Hypsicles Of Alexandria | Encyclopedia.com

(*fl*. Alexandria, first half of second century B.C.)

mathematics, astronomy

Hypsicles is attested, by the more definitive manuscripts, to be the author of what has come to be printed as book XIV of Euclid's *Elements*. In the preface to that book he states that the Basilides of Tyre came to Alexandria, where he engaged in mathematical discussions with Hypsicles' father. Together they studied a tract by Apollonius of Perga on the dodecahedron and the icosahedron inscribed in the same sphere, and found the treatment unsatisfactory.

Later, presumably after his father's death, Hypsicles himself found what would appear to have been a revised, more accurate version in wide circulation. Taken together, these facts suggest that Hypsicles' father was an older contemporary of Apollonius, living at Alexandria. As Apollonius died early in the second century B.C., the middle point of Hypsicles' activities may be placed at about 175 B.C.

The so—called book XIV, like book XIII, is concerned with the inscription of regular solids in a sphere. Hypsicles proves a proposition, which he attributes to Aristaeus (who was probably not the author of *Five Books Concerning Solid Loci*), that the same circle can be described about the pentagonal face of a regular dodecahedron and the triangular face of a regular icosahedron inscribed in the same sphere. He proves, as had Apollonius before him, that the volume of the dodecahedron bears the same relation to the volume of the icosahedron as the surface of the former bears to the surface of the latter, because the perpendiculars to the respective faces are equal; and that both the ratios are equal to the ratio of the side of the inscribed cube to the side of the dodecahedron.

Arabic traditions suggest that Hypsicles also had something to do with the so-called book XV of the *Elements*, whether he wrote it, edited it, or merely discovered it. But this is clearly a much later and much inferior book, in three separate parts, and this speculation appears to derive from a misunderstanding of the preface to book XIV.

One other work by Hypsicles survives, the *Anaphorikos ('Αναφορικός)*, or *On the Ascension of Stars*. Although quite brief, probably truncated, and based on a false assumption, it is noteworthy in being the first work in which the ecliptic is divided into 360 parts or degrees. He writes,

The circle of the zodiac having been divided into 360 equal arcs, let each of the arcs be called a spatial degree, and likewise, if the time taken by the zodiac circle to return from a point to the same point is divided into 360 equal times, let each of the times be called a temporal degree [*Die Aufgangszeiten der Gestirne* 55-59, De Falco, ed., p.36].

This division into 360 parts was almost certainly borrowed from Babylonia, and the *Anaphorikos* is therefore testimony to the existence of links between Greek and Babylonian astronomy in the second century B. C. Hypsicles posits for himself two problems. Given the ratio of the longest day to the shortest day at any place, how long does it take any given sign of the zodia to rise there? Second, how long does it take any given degree in a sign to rise? The practical object of this investigation may have been, as T. L. Heath conjectures, to tell the time at night. But the problem came really within the province of spherical trigonometry, which was not developed until Hipparchus, Ptolemy later solved it with the help of his table of sines (*Syntaxis mathematics*, bk, 2, J. L. Heiberg, ed. [Leipzig, 1898], ch. 8, pp. 134-141); and Hipparchus had no doubt solved it before Ptolemy, for Pappus of Alexandria (*Collection* VI 109, Hultsch ed., 6009. 9-13) refers to calculations "by means of numbers" appearing in Hipparchus' book *On the Rising of the Twelve signs of the Zodiac*. This methods of solution was not open to Hypsicles, which is further confirmation of his date.

The longest day, Hypsicles says, is the time during which Cancer, Leo, Virgo, Libra, Scorpio, and Sagittarius rise (14 hours at Alexandria), and the shortest is the time in which Capricornus, Aquarius, Pisces, Aries, Taurus, and Gemini rise (10 hours); and as their ratio is 7:5, the former signs take 210 temporal degrees and the latter 150. He assumes that the quadrants Cancer-Virgo and Libra-Sagittarius take equal times to rise, 105 temporal degrees, and that the quadrants Capricornus-Pisces and Aries-Gemini each require 75 degrees. He further assumes that the times taken by Virgo, Leo, Cancer, Gemini, Tauras, and Aries form a descending arithmetical series, and that the times for Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces are in the same series.

With the help of three lemmas concerning arithmetical progressions which he has proved at the outset of his book, Hypsicles shows that Virgo and Libra take 38° 20’ to rise, Leo and Scorpio 35° and so on, the common difference being 3° 20’ He goes on the prove that each spatical degree takes 0° 0’ 13“20“’ less (or more) than its predecessor to rise.

As Hypsicles, assumpation that times of rising form an arithmetical progression was erroneous, his results were correspondingly in errors. But his tract was galleant attempt to solve the problem before trigonometry provide the right way. The *Anaphorikos* has probably survived by reasons of having been included in the collection of ancient Greek texts known as The *Little Astronomy*. It was translated into Arabic toward the end of the ninth century; the translation is variously ascribed to Qustã ibn Lüqã and IshãQ ibn Hunayan, but it was in any case considerably altered by latter writers. From Arabic it was translated intoLatin by Gerard of Cremona (*ca* 1150) as *Liber Esculei* De *ascensionibus*. The first printed edition, in Greek and Latin, by Jacobus Mentelisu (Paris, 1657) remained the only one until that on K. Manituius in 1888, and this has in turn been superseded by the criticial editon of De Falco and Krause (1966).

Hypsicles is cited by Diophantus of Alexandria in *De polygonis numeris* (*Diophanti Alexandrini opera amnia*, I, P. Tannery, ed. [Leipzig, 1893-1895], 470, 27-472.4) as the author of the following definition.

If there be as many numbers as we please begining from 1 and increasing by the same common difference, then, when the common difference is 1 the sum of all the numbers is a triangular number; when 2, a square number; when 3, a pentagonal number, and so on, the number of angles being called after the number which exceeds the common difference by 2 and the sides after the number of terms, including 1.

In modern notation, the *nth a*-gonal number (1 being the first) is

From this reference by Diophantus it is presumed that Hypsicles must have written a book, since lost, on numbers. According to Achilles Taitus (*Introductio in Aratum* E. Mass, ed., *Commentariorum in Aratum reliquaiae*, Berlin, 1898, p, 43.9) Hypsicles also wrote a book on the harmony of the spheres; it has not survived.

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