Koch, Helge von | Encyclopedia.com

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(*b*. Stockholm, Sweden, 25 January 1870; *d*. Stockholm[?], 11 March 1924)

mathematics.

Von Koch is known principally for his work in the theory of infinitely many linear equations and the study of the matrices derived from such infinite systems. He also did work in differential equations and in the theory of numbers.

The history of infinitely many equations in infinitely many unknowns is long; special cases of infinite systems were studied by Fourier, who used them naïvely in his celebrated *Théorie analytique de la chaleur;* and there are even earlier examples. Yet despite the many applications in differential equation theory and in geometry, the rigorous study of infinite systems began only in 1884-1885 with the publication by Henri Poincaré of a few special results.

Von Koch's interest in infinite matrices came from his investigations in 1891 into Fuchs's equation:

$D^n + p_2(\chi)\, D^{n-1} + \ldots + p_n(\chi)y = 0,$

were

all of which converge in some annulus *A* with center at the origin. It was known that there existed a solution

which also converged in *A*; but in order explicitly to calculate the coefficient b_k and the exponent ϱ, von Koch was led to an infinite system of linear equations. Here he used Poincare's theory, which forced him to assume some unnaturally restrictive conditions on the original equation.

To remove the restrictions, von Koch published another paper in 1892 which was concerned primarily with infinite matrix theory. He considered the infinite array or matrix

$A = \{A_{i\kappa} : i, \kappa = \ldots, -2, -1, 0, 1, 2, \ldots\}$

and set

$D_m = \det\{A_{i\kappa} : i, \kappa = -m, \ldots, m\}$

The determinant *D* of *A* was defined to be $\lim_{m\to\infty} D_m$ if this limit existed. He then noted that the same array could give rise to denumerably many different matrices—by the use of different systems of enumeration—each with a different main diagonal. He was, however, able to prove that if converged abosutely and also converged absolutely, then D existed and was independent of the enumeration of A. A matrix which satisfied the above hypotheses was said to be in normal form.

Various methods to evaluate *D* were then given by von Koch, all of them analogous to the evaluation of finite determinants. Minors of finite and infinite order were defined, and it was proved that *D* could be evaluated by the method of expansion by minors in a direct generalization to infinite matrices of the Laplace expansion. Finally, he showed that

Here, $a_{pq} = A_{pq} - \delta_{pq}(\delta_{j\kappa} = 1 \text{ if } j = \kappa, \delta_{j\kappa} = 0 \text{ if } j \neq \kappa)$; the largest summation index in each term is to range over all integers; and the others are to range over all integers as indicated. This is particularly interesting because it was the form used by Fredholm in 1903 to solve the integral equation

for the unknown function ϕ, the other functions being supposed known.

Von Koch then went on to prove that if *A* and *B* are in normal form, then the usual product matrix C = *ÁB* can be formed. The matrix C will also be in normal form and det C = (det *A*)(det *B*). He also was able to show that the property of being in normal form is not a necessary condition for *D* to exist and indicated how his theory could be extended to matrices whose entries are functions all analytic in the same disk.

Finally, von Koch applied his results to systems of infinitely many linear equations in infinitely many unknowns. Although he claimed a certain amount of generality, he actually considered only the homogeneous case

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Here the matrix $\{A_{ik}\}$ was supposed to be in normal form, and the only solutions sought were those for which $| x_\varkappa | \leq M$ for $k = -\infty, \ldots, \infty$. He then established that if det $\{A_{ik}\}$ is different from zero, then the only such solution for the above equation is $x_k = 0$ for $\kappa = -\infty \ldots, \infty$. He then showed that if $D = 0$ but $A_{i\kappa} \equiv 0$, there will always exist a minor of smallest order *m* which is not zero. Then if the nonvanishing minor is obtained from $\{A_{ik}\}$ by deleting columns $k_1, k_2, \ldots, k_m$, a solution $\{x_k\}$ can be obtained by assigning arbitrary values to $\chi\kappa_1, \chi\kappa_2 \ldots \chi\kappa_m$ and expressing each of the remaining χ_κ's as a linear combination of $\chi\kappa_1, \chi\kappa_2 \ldots, \chi\kappa_m$ This is similar to the finite case. Von Koch then asserted that analogous results could be obtained for unhomogeneous systems, which is now known to be false unless further restrictions are placed on $\{A_{i\kappa}\}$.

Von koch' work cannot be called pioneering. His results were all fairly readily accessible, although many of the calculations are lengthy. He was aware, through a knowledge of Poincaré's work, of the possibility of obtaining pathological results but did little to explore them. Yet this work can be said to be the first step on the long road which eventually led to functional analysis, since it provided Fredholm with the key for the solution of his integral equation.

BIBLIOGRAPHY

A complete bibliography of von Koch's papers is in *Acta mathematica*, **45** (1925), 345-348. Of particular interest is "Sur les déterminants infinis et les équations differentielles linéaires," in *Acta mathematica*, **16** (1892-1893), 217-295.

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