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(*b*, Lodx, Russian Poland, 29 June 1904; *d*. Uxmal, Mexico, 6 September 1956)

topology.

Although well acquainted with the topology of the Polish school, Hurewicz, the son of an industrialist, began the study of topology under Hans Hahn and Karl Menger in Vienna, where he received the Ph.D. in 1926. After being a Rockefeller fellow at Amsterdam in 1927–1928, he was *Privatdozent* and assistant to L.E.J. Brouwer at the University of Amsterdam from 1928 to 1936. In the latter year Hurewicz took a year's leave of absence to visit the Institute for Advanced Study at Princeton. He decided to stay in the United States, first at the University of North Carolina and then, from 1945, at the Massachusetts Institute of Technology. He died after the International Symposium on Algebraic Topology at the National University of Mexico. While on an excursion he fell from a pyramid he had climbed.

Hurewicz was a marvelously clear thinker, a quality reflected by his style of oral and written communication. This clarity characterizes his early work in set-theory topology. By grasping the essentials and putting them into a larger context, he simplified approaches and generalized theorems and theories. For instance, the switch in dimension theory from subsets of Cartesian to general separable metric spaces is due to Hurewicz. The so-called Sperner proof of the invariance of dimension was independently and simultaneously found and published by Hurewicz.[1] A remarkable result of this first period is his topological embedding of separable metric spaces into compact spaces of the same (finite) dimension.[2]

The next period of Hurewicz's scientific life started with the recognition of spaces of mappings (of one space into another) as a powerful means of topological research; the extensive use of the principle that in complete metric spaces the intersection of overall dense open subsets is itself overall dense[3] is quite characteristic of Hurewicz's thought. In this period he also developed beautiful theorems on dimension-raising mappings,[4] as well as theorems and lucid proofs on embedding finite-dimensional into Cartesian spaces.[5]

For a long time combinatorial methods were belittled as a useful but ugly tool in topology-a necessary evil, as it were. In the early 1930's the desire to change the homological into a homotopical approach was reinforced by Heinz Hopf's homological classification of the mappings of n-dimensional ployhedra into the n-dimensional sphere.[6] Hurewicz was particularly impressed by Karol Borsuk's homotopic characterization of closed sets dividing n-space by essential mappings into the (n–1) sphere.[7] In this respect the last paragraph of one of his papers deserves to be quoted (in translation):

… the part played by mappings on the *n*-sphere in the topological research of the last few years (in particular, in investigations by Hopf and Borsuk). One may expect that a closer study of these mappings (especially by group theory means) will lead to clarifying the relation between homology and homotopy, which would create the possibility to apply set theory methods in those domains which are at present exclusively dominated by combinatorial methods. Among others one should consider that an essential mapping of an *n*-dimensional closed set of a space R on the *n*-sphere is in a sense the set theory analogue of the combinatorial concept of *n*-cycle, whereby mappings that can be continued to the whole R correspond to "bordering" cycles.[8]

As valuable as it may be, Hurewicz's work, as reported so far, is entirely overshadowed by the discoveries made during a short period in the year 1934–1935, which in due course assured him of a place among the greatest topologists: the discovery of the higher homotopy groups and their foremost properties. In hindsight, it all looks so simple: replacing, in Henri Poincaré's definition of the fundamental group, the circles by spheres of any dimension. In fact, the idea was not new, but until Hurewicz nobody had pursued it as it should have been. Investigators did not expect much new information from groups, which were obviously commutative, and in this respect no better than the commutative homology groups. The paragraph quoted may explain why this did not bother Hurewicz, yet the experience acquired in dealing with spaces of mappings gave him a head start.

The wealth of results displayed in the four papers on the topology of deformations[9] is overwhelming. There is little need to go into detail, since most of them are now among the rudiments of homotopy theory, the creation of Hurewicz. Even homological algebra is rooted in this work: In "aspherical" spaces the homology groups are uniquely determined by the fundamental group. Other theorems include the following: If the first n–1 homotopy groups are trivial, the nth homology and homotopy groups are isomorphic. A polyhedron with only trivial homotopy groups is in itself contractible to a point.

Hurewicz's second great discovery (1941) is exact sequences, an almost imperceptible abstract[10] that generated an enormous literature. His work, with others, on fibre spaces has been of lasting importance.[11]

Surprisingly, Hurewicz's bibiography shows a relatively small number of items. His personal influence, however, cannot be overestimated. His knowledge of mathematics went far beyond topology: he lectured in a rich variety of fields. Hurewicz, who never married, was a highly cultured and charming man, and a paragon of absentmindedness, a failing that probably led to his death.

NOTES

1."Über ein topologisches Theorem." in *Mathematische Annalen*. **101** (1929), 210–218.

2. "Theorie der analytischen Mengen," in *Fundamenta mathematicae*, **15** (1930), 4–17.

3. "Dimensionstheorie und Cartesische Räume," in *Koninklijke Akademie van wetenschappen te Amsterdam, Proceedings*. **34** (1931), 399–400.

4. "Über dimensionerhöhende stetige Abbildungen," in *Journal für die reine und angewandte Mathematik*, **169** (1933), 71–78.

5. "Über Abbildungen von endlich dimensionalen Räumen auf teilmengen Cartesischer Räume" in *Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1933), 754–768.

6. Heinz Hopf, "Die Klassen der Abbildungen der n-dimen sionalen Polyeder auf die n-dimensionale Sphare," in *Commentarii mathematici helvetici*. **5** (1933), 39–54.

7. Karol Borsuk, "Über Schnitte der n-dimensionalen Euklidischen Raume," *Mathematische Annalen*. **106** (1935). 239–248.

8. "Uber Abbildungen topologischer Raume auf die n-dimensionale Sphare," in *Fundamenta mathematicae*. **24** (1935). 144–150.

9. "Hoher-dimensionable Homotopiegruppen," *in KoninklijkeAkademie van wetenschappen, te Amsterdam, Proceedings*. **38** (1935) 112–119; "Homotopie und Homologiegruppen," ibid., 521–528; "Klassen und Homologietypen von Abbildungen," *ibid*., **39** (1936). 117– and "Aspharische Raume," *ibid*., 215–224.

10. "On Duality Theorems," *Bulletin of the American Math-ematical Society*, abstract 45–7-329.

11. "Homotopy Relations in Fibre Spaces," *in proceedings of the National Academy of Sciences*. **27** (1941). 60–64, with N.E. Steenrod; "On the Concept of Fiber Space," *ibid*., **41** (1955), 956–961; and "On the Spectral Sequence of a Fiber Space," *ibid*., 961–964, with E. Fadell.

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Solomon Lefschetz, "WitoldHurewicz:ln Memoriam," in *Bulletin of the American Mathematical Society*, **63** (1957), 77–82, includes a complete bibliograhy of Hurewicz's works.

Hans Freudenthal