Christiaan Huygens | Encyclopedia.com

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(*b*. The Hague, Netherlands, 14 April 1629; *d*. The Hague, 8 July 1695)

physics, mathematics, astronomy, optics.

Huygens belonged to a prominent Dutch family. His grandfather, also Christiaan Huygens, served William the Silent and Prince Maurice as secretary. In 1625 his father, Constantijn, became a secretary to Prince Federic Henry and served the Orange family for the rest of his life, as did Christiaan's brother Constantijn.

Along with this tradition of diplomatic service to the house of Orange, the Huygens family had a strong educational and cultural tradition. The grandfather took an active part in the education of his children, and thus Huygens' father acquired great erudition in both literature and the sciences. He corresponded with Mersenne and Descartes, the latter often enjoying his hospitality in The Hague. Constantijn was a man of taste in the fine arts, talented in drawing, a musician and fertile composer, and, above all, a great poet; his Dutch and Latin verse gained him a lasting place in the history of Dutch literature.

Like his father, Constantijn was actively committed to the education of his children. Christiaan and his brother Constatijn were educated at home up to the age of sixteen by both their father and private teachers. They acquired a background in music (Christiaan sang well and played the viola da gamba, the lute, and the harpsichord), Latin, Greek, French, and some Italian, and logic, mathematics, mechanics, and geography. A highly talented pupil, Christiaan showed at an early age the combination of theoretical interest and insight into practical applications and constructions (at thirteen he built himself a lathe) which characterized his later scientific work.

From May 1645 until March 1647 Christiaan studies law and mathematics at the University of Leiden, the latter with Frans van Schooten. He stuided classical mathematics as well as the modern methods of Viéte, Descartes, and Fermat. During this period his father called Mersenne's attention to his son's study on falling bodies, and this opened up a direct correspondence between Christian and Mersenne. Descartes, whose work in these years had a great influence on young Huygens, also showed an interest in and an appreciation of Christiaan's work. From March 1647 until August 1649 Christiaan studied law at the newly founded Collegium Arausiacum (College of Orange) at Breda, of which his father was a curator and where Pell taught mathematics.

Huygens did not, after his studies, choose the career in diplomacy which would have been natural for a man of his birth and education. He did not want such a career, and in any event the Huygens family lost its main opportunities for diplomatic work as a result of the death of William II in 1650. Huygens lived at home until 1666, except for three journeys to Paris and London. An allowance supplied by his father enabled him to devote himself completely to the study of nature. These years (1650-1666) were the most fertile of Huygens' career.

Huygens at first concentrated on mathematics: determinations of quadratures and cubatures, and algebraic problems inspired by Pappus' works. In 1651 the *Theoremata de quadratura hyperboles, ellipsis et circuli* [1] appeared, including a refutation of Gregory of St. Vincent's quadrature of the circle. The *De circuli magnitudine inventa* [2] followed in 1654. In the subsequent years Huygens studied the rectification of the parabola, the area of surfaces of revolution of parabolas, and tangents and quadratures of various curves such as the cissoid, the cycloid (in connection with a problem publicly posed by Pascal in 1658), and the logarithmica. In 1657 Huygens' treatise on probability problems appeared, the *Tractatus de ratiociniis in aleae ludo* [4].

A manuscript on hydrostatics [20] had already been completed in 1650, and in 1652 Huygens formulated the rules of elastic collision and began his studies of geometrical optics. In 1655 he applied himself, together with his brother, to lens grinding. They built microscopes and telescopes, and Huygens, in the winter of 1655-1656, discovered the satellite of Saturn and recognized its ring, as reported in his *De Saturni lunâ observatio nova* [3] and *Systema Saturnium* [6], respectively.

In 1656 Huygens invented the pendulum clock. This is described in 1658 in the *Horologium* [5] (not to be confused with the later *Horologium oscillatorium*) and formed the occasion for the discovery of the tautochronism of the cycloid (1659), and for the studies on the theory of evolutes and on the center of oscillation. Huygens' study of centrifugal force also dates from 1659. In these years he corresponded with increasing intensity with many scholars, among them Gregory of St. Vincent, Wallis, van Schooten, and Sluse. Studies on the application of the pendulum clock for the determination of longitudes at sea occupied much of his time from 1660 onward.

Of the journeys mentioned above, the first, from July until September 1655, brought Huygens to Paris, where he met Gassendi, Roberval, Sorbiére, and Boulliau—the circle of scholars which later formed the Académie Royale des Sciences. He used the opportunity of the stay in France to buy, as did his brother, a doctorate "utriusque juris" in Angers. During his second stay in Paris, from October 1660 until March 1661, he met Pascal, Auzout, and Desargues. Afterward he was in London (until May 1661). There Huygens attended meetings in Gresham College, and met Moray, Wallis, and Oldenburg, and was impressed by Boyle's experiments with the air pump. A third stay in Paris, from April 1663 to May 1664, was interrupted by a journey to London (June to September 1663), where he became a member of the newly founded Royal Society. He then returned to Paris where he received from Louis XIV his first stipend for scientific work.

In 1664 Thévenot approached Huygens to offer him membership in an academy to be founded in Paris; Colbert proposed giving official status and financial aid to those informal meetings of scholars which had been held in Paris since Mersenne's time. In 1666 the Académie Royale des Sciences was founded. Huygens accepted membership and traveled to Paris in May of that year. Thus began a stay in Paris that lasted until 1681, interrupted only by two periods of residence in The Hague because of ill health. Huygens' health was delicate, and in early 1670 he was afflicted by a serious illness. In September, partially recovered, he left for The Hague and returned to Paris in June 1671. The illness recurred in the autumn of 1675, and from July 1676 until June 1678 Huygens again was in The Hague.

As the most prominent member of the Academy, Huygens received an ample stipend and lived in an apartment in the Bibliothéque Royale. In the Academy, Huygens encouraged a Baconian program for the study of anture. He participated actively in astronomical observations (of Saturn, for example) and in experiments with the air pump. He expounded his theory of the cause of gravity in 1669, and in 1678 he wrote the *Traité de la lumière* [12], which announced the wave, or more accurately, the pulse theory of light developed in 1676–1677. In the years 1668–1669 he investigated, theoretically and experimentally, the motion of bodies in resisting media. In 1673 he cooperated with Papin in building *a moteur a explosion*, and from that year onward he was also in regular contact with Leibniz. Huygens began his studies of harmonic oscillation in 1673 and designed clocks regulated by a spring instead of a pendulum, about which a controversy with Hooke ensued. In 1677 he did microscopical research.

In 1672 war broke out between the Dutch republic and Louis XIV and his allies. William III of Orange came to power and Huygens' father and brother assumed prominent positions in Holland. Huygens stayed in Paris, and, although he was deeply concerned with the Dutch cause, proceeded with his work in the Academy under the protection of Colbert. In 1673 he published the *Horologium oscillatorium* [10]. It was his first work to appear after he entered a position financed by Louis XIV, and he dedicated it to the French king. This gesture served to strengthen his position in Paris but occasioned some disapproval in Holland.

Huygens left Paris in 1681, again because of illness. He had recovered by 1683, but Colbert had died meanwhile, and without his support Huygens' nationality, his Protestantism, and his family's ties with the house of Orange would have engendered such strong opposition in Paris that he decided to stay in Holland. His financial position was thus not as secure but he did have an income from his family's landed property. Huygens never married. In the relative solitude of his residence in The Hague and at Hofwijck, the family's country house near Voorburg, he continued his optical studies, constructed a number of clocks, which were tested on several long sea voyages, and wrote his *Cosmotheoros* [14]. From June until September 1689 he visited England, where he met Newton. The *Principia* aroused Huygens' admiration but also evoked his strong disagreement. There is evidence of both in the *Traité de la lumière* [12] and its supplement, the *Discours de la cause de la pesanteur* [13]. Discussions with Fatio de Duillier, correspondence with Leibniz, and the interest created by the latter's differential and integral calculus drew Huygens' attention back to mathematics in these last years.

In 1694 Huygens again fell ill. This time he did not revcover. He died the following summer in The Hague.

Mathematics . The importance of Huygens' mathematical work lies in his improvement of existing methods and his application of them to a great range of problems in natural sciences. He developed no completely new mathematical theories save his theory of evolutes and —if probability may be considered a mathematical concept—his theory of probability.

Huygens' mathematics may be called conservative in view of the revolutionary innovations embodied in the work of such seventeenth-century mathematicians as Viète, Descartes, Newton, and Leibniz. A marked tension is often apparent between this conservatism and the new trends in the mathematics of Huygens' contemporaries. Whereas, for example, Huygens fully accepted Viète's and Descartes's application of literal algebra to geometry, he rejected Cavalieri's methods of indivisibles. In his earlier works he applied rigorous Archimedean methods of proof to problems about quadratures and cubatures. That is, he proved equality of areas or contents by showing, through consideration of a sequence of approximating figures, that the supposition of inequality leads to a contradiction. On the other hand, he accepted Fermat's infinitestimal methods for extreme values and tangents, freely practicing division by "infinitely small"—his terminology—differences of abscissae, which subsequently are supposed equal to zero. Eventually the tediousness of the Archimedean methods of proof forced him to work directly with partition of figures into "infinitely small" or very small component figures; he considered this method to be inconclusive but sufficient to indicate the direction of a full proof. He long remained skeptical about Leibniz' new methods, largely because of Leibniz' secrecy about them.

In his first publication, *Theoremata de quadratura hyperboles*, Huygens derived a relation between the quadrature and the center of gravity of segments of circles, ellipses, and hyperbolas. He applied this result to the quadratures of the hyperbola and the circle. In the *De circuli magnitudine inventa* he approximated the center of gravity of a segment of a circle by the center of the gravity of a segment of a parabola, and thus found an approximation of the quadrature; with this he was able to refine the inequalities between the area of the circle and those of the inscribed. and circumscribed polygons used in the calculations of π. The same approximation with segments of the parabola, in the case of the hyperbola, yields a quick and simple method to calculate logarithms, a finding he explained before the Academy in 1666–1667.

In an appendix to the *Theoremata*, Huygens refuted the celebrated proof by Gregory of St. Vincent (*Opus geometricum* [1647]) of the possibility of the quadrature of the circle. Huygens found the crucial mistake in this very extensive and often obscure work. Gregory had applied Cavalierian indivisible methods to the summation of proportions instead of to line segments. The language of proportions was still sufficiently close to that of arithmetic for Gregory's error not to be a simple blunder, but Huygens was able to show by a numerical example that the application was faulty.

Having heard in Paris about Pascal's work in probability problems, Huygens himself took up their study in 1656. This resulted in the *Tractatus de ratiociniis in aleae ludo*, a treatise that remained the only book on the subject until the eighteenth century. In his first theorems Huygens deduced that the "value of a chance," in the case where the probabilities for *a* and *b* are to each other as p:q, is equal to

He thus introduced as a fundamental concept the expectation of a stochastic variable rather than the probability of a process (to put it in modern terms). Subsequent theorems concern the fair distribution of the stakes when a game is broken off prematurely. The treatise closes with five problems, the last of which concerns expected duration of play.

In 1657 Huygens found the relation between the arc length of the parabola and the quadrature of the hyperbola. His method cannot be extended to a general rectification method, for it depends on a special property of the parabola: if a polygon is tangent to the parabola, and if the tangent points have equidistant abscissae, the polygon can be moved in the direction of the axis of the parabola to form an inscribed polygon. Hygens also employed this property to find the surface area of a paraboloid of revolution. From correspondence he learned about the general rectification method of Heuraet (1657). He found, in 1658, the relation which in modern notation is rendered by $yds = ndx$ (s: arc length; n: normal to the curve (y, x)), with which he could reduce the calculation of surface areas of solids of revolution to the quadrature of the curve $z = n(x)$; he used this relation also in a general rectification method. Some of

these results were published in part 3 of the *Horologium oscillatorium* [10].

In 1659 Huygens developed, in connection with the pendulum clock, the theory of evolutes (Fig. 1). The curve β described by the end of a cord which is wound off a convex curve α is called the evolvent of α, and conversely α is called the evolute of β. In part 3 of the *Horlogium oscillatorium* Huygens showed, by rigorous Archimedean methods, that the tangents to the evolute are perpendicular to the evolvent, and that two curves which exhibit such a relation of tangents and perpendiculars are the evolute and evolvent of one another. Furhter, he gives a general method (proved much less rigourously) of determining from the algebraic equation of a curve the construction of its evolute; the method is equvalent to the determination of the radius of curvature (although Huygens only later interested himself in this as a measure of curvature) and implies, accordingly, a twice repeated determination of tangents by means of Sluse's tangent rule.

Huygens' study on the logarithmica dates from 1661; the results were published in the *Discours*. Huygens introduced this curve (modern $y = ae^x$) as the one in which every arithmetical series of abscissae corresponds to a geometrical series of ordinates. He noted its connection both with the quadrature of the hyperbola and with logarithms and pointed out that its subtangent is constant.

In the last decade of his life Huygens became convinced of the merits of the new Leibnizian differential and integral calculus through the study of articles by the Bernoullis, L'Hospital, and Leibniz, and through correspondence with the latter two. In 1691 he learned how to apply calculus in certain simple cases. Nevertheless, Huygens continued to use the old infinitesimal geometrical methods—which he applied with such virtuosity that he was able to solve most of the problems publicly posed in this period, including Leibniz' isochrome problem (1687), Johann Bernoulli's problem (1693–1694), the tractrix problem (1693),

and the catenary problem (1691–1693). His final solution (1693) of this last problem may serve as an example of the force and style of Hygens' mathematics.

In dealing with the catenary problem, Huygens conceived the chain as as series of equal weights, connected by weightless cords of equal length. It follows from statics that every four subsequent weights *A*, *B*,*C D* (Fig. 2) in the chain are disposed such that the extensions of *AB* and *CD* meet at *H* on the vertical that bisects *BC*. (Huygens had already found this result in 1646 and used it to refuse Galileo's assertion that the catenary is a parabola.) By simple geometry it may now be seen that the tangents of the angles of subsequent cords to the horizontal are in arithmetical progression. Huygens further conceived (Fig.2) the chain $C_1C_2C_3C_4$... (the lower link being horizontal) stretched along the horizontal axis, top become $C_1D_2D_3D_4$... Point P on the vertical through C_1 is chosen such that $\angle C_1PD_2 = \angle C_2C_1B_1$. (As Huygens knew, it can be proved that in the limit C_1 p is equal to the radius of curvature in the vertex of chain.) As the tangents of are obviously in arithmetical progression, $\angle D_1 C_1P$

must be equal to ∠ C_{i+1} C_iB_i. Introducting normals D_i D_i+$_1$ E_i are congruent with C_i C_i+1$B_{i,}$ so that the chain is strtched, as it were, together with its series of characteristics triangles.

Considering the absciassa C_1 B and the ordinate *BC* of a point *C* on the catenary, it is clear that

and that

Huygens now imagines interstices to "infinitely small," so that C_1 coincides with the vertex O of the catenary, and he takes . It is then clear that $\Sigma E_iD_{i+1} = QD$, If $PQ = PO$, so that the ordinate Bc is equal QD. To evaluate the absciss OB, Huygens extends the normals D_iE_i and remarks that they are the tangents of a curve , which has the property that the normals PD_i on its tangents D_i $E_i$meet in one point. This determines the curve as a parabola; by the theory of evolutes $\Sigma D_i\ E_i$ is equal to the arc length of the parabola minus the tangent SD, so that the abscissa OB is equal to — SD. This result, in combination with the previously found equality BC = QD, makes possible the geometrical construction of corresponding ordinates and abscissae of the curve. The construction presupposes the rectification of the parabola, which, as Huygens knew, depends on the quadrature of the hyperbola. Thus his solution of the catenary problem is the geometrical equivalent of the analytical solution of the problem, namely, the equation of the curve involving exponentials.

Statics and Hydrostatics. In the treatment of problems in both statics (the catenary problem, for example) and hydrostatics, Huygens proceeded from the axiom that a mechanical system is in equilibrium if its center of gravity is in the lowest possible position with respect to its restraints. In 1650 he brought together the results of his hydrostatic studies in a manuscript, *De iis quae liquido supernatant* [20]. In this work he derived the law of Archimedes from the basic axiom and proved that a floating body is in a position of equilibrium when the distance between the center of gravity of the whole body and the center of gravity of its submerged part is at a minimum. The stable position of a floating segment of a sphere is thereby determined, as are the conditions which the dimensions of right truncated paraboloids and cones must satisfy in order that these bodies may float in a vertical position. Huygens then deduced how the floating position of a long beam depends on its specific gravity and on the proportion of its width to its depth, and he also determined the floating position of cylinders. The manuscript is of further mathematical interest for its many determinations of centers of gravity and cubatures, as, for example, those of obliquely truncated paraboloids of revolution and of cones and cylinders.

Impact. Huygens started his studies on collision of elastic bodies in 1652, and in 1656 he collected his results in a treatise *De motu corporum ex percussione* [18]. He presented the most important theorems to the Royal Society in 1668, simultaneously with studies by Wren and Wallis; they were published, without proofs, in the *Journal des sçavans* in 1699 [9]. Since Huygens' treatise is a fundamental work in the theory of impact and exhibits his style at its best, it is worth describing in some detail.

Huygens' theory amounted to a refutation of Descartes's laws of impact. Indeed, Huygens' disbelief in these laws was one of the motivations for his study. Descartes supposed an absolute measurability of velocity (that is, a reference frame absolutely at rest). This assumption is manifest in his rule for collision of equal bodies. If these have equal velocities, they rebound; if their velocities are unequal, they will move on together after collision. Huygens challenged this law and in one of his first manuscript notes on the question, remarked that the forces acting between colliding bodies depend only on their relative velocity. Although he later abandoned this dynamical approach to the question, the relativity principle remained fundamental. It appeared as hypothesis III of *De motu corporum*, which asserts that all motion is measured against a framework that is only assumed to be at rest, so that the results of speculations about motion should not depend on whether this frame is at rest in any absolute sense. Huygens' use of this principle in his impact theory may be described algebraically (although Huygens himself, of course, gave a geometrical treatment) as follows: If bodies *A* and *B* with velocities v_A and v_B acquire, after collision, velocities u_A and u_B, then the same bodies with velocities v_A and v_B acquire, after collision, velocities u_A and u_B, then the same bodies with velocities $v_A + v$ and $v_B + v$. Huygens discussed the principle at great length and as an illustration used collision processes viewed by two observers—one on a canal boat moving at a steady rate and the other on the bank.

In the treatise, Huygens first derived a special case of collision (prop. VIII) and extended it by means of the relativity principle to a general law of impact, from which he then derived certain laws of conservation. This procedure is quite contrary to the method of derivation of the laws of impact from the axiomatic conservation laws, which has become usual in more recent times; but it is perhaps more acceptable intuitively. In the special case of prop. VIII the magnitudes of the bodies are inversely proportional to their (oppositely directed) velocities ($m_A{:}m_B$ = ⍰ v_B ⍰ : ⍰ v_A ⍰), and Huygens asserts that in this case the bodies will simply rebound after collision ($u_A = -v_A$, $u_B = -v_B$). To prove this, Huygens assumed two hypotheses. The first, hypothesis IV, states that a body *A* colliding with a smaller body *B* at rest transmits to *B* some of its motion—that is, that *B* will acquire some velocity and *A*'s velocity will be reduced. The second, hypothesis V, states that if in collision the motion of one of the bodies is not changed (that is, if the absolute value of its velocity remains the same), then the motion of the other body will also remain the same.

The role of the concept of motion (*motus*) as used here requires some comment. Descartes had based his laws of impact partly on the theorem that motion is conserved, whereby he had quantified the concept of motion as proportional to the magnitude of the body and to the absolute value of its velocity (m\υ\). Huygens found that in this sense the *quantitas motus* is not conserved in collision. He also found that if the velocities are added algebraically, there is a law of convervation (namely, of momentum) which he formulated as conservation of the velocity of the center of gravity. But for Huygens the vectorial quantity was apparently so remote from the intuitive concept of motion that he did not want to assume its conservation as a hypothesis. Nor could he take over Descartes's quantification of the concept, and thus he used a nonvectorial concept of motion, without quantifying it, restricting himself to one case in which it remains unchanged.

Huygens now deduced from hypotheses III, IV, and V that the relative velocities before and after collision are equal and oppositely directed: $\upsilon_A - \upsilon_B = u_B - u_A$ (prop. IV). To derive proposition VIII, he drew upon three more assertions: namely, Galileo's results concerning the relation between velocity and height in free fall; the axiom that the center of gravity of a mechanical system cannot rise under the influence of gravity alone; and the theorem that elastic collision is a reversible process, which he derived from proposition IV. Huygens considered the velocities υ_A and υ_B in proposition VIII as acquired through free fall from heights h_A and h_B and supposed that the bodies after collision are directed upward and rise to heights h'_A and h'_B. Because the collision is reversible, the centers of gravity of the systems (A, B, h_A, h_B) and (A, B, h'_A, h'_B) must be at the same height, from which it can be calculated that $u_A = -\upsilon_A$ and $u_B = -\upsilon_B$ Proposition VIII is now proved, and by means of the relativity principle the result of any elastic collision can be derived, as Huygens showed in proposition IX. Finally, he deduced from this general law of impact the proposition that before and after collision the sum of the products of the magnitudes and the squares of the velocities of the bodies are equal (conservation of $\Sigma\, m\upsilon^2$).

Optical Techniques. Working with his brother, Huygens acquired great technical skill in the grinding and polishing of spherical lenses. The lenses that they made from 1655 onward were of superior quality, and their telescopes were the best of their time. In 1685 Huygens summarized his technical knowledge of lens fabrication in *Memorien aengaende het slijpen van glasen tot verrekijckers* [17]. In *Astroscopia compendiaria* [11], he discussed the mounting of telescopes in which, to reduce aberration, the objective and ocular were mounted so far apart (up to twenty-five meters) that they could not be connected by a tube but had to be manipulated separately.

Geometrical Optics. As early as 1653 Huygens recorded his studies in geometrical optics in a detailed manuscript, *Tractatus de refractione et telescopiis* [16]. He treated here the law of refraction, the determination of the focuses of lenses and spheres and of refraction indices, the structure of the eye, the shape of lenses for spectacles, the theory of magnification, and the construction of telescopes. He applied his theorem that in an optical system of lenses with collinear centers the magnification is not changed if the object and eye are interchanged to his theory of telescope. He later used the theorem in his calculations for the so-called Huygens ocular, which has two lenses. He began studing spherical aberration in 1665, determining for a lens with prescribed aperture and focal length the shape which exhibits minimal spherical aberration of parralel entering rays. He futher investigated the possibility of compensating for spherical aberration of the objective in a telescope by the aberration of the ocular, and he studied the relation between magnification, brightness, and resolution of the image for telescopes of prescribed length. These results were checked experimentally in 1668, but the experiments were inconclusive, because in the overall aberration effects the chromatic aberration is more influential than the spherical.

About 1685 Huygens began to study chromatic aberration. He did not start from his own experiments, as he usually did, but rather began with the results of Newton's work; he had first heard of Newton's theory of colors in 1672. Huygens confirmed the greater influence of chromatic as compared with spherical aberration, and he thereby determined the most advantageous shapes for lenses in telescope of prescribed length.

About 1677 Huygens studied microscopes, including aspects of their magnification, brightness, depth of focus, and lighting of the object. Under the influence of Leeuwenhoek's discoveries, with his microscope he observed infusoria, bacteria, and spermatozoa. In consequence he became very skeptical about the theory of spontaneous generation.

Astronomy. With the first telescope he and his brother had built, Huygens discovered, in March 1655, a satellite of Saturn, later named Titan. He determined its period of revolution to be about six teen days, and noted that the satellite moved in the same plane as the "arms" of Saturn. Those extraordinary appendages of the planet had presented as-tronomers since Galileo with serious problems of interpretation; Huygents soled these problem with the hypothesis that Saturn is surrounded by a ring. He arrived at this solution partly though the use of better observational equipment, but also by an acute argument based on the use of the Cartesian vortex (the whirl of "celestial matter" around a heavely body supporting its satellites).

Huygens' argument began with the premise that it is a general feature of the solar system that the period of rotation of a heavenly body is much shorter than the periods of revolution of its satellites, and that the periods of inner satellites are smaller than those of outer satellites. This is the case with the sun and the planets, with the earth and the moon, and with Jupiter and its satellites. In the same way the "celestial matter" between Saturn and its satellite must move so that the parts near the planet—including the "arms"—will have a period of revolution about equal to the period of rotation of the planet and much shorter than the sixteen days assigned to the satellite. In the period of Huygens' observations in 1655–1656, no alteration was observed in the aspects of the "arms," a phenomenon which could be explained only if the matter forming the "arms" was distributed with cylindrical symmetry around Saturn, with its axis of symmetry—the axis of the vortex— perpendicular to the plane of the satellite and of the "arms" themselves. Therefore, the "arms" must be considered as the aspect of a ring around Saturn. In his further calculations, Huygens established that this hypothesis could also be used to explain the observed long-term variations in the aspect of the "arms".

In March 1656 Huygens published his discovery of Saturn's satellite in the pamphlet *De Saturni lunâ observatio nova* [3], in which, to secure priority, he also included an anagram for the hypothesis of the ring. (After decoding, this anagram reads "An nulo cingitur, tenui, plano, nusquam cohaerente, ad eclipticam inclinato"—"It is surrounded by a thin flat ring, nowhere touching, and inclined to the ecliptic.") The full theory was published, after some delay (1659), in *Systema Saturnium* [6], together with many other observations on the planets and their satellites, all contributing to an emphatic defense of the Copernican system.

Of Huygens' further astronomical work, one should mention the determination of the period of Mars and the observation of the Orion nebula. He described the latter, in *Systema Saturnium*, as the view through an opening in the dark heavens into a brighter region farther away. He also developed micrometers for the determination of angular diameters of planets.

Pendulum Clock . In the winter of 1656-1657 Huygens developed the idea of using a pendulum as a regulator for clockworks. Galileo had strongly maintained the tautochronism of the pendulum movement and its applicability to the measurement of time. Pendulums were so used in astronomical observations, sometimes connected to counting mechanisms. In cogwheel clocks, on the other hand, the movement was regulated by balances, the periods of which were strongly dependent on the sources of motive power of the clock and hence unreliable. The necessity for accurate measurement of time was felt especially in navigation, since good clocks were necessary to find longitude at sea. In a seafaring country like Holland, this problem was of paramount importance. Huygens' invention was a rather obvious combination of existing elements, and it is thus not surprising that his priority has been contested, especially in favor of Galileo's son, Vincenzio.

There is no question of Huygens' originality, however, if one acknowledges as the essential point in his clock the application of a freely suspended pendulum, whose motion is transmitted to the clockwork by a handle and fork. The first such clock dates from 1657, and was patented in the same year. In the *Horologium* Huygens described his invention, which had great success; many pendulum clocks were built and by 1658 pendulums had been applied to the tower clocks of Scheveningen and Utrecht.

Huygens made many theoretical studies of the pendulum clock in the years after 1658. The problem central to such mechanisms is that the usual simple pendulum is not exactly tautochronous. Its period depends on the amplitude, although when the amplitudes are small this dependence may be neglected. (This problem was recognized in the first applications of Galileo's proposal.) There are three possible solutions. A constant driving force would secure constant amplitude, but this is technically very difficult. The amplitude may be kept small, a remedy Huygens applied in the clock he described in the *Horologium*, but then even a small disturbance can stop the clock. The best method, therefore, is to design the pendulum so that its bob moves in such a path that the dependence of period on amplitude is entirely eliminated. Huygens tried this solution in his first clock, applying at the suspension point of the pendulum two bent metal laminae, or cheeks, along which the cord wrapped itself as the pendulum swung. Thus the bob did not move in a circle but in a path such that—it could be argued qualitatively—the swing was closer to being tautochronous than in the usual pendulum.

In 1659 Huygens discovered that complete independence of amplitude (and thus perfect tautochronism) can be achieved if the path of the pendulum bob is a cycloid. The next problem was what form to give the cheeks in order to lead the bob in a cycloidal path. This question led Huygens to the theory of evolutes of curves. His famous solution was that the cheeks must also have the form of a cycloid, on a scale determined by the length of the pendulum.

Huygens also studied the relation between period and length of the pendulum and developed the theory of the center of oscillation. By this theory the notion of "length" of a pendulum is extended to compound pendulums, so that Huygens could investigate how the period of a pendulum can be regulated by varying the position of an additional small weight on the arm. These studies form the main contents of Huygens' magnum opus, the *Horologium oscillatorium* [10] (1673). After 1673 Huygens studied harmonic oscillation in general, in connection with the tautochronism of the cycloid. He developed the application of springs instead of pendulums as regulators of clocks—a question on which he engaged in priority disputes with Hooke and others. Huygens also designed many other tautochronous balances for clocks.

Huygens considered the determination of longitudes at sea to be the most important application of the pendulum clock. Here the main difficulty was maintaining an undisturbed vertical suspension. Huygens designed various apparatus to meet this problem, some of which were tested on sea voyages after 1663. Huygens discussed these experiments in *Kort Onderwijs aengaende het gebruyck der Horologien tot het vinden der Lenghten van Oost en West*, a manual for seamen on how to determine longitudes with the help of clocks. Clocks tested on later expeditions (for example, to Crete in 1668–1669 and to the Cape of Good Hope in 1686–1687 and 1690–1692) were not really successful.

Simple Pendulum: Tautochronism of the Cycloid . In 1659, in a study done on the ordinary simple pendulum, Huygens derived a relation between the period and the time of free fall from rest along the length of the pendulum. His result, which he published in part 4 of the *Horologium oscillatorium*, is equivalent to . In deriving the relation, Huygens used a certain approximation which discards the dependence of the period on the amplitude. The error thus introduced is negligible in the case of a small amplitude. In a subsequent investigation, Huygens posed the question of what form the path of the pendulum bob should have, so that the approximation assumption would cease to be an approximation and would describe the real situation. He found a condition for the form of the path related to the position of the normals to the curve with respect to the axis; and he recognized this as a property of the cycloid, which he had studied in the previous year in connection with a problem set by Pascal. He thus discovered the tautochronism of the cycloid—"the most fortunate finding which ever befell me," he said later. He published his discovery, with a scrupulously rigorous Archimedean proof, in the second part of *Horologium oscillatorium*.

Center of Oscillation . Huygens began his studies on the center of oscillation in 1659 as part of his work on the pendulum clock. By 1669 he had formulated a general computation rule applicable to all sorts of compound pendulums (*Horologium* *oscillatorium*, part 4). He showed that the period of a compound pendulum depends on the form of the pendulous body and on the position of the axis (Fig. 3). The theory of the center of oscillation determines this dependence by establishing the length λ of the simple pendulum that oscillates isochronously with the compound pendulum. The center of oscillation of the compound pendulum is the point *O* which lies at distance λ from the axis on the line through the center of gravity Z, perpendicular to the axis. If one assumes all the mass of the pendulum to be concentrated in *O*, the simple pendulum thus formed (with the same axis) will have the same period as the compound one.

In determining centers of oscillation Huygens proceeded from two hypotheses. The first, which he also used in deriving laws of impact, assets that the center of gravity of a system, under the sole influence of gravity, cannot rise; the seond, that in the absense of friction the center of gravity of a system will, if the component parts are directed upward after a descent, rise again to its initial height. Huygens further supposed that the latter hypothesis also applies if during the movement the links between the component parts are severed. Huygens' determination of centers of oscillation can now be represented as follows: The compound pendulum (Fig. 3) consists of small parts with weight g_i whose distance to the axis are α_i. The center of gravity Z has distance ζ to the axis; λ is the length of the isochronous simple pendulum, whose bob in initial position (the amplitudes of both pendulums being equal) is at height *h* above its lowest position; passing this lowest position it has velocity *v*. It is now obvious that in moving from the initial to the lowest position, the center of gravity Z descends over , a height to which it will therefore ascend again. Huygens now imagines that at the moment of passing the lowest position, all the linkages between the parts are severed. These parts then have velocities.

with which they can, when directed upward, ascend to heights h_i Now according to Galileo's law of falling bodies, v_i^2 is proportional to h_i; velocity *v* corresponds to height *h,* so that

If all the parts are directed upward and arrested at their highest positions, the center of gravity will be at height ; the second hypothesis asserts that this heigh is equal to .

Thus,

with

hence

This , then, is Huygens' general computation rule for the center of oscillation. More recently, the final term $\Sigma g_i a_i^2$, rendered as $\Sigma m_i a_i^2$ has been called the "moment of inertia," but Huytgens did not give it a separate name. Huygens determined the centers of oscillation of compound pendulums of many types; he applied complicated geometrical transformations to interpret $\Sigma g_i a_i^2$ as being a quadrature, a cubature, or dependent on the center of gravity of certain curvilinear areas or bodies. He also derived the general theorem which asserts that with respect to different parallel oscillation axes of one pendulum, the product ξ(λ-ξ) is constant and that, consequently, if the center of oscillation and the axis are inter changed, the period remains the same.

In the fourth part of *Horologium oscillatorium,* Huygens also discussed the possibility of defining a universal measure of length by using the length of a simple pendulum having a period of one second, an idea he had first developed in 1661. The advantage of such a method of measurement is that it is not affected in the case of bodies subject to wear or decay, while the theory of the center of oscillation makes it easy to verify the measure itself. In this connection Huygens again mentioned the relation between period and time of fall along the pendulum length, which he had determined as being equivalent to He does not, however, touch upon the possibility that the acceleration of free fall is dependent on the geographical position because of the centrifugal force of the earth's rotation. Strangely, he had in 1659 already recognized this possibility, which invalidates his definition of a universal measure of length. But he apparently did not think that the effect occurred in reality, a view which he sustained even after having heard about Richer's observations in Cayenne; indeed, it was only by reports on experiments in 1690–1692 that Huygens was convinced of the actual occurrence of this effect.

Centrifugal Force In 1659 Huygens collected in a manuscript,*De vi centrifuga* [19] (1703), the results of his studies on centrifugal force, which he had taken up in that year in his investigations on the cause of gravity. He published the most important results, without proofs, in*Horologium oscillatorium*. The fundamental concept in Huygens' treatise is the conatus of a body, which is its tendency to motion and the cause of the tension in a cord on which the body is suspended or on which it is swung around. The conatus of a body is measured by the motion that arises if the restraints are removed; that is, in the case of bodies suspended or swung, if the cords are cut. If these motions are similar—, for instance, both are uniformly accelerated—then the two conatus are similar and therefore comparable. If the motions that arise are the same, then the two conatus are equal.

Huygens showed that for bodies suspended on cords and situated on inclined planes, the conatus, measured in this way, are indeed proportionl to the forces which the theory of statics assigns in these cases. He remarked that the motions arising when the restraints are removed must be considered for only a very short interval after this removal, since a body on a curved plane has the same conatus as a body on the corresponding tangent plane; this obtains although the motions which they would perform are approximately the same only in the first instants after release. What was probably the most important result of this study for Huygens himself was his conclusion that centrifugal force and the force of gravity are similar, as is evidenced by the property of horizontal circular motion. After the cutting of the cord, the body will proceed along the tangent with a uniform motion, so that with respect to an observer participating in the circular motion, it will recede in the direction of the cord; it will recede in such a way that, in subsequent equal short-time intervals, the distance between observer and body will increase with increments approximately proportional to the odd numbers 1, 3, 5,....

The motion of the swung body when released is thus similar to the motion of free fall, and the conatus of suspended and swung bodies are therefore similar and comparable. Huygens compared them by calculating for a given radius (length of cord) r, the velocity v wiht which a body must travels the horizontal circle to cause in its cord the same tension as if it were suspended from it. For this to be the case, the spaces traversed in subsequent equal short increments of time in free fall and in release from circular motion must be the same (that is, the conatus must be the same).

Using the law of falling bodies in the form of the relation v (t = 2s (t)/t, it can be deduced that the required velocity v must be the velocity acquired by a body after free fall along distances s=r/2. Huygens then deduced from geometrical arguemtns that the centrifugal conatus is proportional to the square of the velocity and inversely proportional to the square of the velocity and inversley proportional to the radius. These results were later summarized in the formula —which formula, however, differs significantly in its underlying conceptions from Huygens' result, since its standard derivation involves a measure of the force of gravity by the Newtonian expression mg and since it assimilates centrifugal to gravitational force by the common measure involving the second derivative of the distance-time function. In Huygens' treatment, the notion of "acceleration" as a measurable quantity is entirely absent, and the asimilarity of the two different forces is a demonstrandum rather than an axiom.

Fall and Projectiles. In the second part of *Horologium oscillatorium* Huygens gave a rigorous derivation of the laws of unresisted descent along inclined planes and curved paths, these being the laws which he applied in his proof of the tautochronism of the cycloid. In this derivation he made use of an earlier investigation (1646), in which he had dealt with Galileo's law of falling bodies, by considering that such a law has to be scale-free. He also made use of a study of 1659 in which he derived the law of falling bodies from the principles of relativity of motion.

In 1659 Huygens also made experiments concerning the distance which a freely failing body traverses from rest over a period of one second. This is the form in which the physical constant now indicated by the gravitational acceleration g occurs in the work of Huygens and his contemporaries. By means of the relation between period and length of the simple pendulum, derived in the same year, he found for this distance the value of fifteen Rhenish feet, seven and one-half inches, which is very close to the correct value. Huygens published this result in *Horologium oscillatorium*, part 4.

In 1668 Huygens studied fall and projectile motion in resisting media, a subject on which he had already made short notes in 1646 and 1659. He supposed the resistance, that is, the change of velocity induced by the medium in a short time interval, to be proportional to the velocity. By considering a figure in whch the velocity was represented by an area between a time axis and a curve, Huygens was able to interpret vertical segments of the area perpendicular to the axis as the changes in velocity in the corresponding time interval. These changes are calculated as combinations of the acceleration of the curve is known, and Huygens recognized this relation as a property of the logarithmica which he had studied extensively in 1661. In that way he found the velocity-time relation (and consequently the distance-time relation) in this type of reltarded motion without having explicityly introduced acceleration as a distinct quantity.

But by 1669 Huygens had become convinced by experiments that the resistance in such media as air and water is proportional to the square of the velocity. This indusced him to make a new theoretical study of motion in resisting media. Huygens derived a property of the tangents of the curve which represented the velocity-time relation in this case. The determination of the curve was now a so-called inverse tangent problem (equivalent to a first-order differential equaltion). Huygens reduced it to certain quardratures, but no solution as simple as that for the other case of resistance could be found. Huygens published these results in 1690 in a supplement to the *Discours*.

Concepts of Force . Huygens' study of resisted motion shows that, although he did not accept a Newtonian force concept as a fundmental mechanical principle, he was quite able to perform complicated calculations in which this concept occurs implicitly. In that study, however, he left undiscussed the question of the cause of the forces. His researches on harmonic oscillation (1673-1674) illustrate how unnatural it was for Huygens to disregard this question. Huygens' starting point was the tautochronism of the cycloid. He remarked that a force directed along the tangent, whcih can keep a body at a certain point P on a cycloid in equilibrium, is proportional to the arc length between P and the vertex of the cycloid. He concluded from this that, in general, if the force exerted on a body is proportional to its distance to a certain center and directed toward that center, the body will oscillate tautochronously (that is, harmonically) around that center.

Before coming to this conclusion, however, Huygens stated emphatically that in such an instance the foce exerted has to be independent of the velocity of the body (otherwise the property of the force in the case of the cyloid cannot be extended to the case of bodies moving along the curve). He added that this condition of independence will be satisfied if the agent that causes the force (gravity, elasticity, or magnetism, for example) has infinite or very great velocity. This argument appears again in his studies on the cause of gravity. He also expressly formulated the hypothesis that equal forces produce equal motions regardles of their causes. Only under these presuppositions could Huygens accept the conclusion that proportionality of force and distance yields harmonic oscillation. He applied the argument to springs and torsion balances, and he designed numerous ingenious apparatus for tautochronous balances for clocks. He further studied in this connection the vibration of strings.

Huygens also took a critical position toward Leibniz' concept of force. Although in his collision theory he had found that the sum of the products of the quantity of matter and the square of the velocity is conserved, he did not consider mv^2 to be the quantification of a fundamental dynamical entity (what Leibniz called *vis viva*). In Huygens' opinion, Leibniz failed to prove both the existence of a constant *vis viva* and the proportionality of this entity to mv^2. On the other hand, Huygens liked the idea that a force, or "power to lift," is conserved in mechanical systems, as is indicated by a note in his manuscripts of 1693. This is not surprising since the principle on which most of his mechanical theories are founded—namely, that the center of gravity of a mechanical system cannot rise of it own force—can be shown to be equivalent to the principle of conservation of energy. In support of his principle, Huygens sometimes argued that a mechanical *perpetuum mobile* would otherwise be possible, a conclusion he considered absurd. This view is understandable in its turn because (as we have seen) so many of Huygens' basic ideas in mechanics derived from the pendulum and from the Galilean notion of constrained fall.

Mechanistic Philosophy. Huygens' studies on light and gravity (as well as his few researches on sound, magnetism, and electricity) were strongly influenced by his mechanistic philosophy of nature. In the preface of his *Traité de la lumière*, Huygens described a "true philosophy" as one "in which one conceives the causes of all natural effects by reasons of mechanics." In his view, the motions of various particles of matter and their interactions by direct contact are the only valid starting points for philosophizing about natural phenomena. In this he was following Descartes, and if one wants to view this as the essence of Descarte's thought, then Huygens may be called a Cartesian.

There are marked differences between Huygens and Descartes in the actual working out of this philosophy, however. of these, the most important is that Huygens rejected Descartes's complete trust in the power of reason to attain truth. Complete certainty, according to Huygens, cannot be achieved in the study of nature, although there are degrees requires that the philosopher use good sense. Huygens assigned a most important role to experience and experiment in the discovery and verification of theoretical explanations. He also accepted the intercorpuscular vacuum—in regard to which his philosophy is nearer to Gassendi's than to Descartes's.

According to Huygens, the particles of matter move in the vacuum. These particles are homogeneous, being one kind of matter and differing from each other only in shape and size. The *quantitas materiae* is therefore proportional to the content of the particles of, equivalently, to the space occupied by them. The weight of ordinary bodies is proportional to their *quantitas materiae* because the collisions of ethereal particles that cause gravity have effects proportional to the magnitudes of the colliding particles. This may be considered to mark one of the first insights into the difference between mass and weight.

Huygens explained differences in specific gravity of ordinary bodies as differences in the density of matter. The great variety of specific gravities in nature led him to suppose large interspaces, or "pores," between the component particles of bodies and to attribute an important role to the forms of these interspaces. In Huygens' view the particles are completely hard and, in collision, completely elastic. They are indivisible and keep the form in which they were created. They move in right lines or, in the case of vortices, in circles; they can influence each other's motions only by direct contact.

Huygens' mechanistic explanations of natural phenomena thus consisted in showing that given a certain combination of shapes, magnitudes, number, and velocities of particles, processes occur which manifeast themselves macroscopically as the phenomena under consideration.

In the course of working out a pattern of size relations between particles, Huygens came to the conclusion that four or five discrete classes of particles exist. Particles of the same class are approximately equal in form and magnitude. The classes are differentiated by the magnitudes of the particles, those of one class being much smaller than those of the preceding class and much larger than those of the class following.

The particles of the first class are the components of the ordinary bodies and of the air. They move slowly and Huygens used suppositions about their forms in his explanations of cohesion and coagulation. He considered sound to be vibrations in ordinary bodies and in the air. The particles of the second class form the "ether", and the phenomena of light may be explained by shock waves in this medium. In some ordinary bodies, the spaces between the particles of the first class are so formed that the ether particles can traverse them free: These bodies are transparent. The particles of the third class are the carriers of magnetic phenomena, and those of the fourth class form the "subtle matter" which causes gravity. (It is not clear whether Huygens supposed a fifth class between the third and the fourth classes to account for electrical phenomena). Particles of the fourth class move very rapidly in circular paths around the earth; they are so small that they can pass thought the "Pores" of all ordinary bodies and are scarcely hindered by the particles of the other classes. In Huygens' explanation of gravity as caused by the motion of these particles, as well as in his explanation of magnetism, the concept of vortex plays a fundamental role.

Huygens adherence to a strongly geometrical approach to problems in infinitesimal mathematics prevented him from making the definitive innovations in the infinitesimal calculus that Newton and Leibniz did. Similarly, his strict adherence to mechanistic principles prevented his achieving to mechanistic principles prevented his achieving results in mechanics comparable to Newton's revolutionary work. Huygens immediately realized the importance of Newton's *Principia*, but he also strongly opposed Newton's use of attractive force as a fundamental explanatory principle. Force, in the Newtonian sense, could never count as a fundamental mechanical principle for Huygens. The occurrence of such forces always required a further, mechanistic explanation for him.

It is important to emphasize the role of Huygens' mechanistic vision in his studies and the reasons which led him to defend this vision so strongly against Newton. First of all, it is remarkable that in Huygens' early work the mechanistic point of view is of importance only as a source of inspiration rather than as a principle of explanation. The special hypotheses on which Huygens based his studies on collision, centrifugal force, motion of pendulums, and statics were not substantiated by mechanistic arguments, nor did Huygens seem to think this should be done. There is no mechanistic philosophy in the *Horologium oscillatorium*

It would seem that only after his removal to Paris (1666) did Huygens come to emphasize strongly the necessity for strict mechanistic explanations and to combat the supposition of occult qualities-among which he counted attraction-that some of the members of the Academy applied rather freely. His most important reason for taking this position was, no doubt, that he simply could not accept a phenomenon as properly explained if he could not imagine a mechanistic process causing it. As further reasons we must consider the impressive results that he gained precisely by applying this mechanistic point of view. Huygens' discovery of Saturn's ring was directly connected with the vortex theories; and his study of centrifugal force, which showed that the centrifugal tendency (conatus) of particles moving in circles is indeed similar to the centripetal tendency of heavy bodies, supported the explanation of gravity as the effect of a vortex. Finally, Huygens formulated the wave theory of light, which constituted a mechanistic explanation of refraction and reflection, and which he applied in a masterly fashion to the refractive properties of Iceland spar.

The publication, in 1690, of the *Traité de la lumière* and its supplement, the *Discours*, must be seen as Huygens' answer to Newton's *Principia*. In these works Huygens opposed his mechanistic philosophy to Newton's *Philosophia naturalis*. The wave theory of light and its application to the refraction in Iceland spar are an effective mechanistic explanation of the motion of the planets. Huygens' explanation of gravity dealt with fundamental problems that Newton avoided and left unsolved. Finally, Huygens' treatment of motion in resisting media proved that he could achieve the same results as Newton in this difficult subject although with different methods.

Wave Theory of Light. Light, according to Huygens, is an irregular series of shock waves which proceeds with very great, but finite, velocity through the ether. This ether consists of uniformly minute, elastic particles compressed very close together. Light, therefore, is not an actual transference of matter but rather of a "tendency to move," a serial displacement similar to a collision which proceeds through a row of balls. Because the particles of the ether lie not in rows but irregularly, a colliding particle will transfer its tendency to move to all those particle will transfer its tendency to move to all those particle will transfer its tendency to move to all those particles which it touches in the direction of its motion. Huygens therefore concluded that new wave fronts originate around each particle that is touched by light and extend outward from the particle in the form of hemispheres. Single wave fronts originating at single points are infinitely feeble; but where infinitely many of these fronts overlap, there is light—that is, on the envelope of the fronts of the individual particles. This is "Huygens' principle.

"About 1676 Huygens found the explanation of reflection and refraction by means of this principle; his theory connected the index of refraction with the velocities of light in different media. He became completely convinced of the value of his principle on 6 August 1677, when he found the explanation of the double refraction in Iceland spar by means of his wave theory. His explanation was based on three hypotheses: (1) There are inside the crystal two media in which light waves proceed. (2) One medium be haves as ordinary ether and carries the normally refracted ray. (3) In the other, the velocity of the waves is dependent on direction, so that the waves do not expand in spherical form, but rather as ellipsoids of revolution; this second medium carries the abnormally refracted ray. By studying the symmetry of the crystal Huygens was able to determine the direction of the axis of the ellipsoids, and form the refraction properties of the abnormal ray he established the proportion of the abnormal ray he established the proportion between the axes. He also calculated the refraction of rays on plane sections of the crystal other than the natural crystal sides, and verified all his results experimentally.

Although the completeness of Huygen's analysis is impressive, he was unable to comprehend the effect that we now recognize as polarization, which occurs if the refracted ray is directed through a second crystal of which the orientation is varied. Huygens described this effect in his first studies on the crystall, but he could never explain it. These results are in cluded in the *Traité de la lumiére*, which was completed in 1678; Huygens read parts of it to the Academy in 1679.

Gravity. Huygens' explanation of gravity developed the ideas of Descartes. He presupposed a vortex of particles of sublter matter to be circling the earth with great velocity. Because of their circular movement these particles have a tendency (conatus) to move away form the earth's center. They can follow this tendency if ordinary bodies in the vortex move toward the center. The centrifugal tendency of the vortex particles thus causes a centripetal tendency in ordinary bodies, and this latter tendency is gravity. The space which a body of matter vacates, under the influence of gravity, can be taken by an equal quantity of subtle matter. Hence the gravity of a body is equal to the centrifugal conatus of an equal quantity of subtle matter moving very rapidly around the earth.

This argument led Huygens to study centrifugal force in 1959. In his investigations he proved the similarity of the centrifugal and the gravitational conatus, a result that strengthened his conviction of the validity of the vortex theory of gravity. The study also enabled him to work out this theory quantitatively, since given the radius of the earth and the acceleration of gravity he could calculate the velocity of the particles; he found that they circle the earth about seventeen times in twenty-four hours.

Huygens developed this theory further in a treatise presented to the Academy in 1669. Since the cylindrically symmetrical vortices posited by Descrates could explain only a gravity toward the axis, Huygensm imagined a multilaterally moving vortex—in which the particles circle the earth in all directions—by which a truly centrally directed gravity could be explained. The particles are forced into circular paths because the vortex is held within a sphere enveloping the earth, and bounded by "Other bodies," such that the particles cannot leave this space. The boundary of the gravitational vortex was supposed to be somewhere between the earth and the moon, because Huygens thought the moon to be carried around the earth by a uniaxial vortex (the so-called *vortex deferent*). Later, convinced by Newton of the impossibility of such vortices, he supposed the gravity vortex to extend beyond the moon.

Galileo's law of falling bodies requires that the acceleration which a falling body acquires in a unit of time be independent of the velocity of the body. This independence is the greatest obstacle for any mechanistic explanation of gravity, for the accelerations must be acquired during collisions, but the change of velocity of colliding bodies is dependent on their relative velocities. On this problem Huygens argued that, because the velocity of the vortex particles is very great with respect to the velocity of the falling body, their relative velocity can be considered constant. Thus, in effect he argued that Galileo's law of falling bodies holds only approximately for small velocities of the falling body.

Huygens never discussed the fundamental question raised by this explanation of gravity—namely, how, by means of collisions, a centrifugal tendency of the particles of the subtle matter can transfer a centripetal tendency to heavy bodies.

In the *Discours*, the treatise of 1669 is reiterated almost verbatim, but Huygens added a review of Newton's theory of gravitation, which caused him to revise his own theories somewhat. He resolutely rejected Newton's notion of universal attraction, because, as he said, he believed it to be obvious that the cause of such an attraction cannot be explained by any mechanical principle of law of motion. But he was convinced by Newton of the impossibility of the *vortices deferentes*, and he accepted Newton's explanation of the motion of satellites and planets by a force varying inversely with the square of the distance from the central body. According to Huygens, however, this gravity is also caused by a vortex, although he did not dwell on the explanation of its dependence on the distance.

Cosmotheoros . Huygens did not believe that complete certainly could be achieved in the study of nature, but thought that the philosopher must pursue the highest degree of probability of his theories. Clearly Huygens considered this degree to be adequate in the case of his explanations of light and gravity. It is difficult for the historian to assert how plausible, in comparison with those explanations, Huygens considered his theories about life on other planets and about the existence of beings comparable to man. These theories were expounded in his *Κομοζεαος, sive de terris coelestibus, earumque ornatu, conjecturae* [14].

The argument of the book is very methodically set forth, and its earnestness suggests that Huygens did indeed assign a very high degree of probability to these conjectures. Huygens' reasoning is that it is in the creation of life and living beings that the wisdom and providence of God are most manifest. In the Copernican world system—which is sufficiently proved as agreeing with reality—the earth holds no privileged position among the other planets. It would therefore be unreasonable to suppose that life should be restricted to the earth alone. There must be life on the other planets and living beings endowed with reason who can contemplate the richness of the creation, since in their absence this creation would be senseless and the earth, again, would have an unreasonably privileged position. In further discussion of the different functions of living organisms and rational beings, Huygens came to the conclusion that, in all probability, the plant and animal worlds of other planets are very like those of the earth. He also submitted that the inhabitants of other planets would have a culture similar to man's and would cultivate the sciences.

In the second part of *Cosmotheoros*, Huygens discussed the different movements of the heavenly bodies and how they must appear to the inhabitants of the planets. He took the occasion to mention new advances in astronomy. In contrast to most other Huygensian writings, *Cosmotheroros* has had wide appeal and a broad readership, and has been translated into several languages.

Conclusion . In the period bounded on one side by Viète and Descartes and on the other by Newton and Leibniz, Huygens was Europe's greatest mathematician. In mechanics, in the period after Galileo and before Newton, he stood for many years on a solitary height. His contributions to astronomy, time measurement, and the theory of light are fundamental, and his studies in the many other fields to which his universal interest directed him are of a very high order.

But Huygens' work fell into relative oblivion in the eighteenth century, and his studies exerted little influence. There is thus marked discrepancy between Huygen's actual stature as a natural philosopher and the influence he exerted. This is due in part to his extreme reluctance to publish theories which he considered insufficiently developed or which did not meet his high standards of adequacuy and significance. For this reason his work on hydrostatics, collision, optics, and centrifugal force were published too late to be fully influential. It is also clear that Huygens did not attract disciples: he was essentially a solitary scholar.

Other reason for Huygens' limited influence must be sought in the character of his work. His infinitesimal-geometrical mathematics and his studies in mechanics and the theory of light, inspired by his mechanistic philosophy, were culminations that defined limits rather opening new frontiers. Even his early studies in mechanics, based on hypotheses that we can recognize as equivalent to conservation of energy, served as a basis for later work to only a limited extent— although it is true that one may consider the eighteenth-century researches in mechanics, so far as they centered around the Leibnizian concept of *vis viva*, to be continuations of Huygens' approach. The Newtonian notion of force became the fundamental concept in mechanics after publication of the *Principia*; Huygens' work could not easily be incorporated into this new mechanics, and it was only much later that the two different concepts could be synthesized.

Huygens' work nonetheless forms a continuously impressive demonstration of the explanatory power of the mathematical approach to the study of natural phenomena, and of the fertility of its application to the technical arts. His magnum opus, *Horologium oscillatorium*, stands as a solid symbol of the force of the mathematical approach and was recognized as such by Huygens' contemporaries. Compared to the relatively simple mathematical tools which Galileo used in his works, the wealth of mathematical theories and methods that Huygens was able to apply is significant, and herein lies the direct and lasting influence of his work.

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2. *De circuit magnitudine inventa. Accedunt ejusdem problematum quorudam ilustrium constructiones*, Leiden, 1654 (Oeuvres XII).

3. *De Saturni lunâ* observation nova, The Hague, 1656 (*Ieuvres* XV).

4. *Tractatus de rationciniis in aleae ludo*, in F. van Schooten, *Exercitationum mathematicarum libri quinque*, Leiden, 1657 (Latin trans. of [7] by van Schooten).

5. *Horologium*, The Hauge, 1658 (*Oeuvres* XVII).

6. *Systema Saturnium, sive de causis mirandorum Saturni phaenomenôn, et comite ejus planeta novo*, The Hauge, 1659 (*Oeuvres* XV).

7. *Tractaet handelende van Reeckening in Speelen van Gelucl*, in F. van Schooten, *Mathematische Oeffeningen begrepen in viff boecken*, Amsterdam, 1660 (also published separately in the same year; *Oeuvres* XIV).

8. *Kord onderwijs aengaende het gebruyck der Horologien tot het vinden der Lenghten van Oost en West*, 1665 (*Oeuvres* XVII).

9. *Règles du mouvement dans la rencontre des corps, in Journal des scavans*, 1669 (*Oeuvres* XVI).

10. *Horologium oscillatorium, sive de motu pendulorum ad horologia aptato demonstrationes geometricae*, Paris, 1673 (*Oeuvres* XVII); a German trans, in the series *Ostwald's Klassiker der Exakten Wissenschaften*, no. 192 (Leipzig, 1913).

11. *Astroscopia compendiaria, tubi optici molimine liberata*, The Hague, 1684 (*Oeuvres* XXI);

12. *Traité de la lumiè, oú sont expliquées les causes de ce qui lui arrive dans la Reflexion & dans la Refraction, et particulièrement dans l;étrange Refraction du Cristal d;Islande. (Avec un Discours de la Cause de la Pesanteur*), Leiden, 1690 (*Oeuvres* XIX); there is a German trans. in *Ostwald's Klassiker*, no. 20 (Leipzig, 1903).

13. *Discours de la caus de la Pesanteur* appears in [12] (*Oeuvres* XXI).

14. Κοσμουεωρος *sive de terris coelestibus, earumque ornatu, conjecturae* The Hauge, 1698 (*Oeuvres* XXI).

15. B. de Volder and B. Fullenius, ed., *Christiani hugenii Opuscula Posthuma* (Leiden, 1703).

16. *Tractatus de refractione et telescopiis*, MS originating from 1653, was later changed and amplified many times. One version is published under the title *Dioptrica* in the Volder and Fullenius edition and another version in *Oeuvres* XIII.

17. *Memorien aengaende het slijpen van glasen tot verrekijckers*, MS originating from 1685, published in *Oeuvres* XXI. A Latin trans. was published in Volder and Fullenius.

18. *De motu corporum ex percussione*, MS originating from 1656, published in *Oeuvres* XVI A German trans. appeared in *Ostwald's Klassiker*, no. 138 (Leipzig, 1903).

19. *De vi centrifuga*, MS originating from 1659, published in *Oeuvres* XVI, a German trans. existing in *Ostwald*, no. 138, Leipzig, 1903 . Like [18] this is also found in Volder and Fullenius .

20. *De iis quae liquido supernatant*, MS originating from 1650, appears in *Oeuvres* XI.

In his will, Huygens asked Volder and Fullenius to edit some not yet published MSS, which resulted in their posthumous edition [15].

Two further publications of Huygens' writings, edited by G. J. 'sGravesande, are [21] *Christiani hugenii Opera Varia* (Leiden, 1721) and [22] *Christiani Hugenii Opera Relique* (Leiden, 1728). Little more than a century later, P. J. Uylenbroek edited Huygens' correspondence with L'Hospital and Leibniz in [23] *Christiani hugenii aliorumque seculil XVII viroru celebrium exerciationes mathematicae et philosophicae* (The Hauge, 1833).

In 1882, the Netherlands Academy of Science at Amsterdam organized a preparatory committee for a comprehensive ed. of Huygens' works. In 1885 it was agreed that the Society of Sciences of Holland at Haarlem would take responsibility for the publication. The undertaking resulted, after more than sixty years of editorial commitment, in what may be considered the best edition of the works of any scientist, the *Oeuvres complè de christiaan Huygens, publiées par la Société Hollandaise des Sciences*, 22 vols. (the Hauge, 1888-1950).

The first ten vols. comprise Huygens' correspondence, the subsequent ones his published and unpublished scholarly writings, of which the most important are accompanied by a French trans. Vol. XXII containes a detailed biography of Huygens by J. A. vollgraff.

The editors in chief wre, successively, D. Bierens de Haan, J. Bosscha, D. J. Korteweg, and J. A. Vollgraff. Among the many collaborators, C. A. Crommelin, H. A. Lorentz, A. a. Nijland, and E. J. Dijksterhuis may be mentioned. The editors adopted a strict code of anonymity, which was broken only in the last volume.

II. Secondary Literature. While Huygens' work is easily accessible in the *Oeuvres*, there exists relatively little secondary literature about him. We may mention [24]P. Harting, *Christiaan Huygens in zijn leven en werken geschetst* (Groningen, 1868); [25] H. L. Brugmans, *Le sejour de Christiaan Huygens a Paris et ses relations avec les milieux scientifiques francais, suivi de son journal de voyage a Pariset a Londress* (Paris, 1935); and [26] A. Romein Verschoor, "Christiaen Huygens, de ontdekker der waar-schijnlijkheid," in *Erflaters vanonze beschaving* (amsterdam, 1938-1940), written with J. Romein.

The only recent separately published scientific biography of Huygens is [27] A. E. Bell, *Christian Huygens and the Development of Science in the Seventeenth Century* (London, 1947). On the occasion of the completion of the *Oeuvres* edition, there appeared [28] E. J. Dijksterhuis, *Christiaan Huygens* (Haarlem, 1951).

J. A. Vollgraf, who by editing the last seven vols. of the *Oeuvres* acquired a thorough knowledge of Huygens' life and works, has written a book about Huygens which has not been published. The private typescript will be transferred to the Leiden University Library.

H. J. M. Bos.