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(*b*. Liegnitz, Germany [now Legnica, Poland], 7 December 1823; *d*. Berlin, Germany, 29 December 1891)

mathematics.

Kronecker's parents were Isidor Kronecker, a businessman, and his wife, Johanna Prausnitzer. They were wealthy and provided private tutoring at home for their son until he entered the Liegnitz Gymnasium. At the Gymnasium, Kronecker's mathematics teacher was E. E. Kummer, who early recognized the boy's ability and encouraged him to do independent research. He also received Evangelical religious instruction, although he was Jewish; he formally converted to Christianity in the last year of his life.

Kronecker matriculated at the University of Berlin in 1841. He attended lectures in mathematics given by Dirichlet and Steiner; in astronomy, by Encke; in meteorology by Dove; and in chemistry, by Mitscherlich. Like Gauss and Jacobi, he was interested in classical philology, and heard lectures on this subject. He also attended Schelling's philosophy lectures; he was later to make a thorough study of the works of Descartes, Spinoza, Leibniz, Kant, and Hegel, as well as those of Schopenhauer, whose ideas he rejected.

Kronecker spent the summer semester of 1843 at the University of Bonn, having been attracted there by Argelander's astronomy lectures. He also became acquainted with such democrats as Eduard Kinkel, and was active in founding a *Burschenschaft*, a student association. Kronecker's career might thus have been endangered by his political associations. The following autumn he went to Breslau (now Wroclaw, Poland) because Kummer had been appointed professor there. He remained for two semesters, returning to Berlin in the winter semester of 1844-1845 to take the doctorate.

In his dissertation, "On Complex Units," submitted to the Faculty of Philosophy on 30 July 1845, Kronecker dealt with the particular complex units that appear in cyclotomy. He thereby arrived at results and methods closely related to the theory of "ideal numbers" that Kummer was to propound a short time later. (In 1893 Frobenius, in a memorial address on Kronecker, compared this dissertation to a work of "chemistry without the atomic hypothesis.") In evaluating the dissertation, Dirichlet said that in it kronecker demonstrated "unusual penetration, great assiduity, and an exact knowledge of the present state of higher mathematics."

Kronecker took his oral examination on 14 August 1845. Encke questioned him on the application of the calculus of probabilities to observations and to the method of least squares; Dirichlet, on definite integrals, series, and differential equations; August Boeckh, on Greek; and Adolf Trendelenburg, on the history of legal philosophy. He was awarded the doctorate on 10 September.

Dirichlet, his professor and examiner, was to remain one of Kronecker's closest friends, as was Kummer, his first mathematics teacher. (On the occasion of the fiftieth anniversary of the latter's doctorate, in 1881, Kronecker said that Kummer had provided him with the "most essential portion" of his "intellectual life.") In the meantime, in Berlin, Kronecker was also becoming better acquainted with Eisenstein and with Jacobi, who had recently returned from Königsberg (now Kaliningrad, U.S.S.R.) for reasons of health. During the same period Dirichlet introduced him to Alexander von Humboldt and to the composer Felix Mendelssohn, who was both Dirichlet's brother in-law and the cousin of Kummer's wife.

Family business then called Kronecker from Berlin. In its interest he was required to spend a few years managing an estate near Liegnitz, as well as to dissolve the banking business of an uncle. In 1848 he married the latter's daughter, his cousin Fanny Prausnitzer; they had six children. Having temporarily renounced an academic career, Kronecker continued to do mathematics as a recreation. He both carried on independent research and engaged in a lively mathematical correspondence with Kummer; he was not ambitious for fame, and was able to enjoy mathematics as a true amateur. By 1855, however, kronecker's circumstances had changed enough to allow him to return to the academic life in Berlin as a financially independent private scholar.

This was a momentous time for mathematics in Germany. In 1855 Dirichlet left Berlin to go to Göttingen as successor to Gauss; Kummer succeeded Dirichlet in Berlin; and Carl Wilhelm Borchardt became editor of the *Journal für die reine und angewandte Mathematik*, following the death of its founder Crelle. In 1856 Weierstrass was called to Berlin and Kronecker and Kummer soon became friends with Borchardt and Weierstrass.

Although Kronecker had published some scientific articles before he returned to Berlin, he soon brought out a large number of mathematical tracts in rapid succession. Among other subjects he wrote on number theory (one of his earliest interests, instilled in him by Kummer), the theory of elliptical functions, algebra, and, particularly, on the interdependence of these mathematical disciplines. In 1860 Kummer, seconded by Borchardt and Weierstrass, nominated Kronecker to the Berlin Academy, of which he became full member on 23 January 1861.

In the winter semester of the following year Kronecker, at Kummer's suggestion, made use of a statutory right held by all members of the Academy to deliver a series of lectures at the University of Berlin. His principal topics were the theory of algebraic equations, the theory of numbers, the theory of determinants, and the theory of simple and multiple integrals. He attempted to simplify and refine existing theories and to present them from new perspectives. his teaching and his research were closely linked and, like Weierstrass, he was most concerned with ideas that were still in the process of development. unlike Weierstrass—and for that matter, Kummer—kronecker did not attract great numbers of students. Only a few of his auditors were able to follow the flights of his thought, and only a few persevered until the end of the semester. To those students who could understand him, however, Kronecker communicated something of his joy in mathematical discussion. The new ideas that he offered his colleagues and students often received their final formulation in the course of such scholarly exchanges. He was allowed a considerable degree of autonomy in his teaching at Berlin, so much so that when in 1868 he was offered the chair at Göttingen that had been held successively by Gauss, Dirichlet, and Riemann, he refused it.

Kronecker was increasingly active and influential in the affairs of the Academy, particularly in recruiting the most important German and foreign mathematicians for it. Between 1863 and 1886 he personally helped fifteen mathematicians in becoming full, corresponding, or honorary members, or in obtaining a higher degree of membership. The names of these men constitute a formidable catalog; they were, in the order in which Kronecker assisted them, Heine, Riemann, Sylvester, Clebsch, E. Schering, H. J. Stephen Smith, Dedekind, Betti Brioschi, Beltrami, C. J. Malmsten, Hermite, Fuchs, F. Carorati, and L. Cremona. The formal nominations that Kronecker made during this period are of great interest, not least because of their subjectivity. Thus, to give one example, in his otherwise comprehensive evaluation of Dedekind's work (1880), Kronecker, who was then seeking to reduce all mathematical operations to those dealing in positive whole numbers, ignored Dedekind's *Stetigkeit und irrationale Zahlen* of 1872.

Kronecker's influence outside Germany also increased. He was a member of many learned societies, among them the Paris Academy, of which he was elected a corresponding member in 1868, and the Royal Society of London, of which he became a foreign associate in 1884. He established other contacts with foreign scientists in his numerous travels abroad and in extending to them the hospitality of his Berlin home. For this reason his advice was often solicited in regard to filling mathematical professorships both in Germany and elsewhere; his recommendations were probably as significant as those of his erstwhile friend Weierstrass.

Kronecker's relations with Weierstrass had been disintegrating since the middle of the 1870's. They continued to work together, however,; in 1880, following Borchardt's death, Kronecker took over the editorship of the *Journal für die reine und angewandte Mathematik*, in which Weierstrass for a time assisted him. In 1883 Kummer retired from the chair of mathematics, and Kronecker was chosen to succeed him, thereby becoming the first person to hold the post at Berlin who had also earned the doctorate there. He was simultaneously named codirector of the mathematics seminar that Kummer and Weierstrass had founded in 1861. Kronecker continued to lecture, as he had done for twenty years, but now, as a member of the faculty, was able to assume all the rights thereof, including participation in the granting of degrees, the nomination of professors, and the qualifying examinations for university lectures. He was enabled, too, to sponsor his own students for the doctorate; among his candidates were adolf Kneser, Paul Stäckel, and Kurt Hensel, who was to edit his works and some of his lectures.

The cause of the growing estrangement between Kronecker and Weierstrass was the following. The very different temperaments of the two men must have played a large part in it, and their professional and scientific differences could only have reinforced their personal difficulties. Since they had long maintained the same circle of friends, their friends, too, became involved on both levels. A characteristic incident occurred at the new year of 1884-1885, when H. A. Schwarz, who was both Weierstrass'student and Kummer's son-in-law, sent Kronecker a greeting that included the phrase: "He who does not honor the Smaller [Kronecker], is not worthy of the Greater [Weierstrass]." Kronecker read this allusion to physical size—he was a small man, and increasingly self-conscious with age—as a slur on his intellectual powers and broke with Schwarz completely. (Other scholars, among them Hofmann and Helmholtz, maintained lasting good relations with Kronecker by displaying more tact toward his special sensitivities.)

At any rate, personal quarrel became scholarly polemic. Weierstrass, for example, believed (perhaps rightly) that Kronecker's opposition to Cantor's views on "transfinite numbers" reflected opposition to his own work.

The basis of Kronecker's objection to Weierstrass' methods of analysis is revealed in his well-known dictum that "God Himself made the whole numbers—everything else is the work of men." Kronecker believed that all arithmetic could be based upon whole numbers, and whole numbers only; he further classified all mathematical disciplines except geometry and mechanics as arithmetical, a category that specifically included algebra and analysis. He never actually stated his intention of recasting analysis without irrational numbers, however, and it is possible that he did not take his radical notions altogether seriously himself. Weierstrass could not afford to regard Kronecker's demands as merely whimsical; in 1885 he claimed indignantly that for Kronecker it was an axiom that equations could exist only between whole numbers, while he, Weierstrass, granted irrational numbers the same validity as any other concepts.

Kronecker's remarks that arithmetic could put analysis on a more rigorous basis, and that those who came after him would recognize this and thereby demonstrate the falseness of so-called analysis, angered and embittered Weierstrass. He saw in these words an attempt by Kronecker not only to invalidate his whole life's work, but also to seduce the younger generation of mathematicians to an entirely new theory. The two men were further at odds over a Swedish mathematics prize contest and over the editing of Borchardt's works. By 188, Weierstrass had confided to a few close friends that his break with Kronecker was complete; Kronecker, for his part, apparently did not realize how gravely his opinions and activities had wounded Weierstrass, since on several later occasions he still referred to himself as being his friend.

Weierstrass at this time even considered leaving Germany for Switzerland to avoid the constant conflict with Kronecker, but one consideration kept him in Berlin. Kronecker had remained on good terms with Kummer and with Kummer's successor, Fuchs; it was therefore likely that Kronecker would have considerable influence in the choice of Weier-strass' own successor. Weierstrass believed that all his work would be undone by a successor acceptable to Kronecker; for this reason he stayed where he was. In the meantime, new sources of antagonism arose, among them Weierstrass'scruple about the qualifications as a lecturer of kroneckker's protégé Hensel and Kronecker's stated objection to granting an assistant professorship to Weierstrass' pupil Johannes Knoblach. These new difficulties never reached a crucial point, however, since kronecker's wife died on 23 August 1891, and he survived her by only a few months.

Kronecker's greatest mathematical achievements lie in his efforts to unify arithmetic, algebra, and analysis, and most particularly in his work on elliptical functions. his boundary formulas are particularly noteworthy in this regard, since they laid bare the deepest relationships between arithmetic and elliptical functions and provided the basis for Erich Hecke's later analytic-arithmetical investigations. Kronecker also introduced a number of formal refinements in algebra and in the theory of numbers, and many new theorems and concepts. Among the latter, special mention should be made of his theorem in regard to the cyclotomic theory, according to which all algebraic numbers with Abelian and Galois groups (over the rational number field) are rational combinations of roots of unity. His theorem on the convergence of infinite series is also significant.

The most important aspects of Kronecker's work were manifest as early as his dissertation of 1845. In his treatment of complex units, kronecker sought to present a theory of units in an algebraic number field, and, indeed, to present a whole system of units as a group. Twenty-five years later he succeeded in constructing an implicit system of axioms to rule finite Abelian groups, although he did not at that time apply it explicitly to such groups. his work thus lay clearly in the line of development of modern algebra.

For this reason it might be useful to assess Kronecker's position with respect to other mathematicians. one criterion that suggests itself is the application of the algorithm, and while few mathematicians have held unalloyed opinions on this matter, two sharply differentiated positions may be distinguished. One group of mathematicians then—of whom Gauss, Dirichlet, and Dedekind are representative—found the algorithm to be most useful as a concept, rather than a symbol; their work centered on ideas, not calculations. The other group, which includes Leibniz, Euler, Jacobi, and—as kneser demonstrated—kronecker, stressed the technical use of the algorithm, employing it as a means to an end. Kronecker's goal was the perfection of the technique of calculation and he employed symbols to avoid the repetition of syllogisms and for clarity. he termed Gauss's contrasting method of presenting mathematics "dogmatic," although he retained a great respect for Gauss and for his work.

Kronecker's mathematics lacked a systematic theoretical basis, however, and for this reason Frobenius asserted that he was not the equal of the greatest mathematicians in the individual fields that he pursued. Thus, Frobenius considered Kronecker to be inferior to Cauchy and jacobi in analysis; to Riemann and Weierstrass in function theory; to Kummer and Dirichlet in arithmetic; and to Abel and Galois in algebra.

Kronecker was nevertheless preeminent in uniting the separate mathematical disciplines. Moreover, in certain ways—his refusal to recognize an actual infinity, his insistence that a mathematical concept must be defined in a finite number of steps, and his opposition to the work of Cantor and Dedekind—his approach may be compared to that of intuitionists in the twentieth century. Kronecker's mathematics thus remains influential.

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