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(*b*. Königsberg, Germany [now Kaliningrad, U.S.S.R.], 22 April 1811; *d*. Munich, Germany, 4 August 1874)

mathematics.

Hesse was the eldest son of Johann Gottlieb Hesse, a merchant and brewer, and his wife, Anna Karoline Reiter. He grew up in Königsberg, where he had his first contact with the sciences at the Old City Gymnasium. After obtaining his school certificate in 1832, he attended the University of Königsberg, specializing in mathematics and the natural sciences. Hesse studied mainly under C.G.J. Jacobi, who greatly stimulated his mathematical investigations. After taking the examination for headmaster in 1837 and spending a probationary year at the Kneiphof Gymnasium in Königsberg, Hesse made an educational journey through Germany and Italy. In the fall of 1838 he began to teach physics and chemistry at the trade school in Königsberg. In 1840 he graduated from the University of Königsberg and was made a lecturer there on the basis of this thesis, *De octo punctis intersectionis trium superficium secundi ordinis*. After this he lectured regularly, and in 1841 he resigned his position at the trade school. In the same year he married Maria Dulk, daughter of a chemistry professor; they had six children.

In 1845 Hesse was appointed extraordinary professor at Königsberg; he spent a total of sixteen years there as teacher and researcher. During this time nearly all his mathematical discoveries were made, and he published them in Crelle’s *Journal für die reine und angewandte Mathematik*. Among those attending his lectures were Gustav Kirchhoff, Siegfried Heinrich Aronhold, Carl Neumann, Alfred Clebsch. and Sigismund Lipschitz.

Despite recognition of his scientific achievements, it was not until 1855 that Hesse received a call as ordinary professor to the University of Halle. Shortly thereafter he received an appointment to Heidelberg, which he gladly accepted, for Robert Bunsen and his former student Kirchhoff were there. From the winter of 1856 until 1868 Hesse taught in Heidelberg. During this period he wrote the widely read textbooks *Vorlesungen über analytische Geometrie des Raumes* and *Vorlesungen uber analytische Geometrie*. According to Felix Klein, Hesse’s methods of presenting material fortified and disseminated the feeling for elegant calculations expressed in symmetrical formulas. In 1868 Hesse accepted a call to the newly founded Polytechnicum at Munich. But only a few more years of activity were granted him, and he died in 1874 of a liver ailment. At his request, he was buried in Heidelberg, the city that had become his second home. The Bavarian Academy of Sciences, of which Hesse had become a member in 1868, arranged for the publication of his complete scientific works.

Hesse’s mathematical works are important for the development of the theory of algebraic functions and of the theory of invariants. His achievements can be evaluated, however, only in close connection with those of his contemporaries. Hesse was indebted to Jacobi’s investigations on the linear transformation of quadratic forms for the inspiration and starting point of his initial works on the theory of quadratic curves and planes. For proof (again influenced by Jacobi) he used the newly developed determinants, which allowed his presentation to reach an elegance not previously attained. Hesse again presented the results of these first researches when he developed his space geometry in his textbook.

In 1842 Hesse began his investigation on cubic and quadratic curves, which are closely linked to the development of basic concepts of algebra. The starting point was the paper “Über die Elimination der Variabeln aus drei algebraischen Gleichungen zweiten Grades mit zwei Variabeln.” Again the problem can be traced to Jacobi. A treatise on the inflection points of cubic curves immediately followed this work. Within the framework of this treatise is the functional determinant that is named after Hesse and arises from the second partial derivative of a homogeneous function $f(x_1,x_2,x_3)$

This functional determinant has found many applications in algebraic geometry. In linear transformation of the variables x_1, x_2, x_3 into the variables y_1, y_2, y_3, $H'=A^2 . H$, where A is the determinant of the matrix of the transformation and H is a covariant of f. Upon geometrically applying his first fundamental theory of homogeneous forms, Hesse obtained the result that the points of inflection of a curve C^n of the nth order are generally given as the intersection of this curve and a curve of the order 3 (n - 2). These curves can be described by means of the Hessian determinant of C_n. Julius Plucker had previously obtained this result for C_3. With this work Hesse demonstrated how, by geometrical interpretation, the results of algebraic transformations could not only equal, but even surpass, the results of geometers.

Hesse devoted much research effort to the geometrical interpretation of algebraic transformations, admitting that he was stimulated primarily by the geometrical works of Jakob Steiner and by Plücker and Poncelet. Plucker had further discovered that the planar C_3 contains nine points of inflection, which lie on twelve straight lines in groups of three. Hesse proved that these twelve straight lines are arranged in four triple lines, each of which contains all nine points. He further demonstrated that for a complete mathematical solution of the problen: an equation of the fourth degree is necessary; this was later confirmed by Aronhold.

A similar investigation of groupings was necessitated by the twenty-eight double tangents of the planar C_4. Here too Hesse's starting point was the so-called canonical representation of C_4 in the form of a symmetrical determinant of a quadruple series. By this representation of the equation of the curve, the planar problem of the double tangent can be combined with a spatial problem: eight points in space are connected by twenty-eight straight lines. If a group of planes of the second order, infinite in both directions, is drawn through these eight fixed base points, then the parameters of the conical surfaces of this group are sufficient for a condition that can be understood as the given equation of this group. This connection led to the proof that the special case of the equation of C_4 can be represented in thirty-five other ways, all markedly different from the first.

From the beginning, Hesse always sought to arrange his calculations with homogeneous symmetrical starting points, so that the algebraic course of the calculation would be the counterpart of the geometric considerations. His student Alfred Clebsch in particular has used this concept in his own work and has further expanded on it.

In England, Cayley was also working on the theory of homogeneous forms. Rivalry arose when his "Mémoire sur les hyperdéterminants" appeared simultaneously with Hesse's paper.

Hesse's teaching was also influential. In his long years as a lecturer, he continually showed his enthusiasm for mathematics, and his textbooks on analytical geometry must be seen in this context. The special forms of linear equation and of planar equation that Hesse used in these books are called Hesse's normal form of the linear equation and of the planar equation in all modern textbooks in this discipline.

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Karlheinz Hass