Heilbronn, Hans Arnold | Encyclopedia.com

(*b*. Berlin, Germany, 8 October 1908; *d*. Toronto, Canada, 28 April 1975)

Mathematics.

Heilbronn was born the son of Alfred and Gertrud Heilbronn, middle-class Jews who were cousins. After graduating from the Realgymnasium at Berlin-Schmargenhof in 1926 and studying at the universities of Berlin, Freiburg, and Göttingen, in 1930 he became assistant to Edmund Landau at Göttingen. Heilbronn's work on the distribution of primes rapidly brought him renown. He received the doctorate in 1933.

The advance of Hitler to power forced Heilbronn to emigrate to England in 1933, and until 1935 he was supported at the University of Bristol by refugee organizations. In this period he rose to prominence with his proof of Gauss's conjecture that the class number $h(d)$ of the imaginary quadratic field of discriminant $d<0$ tends to infinity as d tends to minus infinity. Erich Hecke had already shown in 1918 that this would follow from a generalized Riemann hypothesis. Heilbronn showed that Gauss's conjecture also follows from the falsity of the same hypothesis. By the "law of the excluded middle" Gauss's conjecture holds, but this argument is ineffective in the logical sense: for given h_o it does not provide a d_o such that $h(d)>h_o$ for all $d<d_o$. Heilbronn's theorem was strengthened and generalized by C.L. Siegel (1935), Richard Brauer (1950), and others, but the first effective version was obtained only in 1983 by Gross and Zagier, using a conditional result of Goldfeld. In 1934 Heilbronn showed (with E.H. Linfoot) that there are at most ten imaginary quadratic fields with class number h(d) = 1. Again the proof was ineffective, and although nine fields with this property were known, the proof gave no means for deciding whether there actually was a tenth. Its existence was claimed to be disproved in 1952 by K.Heegner in a controversial paper (later seen to be basically correct) and by H. M. Stark and A. Baker in 1966–1967.

From 1935 to 1940 Heilbronn was at Cambridge as fellow of Trinity College. This period saw the beginning of his close collaboration with Harold Davenport, which lasted until the latter's death in 1971. In 1936 Heilbronn published a paper simplifying, strengthening, and making generally accessible the methods by which I.M. Vinogradov had greatly improved the estimates of Godfrey Hardy and John Littlewood for such additive problems as Waring's problem; and in a series of papers (1936–1937) with Davenport he considered specific problems of this type. In another area, Heilbronn showed that there are only finitely many real quadratic fields in which the euclidean algorithm holds. His method was not suited to determining them all, which was done only in the 1950's by Davenport and others, using the geometry of numbers. At that time Heilbronn returned to the problem; he showed that only finitely many cyclic cubic fields have a euclidean algorithm and obtained similar results in other cases to which the methods of the geometry of numbers do not apply.

In 1940, at the outbreak of war, Heilbronn was interned for a period as an "enemy alien" but then served with the British services. He was demobilized in 1945 and, after a brief period at University College, London, returned in 1946 to Bristol as reader and became a British citizen; in 1949 he became professor and head of department. He rapidly built up a strong department, particularly a school of number theory. One of his pupils was A. Fröhlich.

At the beginning of the Bristol period, Heilbronn considered the approximations to an arbitrary real number θ by rational fractions whose denominator is a perfect square. He showed that for every $\eta > 0$ there is a $C(\eta) > 0$ with the property that for every real θ and every integer $N \geq 1$ there are integers n.g such that

A weaker form with ⅖ instead of ½ had been given by Vinogradov (1927). Heilbronn asked whether ½ is the best possible constant, but no further progress has been made; there have, however, been a large number of generalizations. The work on euclidean algorithms in cyclic fields was also done at Bristol, but increasingly Heilbronn's preoccupation with policy problems and administrative irritations left him little time for mathematics.

In 1963, feeling that the university was not supporting his department as it should, Heilbronn resigned his chair at Bristol and in 1964 accepted one in Toronto. He moved to Canada with his wife Dorothy Greaves, whom he had married that same year. In 1970 he became a Canadian citizen. In Toronto he built up a thriving research school and took an active part in Canadian mathematics. In what was to be his last published paper, he showed that the question of whether there is a real zero s_0 in $½ < S_0 < 1$ of the zeta-function $\zeta_L(s)$ of an algebraic number field L reduces to the case when L is quadratic. This was subsequently used by Stark to show that the Siegel-Brauer theorem about the class numbers of algebraic number fields (which generalizes Heilbronn's) can be made effective in many cases.

In 1973 Heilbronn had a heart attack and did not recover completely. He died during an operation to implant a pacemaker. During his life Heilbronn was elected to the Royal Society (1951) and to the Royal Society of Canada (1967). He was president of the London Mathematical Society from 1959 to 1961.

BIBLIOGRAPHY

I. Original Works. Heilbronn's papers are brought together in *The Collected Papers of Hans Arnold Heilbronn*, Ernst J. Kani and Robert A. Smith, eds. (New York, 1988). A complete bibliography of his works is in Cassels and Fröhlich (see below).

II. Secondary Literature. A full account of Heilbronn's life and work is J. W. S. Cassels and A. Fröhlich, "Hans Arnold Heilbronn," *in Biographical Memoirs of Fellows of the Royal Society*, **22** (1976), 119–135, reprinted in *Bulletin of the London Mathematical Society*, 9 (1977), 219–232. See also *Proceedings of the Royal Society of Canada*, 4th ser., 13 (1975), 53–56.

For the Gross-Zagier-Goldfeld effective estimate, see D. Zagier, "L-Series of Elliptic Curves, the Birch-Swinnerton-Dyer Conjecture and the Class Number Problem of Gauss, 'in *Notices of the American Mathematical Society*, 31 (1984), 739–743. For recent work on the existence of euclidean algorithms in number fields, see H. W. Lenstra, Jr., "Euclidean Number Fields of large Degree," in *Inventiones Mathematicae*, 38 (1977), 237–254.

J. W. S. Cassels