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(*b*. Luckenwalde, Germany 15 February 1882; *d*. Leipzig, Germany, 6 August 1945)

mathematics.

The son of Hermann Koebe, a factory owner, and Emma (née Kramer) Koebe, Paul Koebe attended a Realgymnasium in Berlin. He started university studies in 1900 in Kiel, which he continued at Berlin University (1900–1905) and Charlottenburg Technische Hochschule (1904–1905). He was a student of H.A. Schwarz; the other referee of his thesis was F.H. Schottky. His habilitation as a *Privatdozent* at Göttingen University took place in 1907. He was appointed a professor extraordinary at Leipzig University in 1910, a professor ordinary at Jena University in 1914, and again at Leipzig in 1926. His numerous papers are all concerned with one chapter of the theory of complex functions, which is best characterized by the headings' conformal mapping' and "uniformization." In fact, in 1907, simultaneously with and independently of H. Poincaré, he accomplished the long-desired uniformization of Riemann surfaces.

The strange story of uniformization has still to be written. Its origin is the parametrization of the algebraic curves (z, w) with $w^2 = (z-a_1)\ (z-a_2)\ (Z-a_3)\ (Z-a_4)$ by means of elliptic functions, achieved by Abel and Jocobi. The attempt to use abelian integrals for general algebraic curves of genus *p* in the same way as elliptic ones had been used in the case of genus one led Jacobi to reformulate the problem: parameterization of the *p*-th power of the Riemann surface by means of a *p*-tuple of functions of *p* variables-the Jacobi inversion problem, which was solved by B. Riemann. By Pointcaré #x2019;s intervention in 1881 and 1882, history took a quite unexpected turn. When studying differential equations, Poincaré discovered automorphic functions, which F. Klein was investigating at the same time. If in C the group *G* is generated by rotations with centers $a_1, a_2, \ldots, a_n+1$ and corresponding rotation angles $2\pi./k_1$ (integral $k_i= 1, \ldots, n$), an automorphic function F of G is easily constructed by Poincaré series. Such an F maps its domain conformably on a Riemann surface branched at the $F(a_i)$ with degrees k_i. Its inverse achieves the "uniformization" of the Riemann surface. Counting parameters and applying "continuity" arguments, Klein (1882) and Poincaré (1884) stated that the scope of this method included Riemann surfaces of all algebraic functions, although their proofs were unsatisfactory. It was not until 1913 that the "continuity method" of proof was salvaged by L.E.J. Brouwer.

At present, if a Riemann surface is to be uniformized, it is wrapped up with, rather than cut up into, a simply connected surface, which is then conformally mapped upon a standard domain (circular disk, plane, or plane closed at infinity), and in this framework automorphic functions are an a posteriori bonus. The idea of uniformizing the universal wrapping rather than the Riemann surface itself goes back to Schwarz. As early as the 1880's various methods were available to solve the boundary value problem of potential theory for simply connected (even finitely branched) domains, or, equivalently, to map them conformally upon the standard domain, as long as the boundary was supposed smooth, say piecewise analytic; and as early as 1886 Harnack's theorem had made convergence proofs for sequences of harmonic functions easy. Using these tools, uniformization could have been achieved in the 1880's were it not for the blockage of this access by automorphic functions.

In 1907 the lock was opened. Koebe and Poincaré simultaneously noticed that if an arbitrary simply connected domain (the universal wrapping of the Riemann surface) has conformally to be mapped on the standard domain, it suffices to exhaust it by an increasing sequence of smoothly bounded ones. As a matter of fact, because of Harnack's theorem, it was preferable to use Green's functions of the approximating domains, with the –singularity at P, which form an increasing sequence u_n. If the are bounded at *P*. it converges toward Green's function *u* for the prescribed domain, which together with its conjugate V solves the mapping problem by exp[-(u+ iv)]. If not (the case of mapping upon the whole plane) the goal is attained by noticing that all schlicht images of the unit circle by *f* with $f(0) = 0, f^1(0) = 1$ contain a circle with center $f(0)$ and a radius independent of *f* (this' Koebe constant' has been proved to be 1/4). The latter remark is a weak form of Koebe's distortion theorem. Which for *schlicht* mappings *f* of the unit circle states for $|z| < r$ an inequality of the form

with Q(r) independent of *f*

Koebe's most influential contribution to conformal mapping on the unit circle was his 1912 proof by *Schmiegung*, which has become so common that textbooks are silent about its authorship. It rests on the remark that $z \rightarrow z^2$ for $|z| \rightarrow 1$ increases the distances from the boundary, and consequently the square root reduces them. To use the square root univalently the given domain D is transformed by linear fractions such as to lie on one sheet of the square root surface, with the branching outside D. Then the square root operation brings the boundary of D nearer to that of the unit circle. This process is repeated with suitable branching points such as to deliver a sequence of mappings converging to that of the unit circle. This square root trick goes back to Koebe's 1907 paper; it seems that Carathéodory suggested it be applied that fundamentally.

Koebe’s mathematical style is prolix, pompous, and chaotic. He tended to deal broadly with special cases of a general theory by a variety of methods so that it is difficult to give a representative selective bibliography. Koebe’s life-style was the same; Koebe anecdotes were widespread in interbellum Germany. He never married.

BIBLIOGRAPHY

I. Original Works. See Poggendorff. The following papers by Koebe are particularly important:’ Über die Uniformisierung beliebiger analytischer Kurven,” in *Nachrichten von der Köninglichen Gesellschaft der Wissenschaftensenschaften zu Göttingen, Math.-Phys. Kl.* (1907), 191–210, 633–669; “über eine neue Methode der konformen Abbildung und Uniformisierung,” *ibid.* (1912). 844–848;’ Über die Uniformisierung der algebraischen Kurven, I, “in *Mathematische Annalen*, 67 (1909), 145–224;’ Allgemeine Theorie der Riemannschen Mannigfaltigkeiten, ’ in *Acta Mathematica*, **50** (1927), 27–157.

II. Secondary Literature. Ludwig Bieberbach, “Das Werk Paul Koebe,” and H. Cremer, ’ Erinnerungen an Paul Koebe.’ *in Jahresbericht der Deutschen Mathematiker-Vereinigung*.70 (1968). 148–158, 158–161; L. E. J. Brouwer. *Collected Works*. H. Freudenthal, ed., Ii (Amsterdam, 1976). 572–576. 583, 585–586; H. Freudental. “Poincaré et les fonctions automorphes,” in *Le livre du centenaire de la naissance de Henri Poincaré* (Paris, 1955), 212–219, and “A Bit of Gossip; Koebe,” *in Mathematical Intelligencer*, 6, no. 2 (1984), 77: Reiner Kühnau. “Paul Koebe und die Funktionentheorie,” in Herbert Beckert and Horst Schuman, eds., 100. *Jahre Mathematisches Seminar der Karl-Marx-universität* (Leipzig, 1981), 183–194; Otto Volk, “Paul Koebe,” in *Neue Deutsche Biographie*, XII (1980).

Hans Freudenthai.