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(*fl. ca.* 1220)

mechanics, mathematics.

Although Jordanus has been justly proclaimed the most important mechanician of the Middle Ages and one of the most significant mathematicians of that period, virtually nothing is known of his life. That he lived and wrote during the first half of the thirteenth century, is suggested by the inclusion of his works in the *Biblionomia*, a catalogue of Richard de Fournival's library compiled sometime between 1246 and 1260.[1] In all, twelve treatises are ascribed to Jordanus de Nemore, whose name is cited four times in this form.[2] Since most of his genuine treatises are included, it seems reasonable to infer that Jordanus' productive career antedated the *Biblionomia*.

The appellation "Jordanus de Nemore" is also found in a number of thirteenth-century manuscripts of works attributed to Jordanus. The meaning and origin of "de Nemore" are unknown. It could signify "from" or "of Nemus," a place as yet unidentified (the oft-used alternative "Nemorarius," frequently associated with Jordanus, is apparently "a later derivation from "Nemore")", or it may have derived from a corruption of "de numeris" or "de numero" from Jordanus' arithmetic manusripts.[3]

Identification of Jordanus de Nemore with Jordanus de Saxonia (or Jordanus of Saxony), the master general of the Dominican order from 1222 to 1237, has been made on the basis of a statemetn by Nicholas Trivet (in a chronicle called *Annales sex regum Angliae*) that Jordanus of Saxony was an outstanding scientist who is said to have written a book on weights and a treatise entitled *De lineis datis*.[4] Although a late manuscript of a work definitely written by Jordanus de Nemore is actually ascribed to "Jordanus de Alemannia" (Jordanus of Germany, and therefore possibly Jordanus of Saxony), no mathematical or scientific works can be assigned to Jordanus of Saxony, whose literary output was seemingly confined to relgon and grammar. At no time, moreover, was Jordanus of Saxony called Jordanus de Nemore or Nemorarius. Finally, if Jordanus de Nemore lectured at the University of Toulouse, as one manuscript indicates,[5] this could have occurred no earlier than 1229, the year of its foundation. As master general of the Dominican order during the years 1229-1237, the year of his death, Jordanus of Saxony could hardly have found time to lecture at a university. For all these reasons it seems implausible to suppose that Jordanus of Saxony is identical with Jordanus de Nemore.

It was in mechanics that Jordanus left his greatest legacy to science. The medieval Latin "science of weights" (Scientia de ponderibus), or statics, is virtually synonymous with his name, a state of affairs that has posed difficult problems of authorship. So strongly was the name of Master Jordanus identified with the science of weights that manuscripts of commentaries on his work, or works, were frequently attributed to the master himself. Since the commentaries were in the style of Jordanus, original works by him are not easily distinguished. At present only one treatise, the *Elementa Jordani super demonstrationem ponderum*,, may be definitely assigned to Jordanus. Whether he inherited the skeletal frame of the *Elementa* in the form of its seven postulates and the enunciations of its nine theorems, for which he then supplied proofs, is in dispute.[6] Indisputable, however, is the graet significance of the treatise. here, under the concept of "positional gravity" (*gravitas secundum situm*), we find the introduction of component forces into statics. The concept is expressed in the fourth and fifth postulates, where it is assumed that "weight is heavier positionally, when, at a given position, its path of descent is less oblique" and that "a more oblique descent is one in which, for a given distance, there is a smaller component of the vertical."[7] In a constrained system the effective weight of a suspended body is proportional to the directness of its descent, directness or obliquity of descent being measured by the projection of any segment of the body's arcal path onto the vertical drawn through the fulcrum of the lever or balance. It is implied that the displacement which measures the positional gravity of a weight can be infinitely small. Thus, by means of a principle of virtual displacement (since actual movement cannot occur in a system in equilibrium, positional gravity can be measured only by "virtual" displacements) Jordanus introduced infinitesimal considerations into statics.

These concepts are illustrated in Proposition 2, where Jordanus demonstrates that "when the beam of a balance of equal arms is in horizontal position, then if equal weights are suspended from its extremities, it will not leave the horizontal position; and if it should be moved from the horizontal position, it will revert to it."[8] If the balance is depressed on the side of B (see Figure 1), Jordanus argues that it will return to a

horizontal position because weight *c* at C will be positionally heavier than weight *b* at B, a state of affairs which follows from the fact that if any two equal arcs are measured downward from *C* and *B*, they will project unequal intercepts onto diameter *FRZMAKYE*. If the equal arcs are *CD* and *BG*, Jordanus can demonstrate (by appeal to his *Philotegni*, or *De triangulis*, as it was also called) that the intercept of arc *CD*—*ZM*—is greater than the intercept of arc *BG*—*KY*—and the "positionally heavier *c* will cause *C* to descend to a horizontal position. The concept of positional heaviness, although erroneous when applied to arcal paths, may have derived ultimately from application of an idea in the Pseudo-Aristotelian *Mechanica*, where it was argued that the further a weight is from the fulcrum of a balance, the more easily it will move a weight on the other side of the fulcrum, since "a longer radius describes a larger circle. So with the exertion of the same force the motive weight will change its position more than the weight which it moves, because it is further from the fulcrum."[9] It was by treating the descent of *b* independently from the ascent of *c* that Jordanus fell into error. A comparison of the ratio of paths formed by a small descent of *b* and an equal ascent of *c* with the ratio of paths formed by a small descent of *c* and an equal ascent of *b* would have revealed the equality of these ratios and demonstrated the absence of positional advantage. As we shall see below, however, when the concept of positional gravity was applied to rectilinear, rather than arcal, constrained paths, perhaps by Jordanus himself, it led to brilliant results.

More important than positional gravity is Jordanus' proof of the law of the lever by means of the principle of work. In Figure 2, *ACB* is a balance beam and

a and *b* are suspended weights. If we assume that $b\text{-}a = AC/BC$, no movement of the balance will occur. The demonstration takes the form of an indirect proof. It is assumed that the balance descends on *B*'s side so that as *b* descends through vertical distance *HE*, it lifts *a* through vertical distance *DG*.If a weight *l*, equal to *b*, is now suspended at *L*, Jordanus shows, on the basis of similar triangles *DCG* and *ECH*, that $DG/EH = b\text{-}a$. On the assumption that $CL = CB$ and drawing perpendicular *LM*, he concludes that $LM = EH$. Therefore $DG\text{-}LM = b/a = l\text{-}a$. At this point the principle of work is applied, for "what suffices to lift *a* to *D*, would suffice to lift *l* through the distance *LM*. Since therefore *l* and *b* are equal, and *LC* is equal to *CB*, *l* is not lifted by *b*; and, as was asserted, *a* will not be lifted by *b*."[10] If a weight is thus incapable of lifting an equal weight the same distance that it descends, it cannot raise a proportionally smaller weight a proportionally greater distance.

The principles of positional gravity and work were superbly employed in the *De ratione ponderis*, which contains forty-five propositions and is probably the most significant of all medieval statical treatises. If, as the manuscripts indicate, it was by Jordanus himself[11] (although there is some doubt about this),[12] not only did Jordanus extend his own concept of positional gravity to rectilinear paths (the incorrect application to arcal paths was, however, retained in a few propositions) but he also applied that concept, in conjunction with the principle of work, to a formulation of the first known proof—long before Galileo—of the conditions of equilibrium of unequal weights on planes inclined at different angles. Paradoxically, in Book I, Proposition 2, the *De ratione ponderis* included reasoning which, if rigorously applied, would have destroyed the notion that an elevated weight has greater positional gravity with which to restore the equilibrium of a balance.[13]

In Book I, Proposition 9, Jordanus (for convenience we shall assume his authorship) shows that positional gravity—the heaviness or force of a weight—along an inclined plane (see Figure 3) is the same at any

point. Thus a given weight at *D* or *E* will possess equal force, since for equal segments of the inclined path *AB*—*DF* and *EG*—equal segments of the vertical *AC* will be intercepted—*DK* and *EM*

On the basis of Postulates 4 and 5 of the DE *Elementa*, which are also Postulates 4 and 5 of the *De ratione ponderis*, and Book I, Proposition 9, the inclined plane proof is enunciated in Book I, Proposition 10, as follows: "If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, they will be of equal force in descending."[14] Jordanus demonstrates that weights *e* and *h*, on differently inclined planes, are of

equal force. He first assumes that a weight *g*, equal to *e*, is on another plane, *DK*, whose obliquity is equal to that of *DC* and then assumes that if *e* moves to *L* through vertical distance *ER*, it will also draw *h* up to *M*. should this occur, however, it would follow by the principle of work that what is capable of moving *h* to *M* can also move *g* to *N*, since it can be shown that $MX/NZ = g/h$. But *g* is equal to *e* and at the same inclination; hence, by Book I, Proposition 9, they are of equal force because they will intercept equal segments of vertical *DB*. therefore *e* is incapable of raising *g* to *N* and, consequently, unable to raise *h* to *M*. By substituting a straight line for an arcal path and utilizing Postulate 5 of the *Elementa* (see above), Jordanus was, in modern terms, measuring the obliquity of descent, or ascent, by the sine of the angle of inclination. The force along the rectilinear oblique path, or incline, is thus equivalent to

$F = W \sin a,$

where *W* is the free weight and *a* is the angle of inclination of the oblique path.

The principle of work, which was but a vague concept prior to Jordanus, was not only used effectively in the proof of the inclined plane and, as indicated above, in the law of the lever in the *Elemenenta*, a proof repeated in Book I, Proposition 6, of the *De ratione ponderis*, but was also applied successfully to the first proof of the bent lever in Book I, Proposition 8, of the *De ratione ponderis*, which reads; "If the arms of a balance are unequal, and form an angle at the axis of support, then, if their ends are equidistant from the vertical line passing through the axis of support, equal weights suspended from them will, as so placed, be of equal heaviness."[15] In this proof there is also an anticipation of the concept of static moment, that the effective force of a weight is dependent on the weight and its horizontal distance from a vertical line passing through the fulcrum.[16]

Over and above his specific contributions to the advance of statics, Jordanus marks a significant departure in the development of that science. He joined the dynamical and philosophical approach characteristic of the dominant Aristotelian physics of his day with the abstract and rigorous mathematical physics of Archimedes. Thus the postulates of the *Elementa* and *De ratione* *ponderis* were derived from, and consistent with, Aristotelian dynamical concepts of motion but were arranged in a manner that permitted the derivation of rigorous proofs within a mathematical format modeled on Archimedean statics and Euclidean geometry.

The extensive commentary literature on the statical treatises ascribed to Jordanus began in the middle of the thirteenth century and continued into the sixteenth. Through printed editions of the sixteenth century, the content of this medieval science of weights, identified largely with the name of Jordanus, became readily available to mechanicians of the sixteenth and seventeenth centuries. Dissemination was facilitated by works such as Peter Apian's *Liber Iordani Nemorarii. . .de ponderibus propositiones XIII et earunaem demonstrationes* (Nuremberg, 1533); Nicolo Tartaglia's *Questii ed invenzioni diverse* (Venice, 1546, 1554, 1562, 1606; also translated into English, German, and French), which contained a variety of propositions from Book I of the *De ratione ponderis*; and *Jordani Opusculum de ponderositate* (Venice, 1565), a version of the *De ratione ponderis* published by Curtius Trojanus from a copy owned by Tartaglia, who had died in 1557.

Concepts such as positional gravity, static moment, and the principle of work, or virtual displacement, were now available and actually influenced leading mechanicians, including Galileo, although some preferred to follow the pure Greek statical tradition of Archimedes and Pappus of Alexandria. In commenting on Guido Ubaldo del Monte's *Le mechaniche* (1577), which he himself had translated into Italian, Filippo Pigafetta remarked that Guido Ubaldo's more immediate predecessors

. . .are to be understood as being the writers on this subject cited in various places by the author [Guido Ubaldo], among them Jordanus, who wrote on weights and was highly regarded and to this day has been much followed in his teachings. Now our author [Guido Ubaldo] has tried in every way to travel the road of the good ancient Greeks, . . . in particular that of Archimedes of Syracuse . . .and Pappus of Alexandria . . .[17]

Guido Ubaldo's attitude was costly, for it led him to reject Jordanus' correct inclined-plane theorem in favor of an erroneous explanation by Pappus.

Since few editions of his mathematical treatises have been published, and critical studies and evaluations are largely lacking, Jordanus' place in the history of medieval and early modern mathematics has yet to be determined. Treatises on geometry, algebra, proportions, and theoretical and practical arithmetic have been attributed to him.

The *Liber Philotegni de triangulis*, a geometrical work extant in two versions, represents medieval Latin geometry at its highest level. In the four books of the treatise we find propositions concerned with the ratios of sides and angles; with the division of straight lines, triangles, and quadrangles under a variety of given conditions; and with ratios of arcs and plane segments in the same circle and in different circles. The fourth book contains the most significant and sophisticated propositions. In IV.20 Jordanus presents three solutions for the problem of trisecting an angle, and IV.22 offers two solutions for finding two mean proportionals between two given lines. A proof of Hero's theorem on the area of a triangle—$A=$, where s is the semiperimeter and a, b and c are the sides of the triangle—may also have been associated with the *De triangulis*.Jordanus drew his solutions largely from Latin translations of Arabic works, which were themselves based on Greek mathematical texts. He did not always approve of these proofs and occasionally displayed a critical spirit, as when he deemed two proofs of the trisection of an angle based on mechanical means inadequate and uncertain (although no source is mentioned, they were derived from the *Verba filiorum* of the Banū Mūsā) and offered what is apparently his own demonstration,[18] in which a proposition from Ibn al-Haytham's *Optics* is utilized. In IV.16 a nonArchimedean proof of the quadrature of the circle may have been original. It involves "finding a third continuous proportional." Here is the proof.[19]

To Form a Square Equal to a Given Circle.

For example, let the circle be *A [see Figure 5].*

Disposition: Let another circle *B* with its diameter be added; let a square be described about each of those circles. And the circumscribed square [in each case] will be as a square of the diameter of the circle. Hence, by [Proposition] XII.2 [of the *Elements*], circle *A*-circle *B* = square *DE*-square FG. Therefore, by permutation, $DE/A = FG/B$. Let there be formed a third surface *C*, which is a [third] proportional [term] following *DE* and *A*. Now *C* will either be a circle or a surface of another kind, like a rectilinear surface. In the first place, let it be a circle which is circumscribed by square *HK*. And so, $DE/A = A/C$ but also, by [proposition] XII.2 [of the *Elements*], $DE/A = HK/C$. Therefore, *HK* as well as *A* is a mean proportional between *DE* and *C*. Therefore, circle *A* and square *HK* are equal, which we proposed.

Next, let *C* be some [rectilinear] figure other than a circle. Then let it be converted into a square by the last [proposition] of [Book] II [of the *Elements*], with its angles designated as R, S, Y, and X. And so, since *DE* is "the larger extreme (among the three proportion terms) DE is" greater than a side [of *RY*]. Therefore, let *MT*, equal to *RX*, be cut from *MD*. Then a parallelogram *MN*—contained by *ME* and *MT*—is described. Therefore, *MN* is the mean proportional between *DE* and *RY*, which are the squares of its sides, since a rectangle is the mean proportional between the squares of its sides. But circle *A* was the mean proportional between them [i.e., between square *DE* and *C* (or square RY)]. Therefore, circle *A* and parallelogram *MN* are equal. Therefore, let *MN* be converted to a square by the last [proposition] of [Book] II [of the *Elements*], and this square will be equal to the given circle *A*, which we proposed.

The *De numeris datis*, Jordanus' algebraic treatise in four books, which was praised by Regiomontanus, was more formal and Euclidean than the algebraic treatises derived from Arabic sources. Indeed it has recently been claimed[20] that in the *De numeris datis* Jordanus anticipated ViÈte in the application of analysis to algebraic problems. This may be seen in Jordanus' procedure, where he regularly formulated problems in terms of what is known and what is to be found (this is tantamount to the construction of an equation), and subsequently transforms the initial equation into a final form from which a specific computation is made with determinate numbers that meet the general conditions of the problem.

The general pattern of every proposition is thus (1) formal enunciation of the proposition; (2) proof; and (3) a numerical example, which is certainly non-Euclidean and was perhaps patterned after Arabic algebraic treatises. In this wholly rhetorical treatise, Jordanus used letters of the alphabet to represent numbers. An unknown number might be represented as *ab* or *abc*, which signify $a + b$ and $a + b + c$ respectively. Occasionally, when two unknown numbers are involved, one would be represented as *ab*, the other as *c*.

Typical of the propositions in the De numeris datis are Book I, Proposition 4, and Book IV, Proposition 7. In the first of these, a given number, say s.[21] is divided into numbers x and y, whose values are to be determined. It is assumed that g, the sum of x^2 and y^2, is also known. Now $s^2 - g = 2\,xy = e$ and $g - e = h = (x - y)^2$. Therfore $(x - y) = \sqrt{h} = d$.Since d is the difference between the unknown numbers x and y, their values can be determined by Book I, Proposition 1, where Jordanus demonstrated that "if a given number is divided in two and their difference is given, each of them will be given," The numbers are found from their sum and difference. Since $x + y = s$ and $x - y = d$, it follows that $y - d$, it follows that $y + d = x$ and, therefore, $2\,y + d = x + y = s$; hence $2\,y = s - d$, $y = (s - d)$-2, and $x = s - y$. Should we be given the ratio between x and y, say r, and the product of their sum and difference, say p, the values of x and y are determinable by Book IV, Proposition 7, as follows: since x- $y = r$, $x^2/\,y^2 = r^2$; moreover, since $(x + y)\,(x - y) = p$, therefore $x^2 - y^2 = p$. Now $x^2 = r^2y^2$, so that $(r^2 - 1)\,y^2 = p$ and In the numerical example $r = 3$ and $p = 32$, which yields $y = 2$ and $x = 6$.

In the *Arithmetica*the third and probably most widely known of his three major mathematical works, Jordanus included more than 400 propositions in ten books which became the standard source of theoretical arithmetic in the Middle Ages. Proceeding by definitions, postulates, and axioms the *Arithmetica* was modeled after the arithmetic books of Euclid's *Elements*a treatise which Jordanus undoubtedly used, although the proofs frequently differ. Jordanus' Arithmetica contrasts sharply with the popular, non- formal, and often philosophical Arithmetica of Boethius. A typical proposition, which has no counterpart in Euclid's *Elements*, is Book I, Proposition 9:

The [total sum or] result of the multiplication of any number by however many numbers you please is equal to [est quantum] the result of the multiplication of the same number by the number composed of all the others.

Let *A* be the number multiplied by *B* and *C* to produces *D* and *E* [respectively]. I say that the composite [or sum] of *D* and *E* is produced by multiplying *A* by the composite of *B* and *C*. For it is obvious by Definition [7] that *B* measures [*numerat*] *D A* times and that *C* measures *E* by the same number, namely, *A* times. By the sixth proposition of this book, you will easily be able to argue this.[22]

Thus Jordanus proves that if $A \cdot B = D$ and $A \cdot C = E$, then $D + E = A\,(B + C)$. By Definition 7, D- $B = A$ and $E/C = A$. And since D and E are equimultiples of B and C, respectively, then, by Proposition 6, it follows that B + C = 1/n (D + E) and, assuming $n = A$, we obtain $A\,(B + C) = D + E$.

The Arabic number system also attracted Jordanus' attention—if the *Demonstratio Jordani de algorismo* and a possible earlier and shorter version of it, the *Opus numerorum*, are actually by Jordanus. Once again Jordanus proceeded by definitions and propositions in a manner that differed radically from Johannes Sacrobosco's *Algorismus vulgaris*, or *Common Algorism*. Unlike Sacrobosco, Jordanus described the arithmetic operations and extraction of roots succinctly and formally and without examples. Among the twenty-one definitions of the *Demonstratio Jordani* are those for addition, doubling, halving, multiplication, division, extraction of a root (these definitions are illustrated as propositions), simple number, composite number, digit, and article (which is ten or consists of tens). Propositions equivalent to the following are included:

3. If a: $b = c$: d, then $a \cdot 10^n$: $b = c \cdot 10^n$: d

12. $1 \cdot 10^n + 9 \cdot 10^n = 1 \cdot 10^{n+1}$

19. $a \cdot 10^n + b \cdot 10^n = (a + b)\,10^n$

32. If $a_1 = a \cdot 10$, $a_2 = a \cdot 100$, $a_3 = a \cdot 1{,}000$, then $(a \cdot a_1/\,a^2 = (a_1 \cdot a_2)$- $a^2{}_1 = (a_2 \cdot a_3)/\,a^2{}_2 = \ldots$

An algorithm of fractions, called *Liber or Demonstratio de minutiis* in some manuscripts, may also have been written by Jordanus. It describes in general terms arithmetic operations with fractions alone and with fractions and integers. He also composed a *Liber de proportionibus*, a brief treatise containing propositions akin to those in Book V of Euclid's *Elements*.

NOTES

1. Marshall Clagett, *The Science of Mechanics in the Middle Ages*, pp. 72-73.

2. Leopold Delisle, *Le cabinet des manuscrits de la BibliothÈque nationale*, II (Paris, 1874), 526, 527.

3. This has been suggested by O. Klein, “Who Was Jordanus Nemorarius?,” in *Nuclear Physics*, **57** (1964), 347.

4. Maximililan Curtze, “Jordani Nemorarii Geometria vel De triangulis libri IV,” in *Mitteilungen des Coppernicus-Vereins*, **6** (1887), iv, n. 2.

5. *Ibid*., p. vi.

6. Calgett, op. *cit*., p. 73.

7. Translated by E. A. Moody and M. Clagett, *The Medieval Science of Weights*, p. 129.

8. *Ibid*., p.131.

9. 850b. 4-6 in the translation of E. S. Forster (Oxford, 1913).

10. Moody and Clagett, trans., op, *cit*., pp. 139, 141.

11. A position adopted by Moody, *ibid*., pp. 171-172.

12. Joseph E. Brown, *The Scientia de ponderibus in the Later Middle Ages*, pp. 64-66.

13. Clagett, op. *cit*., pp. 76-77.

14. Moody and Clagett, trans., op, *cit*., p. 191.

15. *Ibid*., *pp* . 185, 187 .

16. Clagett, op. *cit*., p. 82.

17. Translated by Stillman Drake in Mechanics in *Sixteenthcentury Italy*, trans. and annotated by Stillman Drake and I. E. Drabkin (Madison, wis., 1969), p.295.

18. Marshall Clagett, *Archimedes in the Middle Ages,* I, 675.

19. Trans. by Clagett, *ibid*., pp. 573-575.

20. Barnabas B. Hughes (ed. and trans), *The De numeris datis of Jordanus de Nemore*, pp. 50-52.

21. In his ed. of the *De numeris datis*, Curtze altered the letters in presenting the analytic summaries of the propositions; in a few instances I have altered Curtze”s letters.

22. My translation from Edward Grant, ed., *A Source Book in Medieval Science* (in press).

BIBLIOGRAPHY

I. Original Works.. In Richard Fournival’ *Biblionomia* twelve works are attributed to jordanus In codex 43 we find (1) The *Philotegni, or De triangulis*; (2) *De ratione ponderum* (3) *De ponderum proportione*; and (4) *De quadratura circuli*. In codex 45 three works are listed (5) *Practica, or Algorismus*;(6) Practica de minutiis; and (7) Experimenta super algebra. Codex 47 contains the lenghy (8) *Arithmetica*.Codex 48 includes (9) *De numeris datis*;(10) *Quedam experimenta super progressione numerorum*; and (11) *Liber de proportionibus*. Codex 59 includes a treatise called (12) *Suppletiones plane spere*.

Numbers (2) and (3) are obviously statices. The *De ratione ponderum* is probably the *De ratione ponderis* edited and translated by E. A. Moody in E. A. Moody and M. Clagett, *The Medieval Science of Weights (Scientia de ponderibus)* (Madison, Wis., 1952);its attribution to Jordanus has been questioned by Joseph E. Brown, *The Scientia de ponderibus in the Later Middle Ages* (ph.D. diss., University of Wis., 1967), pp. 64-66. In the same volume Moody has also edited and translated the *Elementa* Jordani super demonstrationem ponderum, a genuine work of Jordanus, which perphaps corresponds to the *De ponderum proportione of the Biblionomia*.

The *Philotegni, Or De triangulis*, exists in two versions. The longer, and apparently later, version was published by Maximilian Curtze, "Jordani Nemorarii Geometria vel De triangulis libri IV"in *Mitteilunge des Copperniucs-Vereins* Für Wissenschaft und Kunst zu Thorn **6** (1887), from MS Dresden, Sachsisc Landesbibliothek, Db 86, fols. 50r-61v. Utilizing additional MSS, Marshall Clagett reedited and translated Props. IV.16 (quadrature of the circle), IV.20 (trisection of an angle), and IV. **22** (finding of two mean proportionals) in his *Archimedes in the Middle Ages*, I, The Arabo-Latin Tradition (Madison, Wis., 1964), 572-575, 672-677, and 662-663, respectively A shorter version, which lacks Props. II.9-12, 14-16, and IV.10 and terminates at IV.9 or IV.11, has been identified by Clagett. Both versions wil be reedited by Clagett in vol. IV of his *Archimedes in the Middle Ages*. of the 17 MSS of the two versions which Clagett has found thus far, we may note, in addition to the Dresden MS used by Curtze, the following: Paris, BN lat. 7378A, 29r-36r; London, Brit. Mus., Sloane 285, 80r-92v; Florence, Bibl. Naz. Centr., Conv. Soppr. J. V. 18, 17r-29v; and London, Brit. Mus., Harley 625, 12r-130r.

The De *quadratura circuli* attributed to Jordanus as a separate treatise in Fournival's *Biblionomia* may be indentical with BK. IV, Prop. 16 of the *De triangulis*, which bears the title "To From a Square Equal to a Given Circle" (quoted above; for the Latin text, see Clagett, *Archimedes in the Middle Ages*, I, 572, 574). In at least one thirteenthcentury **MS** (Oxford, Corpus Christi College 251, 84v) the proposition stands by itself completely separated from the rest of the De triangulis, an indication that it may have circulated independently (for other MSS, see Clagett, *Archimedes*, I, 569).

The De numeris datis has been edited three times. It was first published on the basis of a single fourteenth-century MS, Basel F. II. 138v-145v, by P. Treutlein, "Der Traktat des Jordanus Nemorarius "De numeris datis" in *Abhandlungen zur Geschichte der Mathematik*, no. 2 (Leipzig, 1879), pp. 125-166. Relyin on MS Dresden Db86 supplemented by MS Dresden C80, Maximilian Curtze reedited the De numeris datis and subdivided it into four books in "Commentar zu dem 'Tractatus de numeris datis' des Jordanus Nemorarius "in Zeitschrift fü Mathematik und Physik, hist.-lit. Abt., **36** (1819), 1-23, 41-63, 81-95, 121-138. In MS Dresden C80 Curtze found additional propositions (IV.16-IV.35) beyond the concluding propositiopn in Treuleins's ed. The additional proposition included on proofs but only the enunciations of the propositions followed immediately by a single numerical example for each. That these extra propositions formed a genuine part of the *De numeris datis* was verified by MSS Vienna 4770 and 5303, which included not only the additional propositions but also their proofs. Using MS Vienna 4770, from which 5303 was copied, R. Daublebsky von Sterneck published complete versions of Props. IV. 15-IV.35 and also supplied corrections and additions to a few propositions in Bk., I in Zur Vervollstündigung der Ausgaben der Schrift des Jordanus Nemorarius: 'Tractatus de numerise datis'" in *Monatshefte für Mathematik und Physik*, **7** (1896), 165-179. A third ed., with the first English trans., has been completed by Barnabas Hughes: *The De numeris datis of Jordanus de Nemore, a Critical Edition, Analysis, Evaluation and Translation* (Ph.D.diss., Stanford University, 1970). Hughes's through study also includes pp.104) a history of previous editions, as well as a description of twelve MSS, whose relationships are discussed in detail.

A Russian translation from Curtze's edition was made by S. N.sreÍder, "The Beginnings of Algebra in Medieval Europe in the Treatise De numeris datis of Jordanus de Nemore" in *Istoriko-Matematicheskie issledovaniya*, **12** (1959), 679-688.

As yet there is no ed. of the ten-book *Arithmetica,* although the enunciations of the propositions were published by Jacques LefÈvre d'Etaples (Jacobus Faber Stapulensis), who supplied his own demonstrations and comments in *Arithmetica (lordani Nemorarii) decem libris demonstrata* (Paris, 1496, 1503, 1507, 1510, 1514). At least sixteen complete or partial *MSS* of it are presently known, among which are two excellent and complete thirteenthcentury *MSS*: Paris, *BN* 16644, 2r-93v; and Vat. lat., Ottoboni *MS* 2069, lr-51v.

The Latin text of the definitions and enunciations of the 34 propositions of the *Demonstratio Jordani de algorismo* were published by G. Eneström, "über die 'Demonstratio Jordani de algorismo,'" in *Bibliotheca mathematica,* 3rd ser., 7 (1906-1907), 24-37, from *MSS* Berlin, lat. 4° 510, 72v-77r (königliche Bibliothek, renamed Preussische Staatsbibliothek in 1918; the fate of this codex after World War II when the basic collection was divided between East and West Germany, is unknown to me) and Dresden Db 86, 169-175r The *Demonstratio* appears to be an altered version of a similar and earlier work beginning with the words "Communis et consuetus. . .," which Eneström called *Opus numerorum*. The Latin text of its introduction and a comparison of its propositions with those of the *Demonstratio Jordani* were published by Eneström as "über eine dem Jordanus Nemorarius zugeschriebene kurze Algorismusschrift," in *Bibliotheca mathematica,* 3rd ser., 8 (1907-1908). 135-153. He relied primarily on *MS* Vat. lat. Ottob. 309, 114r-117r, supplemented by *MSS* Vat. lat. Reg. Suev. 1268, 69r-71r; Florence, Bibl. Naz. Centr., Conv. Soppr. J. V.18 (cited by Eneström as San Marco 216, its previous designation), 37r-39r; "and Paris Mazarin 3642-96r 105r". Since the "Demonstratio Jor *dani was definitely" ascribed to Jordanus*, and the *Opus numerorum* seemed an earlier version of it, Eneström conjectured that the *Opus* was a more likely candidate for Jordanus' original work, while the *Demonstratio Jordani,* which omits most of the introduction but expands the text itself, may have been revised by Jordanus or someone else.

Each of these two treatises has associated with it a brief work, attributed in some *MSS* to Jordanus, on arithmetic operations with fractions. The treatise associated with the *Opus numerorum*, which Eneström, calls *Tractatus minutiarum* contains an introduction and 26 highly abbreviated propositions; the work on fractions associated with the *Demonstratio Jordani de algorismo,* which Eneström calls *Demonstratio de minutiis*, consists of an introduction and 35 propositions. Although the introductions difer, all 26 propositions of the *Tractatus minutia* have, according to Eneström, identical counterparts in the longer *Demonstratio de minutiis* In "Das Bruchrechnen des Jordanus Nemorarius," in *Bibliotheca mathematica* 3rd ser., 14 (1913-1914), 41-54, Eneström includes a list of *MSS* for both treatises (pp. 41-42), the Latin texts of the introductions the texts of the enunciations of the propositions, and analytic representations of the propositions, By analogy with his reasoning about the relations obtaining between the *Opus numerorum* and *Demonstratio Jordani de algorismo* Eneström conjectures that Jordanus is the auther of the *Tractatus minutiarum*, the briefer treatise associated with the *Opus numerorum*. One of the *MSS* is Bibl. Naz. centr., Conv. Soppr. J. V. 18. 39r-42r, which follows immediately after the *Opus numerorum* in the same codex cited above; correspondingly, *MS* Berlin, lat. 4 510, 72r-77r, of the *Demonstratio Jordani de algorismo* is followed immediately by a version of the *Demonstratio de* minutiis on fols. 77r-81v, a relation which also seems to obtain in Bibl. Naz. centr., Conv. soppr. *J. I.* 32, 113r-118v 118v-124r. Whether the two algorithm treatises and the two associated treatises on fractions bear any relation to works (5), (6) (7), or (10), cited above from the *Biblionomia,* has yet to be determined and may indeed be impossible to determine. The *Algorismus demonstratus* published in 1534 by J. Suchöner and formerly ascribed to Jordanus, was composed by a Master Gernardus, who is perhaps identical with Gerard of Brussels.

The Liber *de proportionibus* mentioned in the Biblionomia, is probably a brief work by Jordanus beginning with the words "Proportio est rei ad rem determinata secundum quantitatem habitude...." *A* seemingly complete *MS* of it is Florence, Bibl. Naz. centr. Conv. Soppr. *J.V.* 30, 8r-9v. Other *MSS* are listed in L. Thorndike and P. kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin* rev. ed. (Cambridge, Mass., 1963), col. 1139. The *Suppletiones plane spere* of the *Biblionomia* is probably a commentary on Ptolemy's *Planisphaerium*. According to G. Sarton, *Introduction to the History of Science* 3 vols. in 5 pts., II, pt. 2 (Baltimore, 1931), 614, it is "a treatise on mathematical astronomy, which contains the first general demonstration of the fundamental property of stereographic projection i.e., that circles are projected as circles (ptolemy had proved it only in special cases)" In Thorndike and Kibre op cit., Jordanus' Planisphaerium is listed under three separate and different incipits (see cols. 1119, 1524, and 1525, where *MSS* are lished for each). An edition appeared at Venice in 1558, under the title *Ptolemaei Planisphaerium: Iordani Planisphaerium; Federici Commandi Urbinatis in Ptolemaei Planisphaerium commentarius A* work on isoperimetric figures, *De figuris ysoperimetris*, is also attributed to Jordanus; *MSS* Florence, Bibl. Naz. Centr., Conv. Soppr. *J. V.* 30, 12v (a fragment) and Vienna 5203, 142r-146r, the latter actually copied by Regiomontanus who was also acquainted with Jordanus' *De triangulis, Planisphaerium, Arithmetica, De numeris datis* and *De proportionibus;* the enunciations of the eight propositions in the Vienna *MS* were published by Maximilian Curtze, "Eine Studienreise," in *Zentralblatt für Bibliothekswesen* 16 (1899), 264-265.

II. Secondary Literature. The most significant studies on Jordanus are monographic in character and have been cited above, since they are associated with editions and translations of his works. No general appraisal and evaluation of his scientific works has yet been published. To what has already been cited the following may be added; O. Klein, "Who was Jordanus Nemorarius?," in *Nuclear Physics* 57 (1964), 345-350; Benjamin Ginzberg, "Duhem and Jordanus Nemorarius," in *Isis* 25 (1936), 341-362, which seeks to refute Duhem's claims for medieval science and for Jordanus'statics in particular (Ginzberg seriously misread Duhem and was also ignorant of Jordanus' subsequent impact on later statics, believing mistakenly that all of it was rediscovered); M, Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis., 1959), ch.2, which is a summary of medieval contributions in statics, including source selections from the works of Jordanus; Joseph E. Brown, *The Scientia de ponderibus in the Later Middle Ages* (Ph. D. diss., University of Wis., 1967), which includes summaries and evaluations of the major principles in Jordanus'statical treatises and their subsequent influence in the commentary literature; and G., Wertheim, über die Lösung einiger Aufgaben in *De numeris daits*," in *Bibliotheca mathematica*, **1** (1900), 417-420. Additional bibliography is given in Sarton, *op. cit*., II, pt. 2, 614-616.

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