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(*b*. Buk, Posen, Germany [now Poznan, Poland], 20 September 1887; *d*. Copenhagen, Denmark, 13 February 1947)

mathematics.

Hecke was the son of Heinrich Hecke, an architect. He attended elementary school in Buk and high school in Posen, then studied from 1905 to 1910 at the universities of Breslau, Berlin, and Göttingen. At Berlin he worked mainly with Edmund Landau, and at Göttingen with David Hilbert. In 1910 he obtained his Ph.D. at Göttingen. Subsequently he became Hilbert's and Felix Klein's assistant and was made *Privatdozent* in 1912. In 1915 Hecke became professor at Basel. He went to Göttingen in 1918 and in 1919 to the recently established University of Hamburg, where he was a professor until his death from cancer. He was married and had a son who died young. Hecke was a member of the editorial staff of several mathematical journals and belonged to well-known learned societies.

Most of Hecke's work dealt with analytic number theory, continuing the research of Riemann, Dedekind, and Heinrich Weber. It was Hilbert who influenced him on the subject of his thesis and some additional works on an analogue of complex multiplication, namely, the construction of class fields over real quadratic number fields by adjoining certain values of Hilbert's modular functions. The findings did not meet Hecke's expectations but nevertheless yielded new results, such as an attack on the proof of the functional equation of the Dedekind zeta function. Hecke proved in 1917 that this function can be continued throughout the complex s-plane to a single pole at $s = 1$, where it is "regular" and sufficient for a functional equation of the Riemann zeta-function type.

From this Hecke deduced the decomposition laws of divisors of discriminants for the class fields of complex multiplication. He also defined the generalized Dirichlet L-series for algebraic number fields and derived a functional equation for it. The analogue of the Dirichlet prime number law for number fields followed. Further development of these methods led him to the creation and study of zeta functions $\zeta(s, \lambda)$ with characters λ; that is, Hecke's L-series, which are of fundamental importance to advanced analytic number theory. This research was continued in various directions by Emil Artin, C. L. Siegel, and J. T. Tate.

After these studies and certain related works Hecke turned in 1925 to elliptic modular functions. He systematically applied quadratic number fields to the construction of modular functions. For imaginary quadratic fields he gave an extension of a class of functions known to Klein; for real quadratic fields there arose a new type of function. Hecke was led to these through his functions $\zeta(s, \lambda)$. Hecke then dealt with the Eisenstein series of higher order, especially the partial values of the Weierstrass functions $p(z)$ and $\zeta(z)$. He determined the periods of the Abelian integrals which are received through integration of the p partial values and certain series of the imaginary quadratic number fields. The problem of the periods of the Abelian integral of the first kind concerned him again and again, especially in connection with the representation theory of finite groups.

In 1936 Hecke systematically investigated the connection, restored by the gamma integral and the Mellin integral, of the Dirichlet series with a functional equation of Riemannian type and functions belonging to a certain automorphic group. Through his "operator" T_n Hecke established a theory for the investigation of relations of modular functions to Dirichlet series with Euler's product development. He discovered new connections between prime numbers and analytic functions and new rules for the representation of natural numbers through positive integral quadratic forms of an even number of variables. In 1939 Hans Petersson proved a rule, already anticipated by Hecke, concluding a part of this theory.

Some of Hecke's works are in another field. They are related to his approach to physics, especially the kinetic theory of gases.

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Bruno Schoeneberg