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(*b*. Amsterdam, Netherlands, 9 May 1898; *d*. Lugano, Switzerland, 9 July 1980)

logic, mathematics.

Heyting was the eldest child of Johannes Heyting and Clarissa Kok. Both parents were schoolteachers; his father, a man of considerable intellectual gifts, later became principal of a secondary school.

Originally Heyting was to become an engineer, but later it was decided that he should go to the university. In 1916 he enrolled as a student of mathematics at the University of Amsterdam. The funds to pay for his studies were earned by Heyting and his father by supervising the homework of high school students. Two of his teachers at the university, L. E. J. Brouwer and, to a lesser extent. Gerrit Mannoury, shaped and determined his future scientific interests: the greater part of Heyting's work is devoted to intuitionism. Brouwer's philosophy of mathematics, although at certain points his views are closer to the ideas of Mannoury.

After receiving the equivalent of the M.Sc. in 1922, Heyting became a teacher at two secondary schools in Enschede, an industrial town in the eastern Netherlands, far removed from any of the Dutch universities. In his leisure hours he worked on his dissertation, which dealt with the axiomatics of intuitionistic projective geometry. In 1925 he received his doctorate under Brouwer, cum laude.

Heyting's reputation grew rapidly, and in 1937 he was appointed lector at the University of Amsterdam, having been admitted the year before as *Privatdocent*. In 1948 he became a full professor, and in 1968 professor emeritus. In 1942 he was elected a member of the Royal Dutch Academy of Sciences.

Heyting was retiring and modest, lacking all ostentation. his interests were very wide-ranging and varied: music, literature, linguistics, philosophy, astronomy, and botany; he also was fond of walking, cycling, and gardening. As a teacher and lecturer he impressed his students and his international audiences at congresses with his exceptionally clear presentations. In 1929 Heyting married Johanne Friederieke Nijenhuis; they had eleven children. The couple were divorced in 1960, and in 1961 he married Joséphine Frédérique van Anrooy.

In 1927 the Dutch Mathematical Association published a prize question that asked for a formalization of Brouwer's intuitionistic theories. Heyting's answer was awarded the prize early in 1928; a revised and expanded version of his essay was published in 1930. This work made Heyting's name well known among logicians and philosophers of mathematics. It also marked the beginning of a lifelong friendship with Heinrich Scholz at Münster, not far from Enschede. Who put his extensive library at Heyting's disposal. Scholz held the only chair of mathematical logic in Germany.

In Brouwer's intuitionism, mathematics consists in the mental construction of mathematical systems, an activity that is supposed to be carried out in the mind of an idealized mathematician, in principle without the use of language; language enters only in attempts to suggest similar constructions to other persons. Something is true in intuitionistic mathematics only if it can be shown to hold by means of a construction.

Brouwer's presentation of his views was deliberately antiformal, in a highly personal style, and often difficult to understand. Heyting's formalization, partially anticipated by V. Glivenko and Andrei N. Kolmogorov, made comparison with formalized traditional mathematics possible. ThoughHeyting's work has led some into the mistake of identifying intuitionism with his formalization, in the long run the study of intuitionistic formalisms has greatly helped the understanding of the basic intuitionistic concepts.

Around 1930 Heyting also formulated his explanation of the meaning of the intuitionistic logical operations, based on constructive proof or construction as a primitive notion. Though the germs of such an explanation can already be found in Brouwer's writings, this was an important step forward. (Kolmogorov independently gave, in 1932, a closely related interpretation of intuitionistic logic as a calculus of problems.)

Heyting also continued his work, begun with his dissertation, on Brouwer's program of the reconstruction of actual pieces of mathematics along intuitionist lines; in 1941 he published a pioneering paper on intuitionistic algebra, and in the 1950's he investigated the intuitionistic theory of Hilbert space. Some of his students who wrote dissertations under his direction also contributed to the program: J. G. Dijkman (theory of convergence, 1952), B. Van Rootselaar (measure theory, 1954), D. Van

Dalen (affine geometry, 1963), Ashvinikumar (Hilbert space, 1966), A. S. Troelstra (general topology, 1966), and C. G. Gibson (Radon integral, 1967). In the period of his professorship Heyting also published textbooks in projective geometry, one of which was *Axiomatic Projective Geometry* (1963).

Heyting always saw the creation of a better understanding and appreciation of Brouwer's ideas as one of his principal tasks, and thus many of his talks at international meetings and his published writings are devoted to expositions and defense of the basic ideas of intuitionism, in a style that was never dogmatic or polemical.

There are differences in outlook between Brouwer and Heyting, however; in particular, Heyting frankly recognized the formal-theoretical element introduced into intuitionistic mathematics by the (in practice) inescapable use of language, an aspect suppressed in most of Brouwer's writings although Brouwer was aware of it. He also did not share Brouwer's pessimistic views on language as a means of communication, and accordingly he valued positively the use of formalization and axiomatization for intuitionism.

In 1934 Heyting wrote a short monograph titled *Intuitionism and Proof Theory*, a concise and wellwritten survey in which the viewpoints of intuitionism and formalism are clearly described and contrasted. In 1956 Heyting published his very successful *Intuitionism:* An *Introduction*, from which many logicians and mathematicians learned about intuitionism. It is certainly in large measure due to Heyting that intuitionism is still very much alive today; without his efforts the'intuitionistic revolution' might well have dwindled away and Brouwer's ideas would have become part of the past.

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