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(*b*. Quimper, France, 22 November 1840; *d*. Paris, France, 21 December 1912)

*mathematics*.

Lemoine can be characterized as an amateur mathematician and musician whose work was influential in both areas. Like a number of other famous French mathematicians, he was educated at the École Polytechnique in Paris, from which he was graduated in 1860. He taught there but was forced to resign after five or six years because of poor health. Subsequently he was a civil engineer, and he eventually became chief inspector for the department of gas supply in Paris. His avocations remained mathematics and music.

In 1860, while he was still at the École Polytechnique, Lemoine and other teachers formed a chamber music group, nicknamed “La Trompette.” Camille Saint-Saëns wrote pieces for it.

Lemoine’s major mathematical achievements were in geometry. He and John Casey are generally credited with having founded the newer geometry of the triangle.

In 1873, at the meeting of the Association Française pour l’Avancement des Sciences held in Lyons, Lemoine presented a paper entitled “Sur quelues proprié tés d’un point remarquable du triangle.” In this paper he called attention to the point of intersection of the symmedians of a triangle and described some of its more important properties. He also introduced the special circle named for him.

The point of concurrence of the symmedians of a triangle is called the Lemoine point (in France), the Greble point (in Germany, after E. W. Grebe), or, most generally, the symmedian point. The last term was coined by the geometer Robert Tucker, of the University College School in London, in the interest of uniformity and amity. It is generally symbolized by *K*. The symmedian point had appeared in the work of geometers before Lemoine, but his was the first systematic exposition of some of its interesting properties. Lemoine’s concern with the problem of simplifying geometric construcitons led him to develop a theory of constructions, which he called geometrography. He presented this system at the meeting of the Association Francaise pour l’Avancement des Sciences that was held at Oran, Algeria, in 1888.

Briefly, Lemoine reduced geometric constructions to five elementary operations: (1) placing a compass end on a given point; (2) placing a compass end on a given line; (3) drawing a circle with the compass so placed; (4) placing a straightedge on a given point; (5) drawing a line one the straightedge has been placed. The number of times any one of these five operations was performed he called the “simplicity” of the construction. The number of times operation (1),(2),or(4) was performed he called the “exactitude” of the construction. By a suitable examination of the operations involved in a construction, it is usually possible to reduce the simplicity. For example, it is possible to reduce the simplicity of the construction of a circle tangent to three given circles (Apolloniu’s problem) from over 400 steps to 199 steps.

This system had a mixed reception in the mathematical world. It appears to help reduce the number of steps required for constructions but generally requires more geometrical sophistication and ingenuity on the constructor. It is now generally ignored.

Lemoine also wrote on local probability and on transformations involving geometric formulas. Concerning these transformations, he showed that it is always possible, by a suitable exchange of line segments, to derive from one formula a second formula of the same nature. Thus, from the formula for the radius of the incircle of a triangle, it is possible to derive formulas for the radii of the excircles of the same triangle.

His mathematical work ceased after about 1895, but Lemoine’s interest in the field continued. In 1894 he helped C. A. Laisant to found the periodical *Intermédiare des mathématiciens* and was its editor for many years.

Leomine’s reputation rests mainly on his work with the symmedian point. Briefly (see Figure 1), if, in triangle *ABC*, cevian *AM* is the median from vertex *A* and cevian *AD* is the bisector of the same angle, then cevian *AS*, which is symmetric with *AM* in respect to *AD*, is the symmedian to side *BC*. Among the results presented by Lemoine were that the three symmedians of a triangle are concurrent and that each symmedian divides the side to which it is drawn in the ratio of the squares of the other two sides.

If (see Figure 2) lines are drawn through the given symmedian point *K* parallel to the sides of the given triangle, these lines meet the sides of the triangle in six points lying on a circle. This circle is called the first Lemoine circle. The center of the circle is the midpoint of the line segment, *OK*, joining the circumcenter and symmedian point of the triangle. The distances of *K* from the sides of the triangle are proportional to the lengths of the sides. The line segments cut from the sides by the first Lemoine circle have lengths proportional to the cubes of the sides. If antiparallels are drawn to the sides through *K,* they meet the sides of the triangle in six concyclic points. The circle thus determined, called the second Lemoine circle, has its center at *K*. Since their introduction by Lemoine, many properties of the symmedian point and the Lemoine circles have been discovered.

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