Landen, John | Encyclopedia.com

(*b*. Peakirk, near Peterborough, England, 23 January 1719; *d*. Milton, near Peterborough, 15 January 1790)

mathematics.

Landen was trained as a surveyor and from 1762 to 1788 was land agent to William Wentworth, second Early Fitzwilliam. He lived a quiet rural life with mathematics as the occupation of his leisure, taking up those topics which caught his fancy. He contributed to the *Ladies' Diary* form 1744 and to the *Philosophical Transactions of the Royal Society*; he published his *Mathematical Lucubrations* in 1755 and the two-volume *Mathematical Memoirs* in 1780 and 1790; the latter volume was placed in his hands from the press the day before he died. He was elected a fellow of the Royal Society in 1766.

Landen wrote on dynamics, in which he had the temerity to differ with Euler and d'Alembert, and on the summation of series. He also tried to settle the arguments about the validity of limit processes used as a basis for the calculus by substituting a purely algebraic foundation.

Landen's name is perpetuated by his work on elliptic arcs (*Philosophical Transactions*, 1755). Giulio Carlo Fagnano dei Toschi had obtained elegant theorems about arcs of lemniscates and ellipses. Landen's development expressed the length of a hyperbolic arc in terms of lengths of arcs in two ellipses. The connection in size between these ellipses permits Landen's work to be seen as a relation between two elliptic integrals. In Legendre's notation, if

then in Landen's transformation

$F(\phi, k) = \frac{1}{2}(1 + k_1) F(\phi_1, k_1)$,

where, writing as usual, the new ϕ_1, k_1 are expressed in terms of ϕ, k by the relations

$\sin\phi_1=(1 +k')$, $\sin \phi\cos\phi$,

$k_1 =(1-k')/(1+k')$.

By considering an iterated chain of such transformations, Legendre obtained a method for the rapid computation of elliptic integrals, of which Gauss's method of the arithmetico-gemometric mean is another form. The Landen transformation can also be shown as a relation between elliptic functions; in the Jacobian notation,

$\mathrm{sn}\{(1+k') u, k_1\}=(1+k')\ \mathrm{sn}\ (u, k)\ \mathrm{cd}\ (u, k)$

An interest in integration, or "fluents," led Landen to discuss (*Philosophical Transactions*, 1760, and later) the dilogarithm

(the notation is modern). He obtained several formulas and numerical values that were found at almost the same time by Euler. In the first volume of the *Memoirs* he initiated discussion of the function (now sometimes called the trilogrithm)

deriving functional relations and certain numerical results, work followed up by Spence (1809) and Kummer (1840).

BIBLIOGRAPHY

I. Original Works. Landen's books are *Mathematical Lucubrations* (London, 1755); and *Mathematical Memoirs*, 2 vols. (London, 1780–1790). Articles are "A New Method of Computing the Sums of Certain Series," in *Philosophical Transaction of the Royal Society*, **51** , pt. 2 (1760), 553–565, and "An Investigation of a General Theorem for Finding the Length of Any Conic Hyperbola...," *ibid*., **65** , pt. 2 (1775), 283–289.

II. Secondary Literature. A short biography is C. Hutton, "John Landen," in *A Mathematical and Philosophical Dictionary*, II (London, 1795), 7–9. For the life and work of Fagnano and of Landen, see G. N. Watson, "The Marquis and the Land-Agent,"

in *Mathematical Gazette*, **17** (Feb. 1933). Landen's transformation is discussed in any standard text on elliptic functions. For the dilogarithm and its generalizations, see L. Lewin, *Dilogarithms and Associated Functions* (London, 1958).

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