to Cambridge in 1931, succeeding E. W. Hobson as Sadleirian professor of pure mathematics; he held this chair until his retirement in 1942.

Besides Littlewood and Ramanujan, Hardy collaborated with many other mathematicians, including E. C. Titchmarsh, A. E. Ingham, E. Landau, G. Pólya, E. M. Wright, W. W. Rogosinski, and M. Riesz. He had an exceptional gift for working with others, as he had for leading young men in their early days of research.

Hardy had one ruling passion—mathematics. Apart from that his main interest was in ball games, particularly cricket, of which he was a stylish player and an expert critic. Some of his interests and antipathies are revealed by this list of six New Year wishes which he sent on a postcard to a friend in the 1920's: (1) prove the Riemann hypothesis; (2) make 211 not out in the fourth innings of the last test match at the Oval; (3) find an argument for the nonexistence of God which shall convince the general public: (4) be the first man at the top of Mt. Everest; (5) be proclaimed the first president of the U.S.S.R. of Great Britain and Germany; (6) murder Mussolini.

Hardy was generally recognized as the leading English pure mathematician of his time. His writings attest both his technical power and his mastery of English prose. The photographs in *Collected Papers* show his finely cut features and something of his physical grace. His liveliness and enthusiasm, are vivid in the memory of all who knew him. He received awards from many universities and academies, being elected in 1947 *associé étranger* of the Paris Academy of Sciences—of whom there are only ten from all nations in all subjects.

BIBLIOGRAPHY

I. Original Works. Hardy published, alone or in collaboration, about 350 papers. A complete list is in *Journal of the London Mathematical Society*. **25** (1950), 89–101. Collected papers are being published in 7 vols. (Oxford, 1966–), edited, with valuable comments, by a committee appointed by the London Mathematical Society..

Hardy wrote four tracts published at Cambridge: *The Integration of Functions of a Single Variable* (1905); *Orders of Infinity* (1910); *The General Theory of Dirichlet's Series* (1915), written with M. Riesz; and *Fourier Series* (1944), written with W. W. Rogosinski. The last, in particular, is a model of concise lucidity.

Hardy underlined the neglect of analysis in England by writing in the preface to the 1st ed. of *A Course of Pure Mathematics* (Cambridge, 1908; 10th ed., 1952): "I have indeed in an examination asked a dozen candidates, including several future senior wranglers to sum the series $1 + x + x^2$ and not received a single answer that was not practically worthless." His book changed all that. *Inequalities* (Cambridge, 1934), written with J. E. Littlewood and G. Póya, is a systematic account and includes much material previously accessible only in journals. *The Theory Numbers* (Oxford, 1938), written with E. M. Wright, includes chapters on a variety of topics.

Other works include *A Mathematicians Apology* (Cambridge, 1940; repr. 1967 with a foreword by C. P. Snow); *Ramanujan* (Cambridge, 1NO), twelve lectures on his life and work; *Bertrand Russell and Trinity* (Cambridge, 1970), an account of a 1914–1918 controversy, showing Hardy's sympathy with Russell's opposition to the war. See especially *Divergent Series* (Cambridge, 1948), completed by Hardy shortly before his death. According to Littlewood in his foreword, "All his books gave him some degree of pleasure, but this one, his last, was his favourite."

II. Secondary Lithrature. Notices on Hardy are in *Nature*, **161** (1948), 797; *Obituary Notices Fellows of the Royal Society of London*, **6** (1949), 447–470, with portrait; *Journal of the London Mathematical Society*, **25** (1950), 81; and *Dictionary of National Biography 1941–1950* (Oxford, 1959), 358–360.

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