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(*b*. St.-Omer, Pas-de-Calais, France, 24 March 1809;*d*. Paris, France, 8 September 1882),

mathematics.

Liouville is most famous for having founded and directed for almost forty years one of the major mathematical journals of the nineteenth century, the *Journal de Liouville*. He also made important contributions in pure and applied mathematics and exerted a fruitful influence on French mathematics through his teaching.

The few articles devoted to Liouville contain little biographical data. Thus the principal stages of his life and career must be reconstructed on the basis of original documentation. There is no exhaustive Hst of Liouville's works, which are dispersed in some 400 publications—the most nearly complete is that in the Royal Society's *Catalogue of Scientific Papers*. His work as a whole has been treated in only two original studies of limited scope, those of G. Chrystal and G. Loria. On the other hand, certain of Liouville's works have been analyzed in greater detail, such as those on geometry. In view of the limited space available, this study cannot hope to provide a thorough account of Liouville's work but will attempt instead to present its major themes.

Life. Liouville was the second son of Claude-Joseph Liouville (1772-1852), an army captain, and Thérèse Balland, both originally from Lorraine. Liouville studied in Commerey and then in Tout. In 1831 he married a maternal cousin, Marie-Louise Balland (1812-1880); they had three daughters and one son. Liouville lived a calm and studious life, enlivened by an annual vacation at the family house in Toul. His scientific career was disturbed only by a brief venture into politics, during the Revolution of 1848. Already known for his democratic convictions, he was elected on 23 April 1848 to the Constituent Assembly as one of the representatives from the department of the Meurthe. He voted with the moderate democratic party. His defeat in the elections for the Legislative Assembly in May 1849 marked the end of his political ambitions.

Admitted to the École Polytechnique in November 1825, Liouville transferred in November 1827 to École des Ponts et Chaussées, where, while prepari for a career in engineering, he began original research in mathematics and mathematical physics. Between June 1828 and November 1830 he presented before the Académie des Sciences seven memoirs, two of which dealt with the theory of electricity, three with the analytic theory of heat, and two with mathematical analysis. Although Academy reporters expressed certain reservations, these first works were on the whole very favorably received; and their partial publication in the *Annales de chimie et de physique*, in Gergonne's *Annales*,[1] and in Férussac's *Bulletin* gained their author a certain reputation. In order to secure as much freedom as possible to pursue his research, Liouville soon thought of changing professions. In 1830, upon graduating from the École des Ponts et Chaussées, he refused the position of engineer that he was offered, hoping that his reputation would permit him to obtain a teaching post fairly soon.

In November 1831 Liouville was selected by the Council on Instruction of the École Polytechnique to replace P. Binet as *répétiteur* in L. Mathieu's course in analysis and mechanics. This was the beginning of a brilliant career of some fifty years, in the course of which Liouville taught pure and applied mathematics in the leading Paris institutions of higher education.

In 1838 Liouville succeeded Mathieu as holder of one of the two chairs of analysis and mechanics at the École Polytechnique, a position that he resigned in 1851, immediately after his election to the Collège de France. From 1833 to 1838 Liouville also taught mathematics and mechanics, but at a more elementary level, at the recently founded École Centrale des Arts et Manufactures. In March 1837 he was chosen to teach mathematical physics at the Collège de France as *suppléant* for J. B. Biot. He resigned in March 1843 to protest the election of Count Libri-Carrucci to the chair of mathematics at that institution.

Liouville did not return to the Collège de France until the beginning of 1851, when he succeeded Libri-Carrucci, who had left France.[2] This chair had no fixed program, and so for the first time Liouville could present his own research and discuss current topics. He took advantage of this to present unpublished works, some of which were developed by his students before he himself published them.[3] Appreciating the interest and flexibility of such teaching, he remained in the post until 1879, when he arranged for O. Bonnet to take over his duties.

Liouville also wished to teach on the university level. Toward this end he had earned his doctorate with a dissertation on certain developments in Fourier series and their applications in mathematical physics (1836). He was therefore eligible for election, in 1857, to the chair of rational mechanics at the Paris Faculty of Sciences, a position vacant since the death of Charles Sturm. In 1874, he stopped teaching there and arranged for his replacement by Darboux.

Parallel with this very full academic career, Liouville was elected a member of the astronomy section of the Académie des Sciences in June 1839, succeeding M. Lefrançois de Lalande; and in 1840 he succeeded Poisson as a member of the Bureau des Longitudes. From this time on, he participated regularly in the work of these two groups. For forty years he also passionately devoted himself to another particularly burdensome task, heading an important mathematical journal. The almost simultaneous demise, in 1831, of the only French mathematical review, Gergonne's *Annales de mathématiques pures et appliquées*, together with one of the principal science reviews, Férussac's *Bulletin des sciences mathématiques, astronomiques, physiques et chimiques*, deprived French-language mathematicians of two of their favorite forums. Liouville understood that the vigor of French mathematical writings demanded the creation of new organs of communication. Despite his youth and inexperience in the problems of editing and publishing, he launched the *Journal de mathématiques pures et appliquées* in January 1836. He published the first thirty-nine volumes (the twenty of the first series [1836-1855] and the nineteen of the second series [1856-18741), each volume in twelve fascicles of thirty-two to forty pages. Finally, at the end of 1874, he entrusted the editorship to H. Résal.

Open to all branches of pure and applied mathematics, the publication was extremely successful and was soon called the *Journal de Liouville*. Its first volume contained articles by Ampère, Chasles, Coriolis, Jacobi, Lamé, V.-A. Lebesgue, Libri-Carrucci, Plücker, Sturm, and Liouville himself. Although not all the tables of contents arc as brilliant as that of the first, the thirty-nine volumes published by Liouville record an important part of the mathematical activity of the forty years of the mid-nineteenth century. In fact, Liouville secured regular contributions from a majority of the great mathematicians of the era and maintained particularly warm and fruitful relations with Jacobi, Dirichlet, Lamé, Coriolis, and Sturm. At the same time he sought to guide and smooth the way for the first works of young authors, notably Le Verrier, Bonnet, J. A. Serret, J. Bertrand, Hermite, and Bour. But his outstanding qualities as an editor are most clearly displayed in his exemplary effort to assimilate as perfectly as possible the work of Galois. He imposed this task on himself in order to establish in an irreproachable fashion the texts he published in volume 11 of the *Journal de mathématiques*.

During the first thirty years of his career Liouville, while maintaining a very special interest in mathematical analysis, also did research in mathematical physics, algebra, number theory, and geometry. Starting in 1857, however, he considerably altered the orientation of his studies, concentrating more and more on particular problems of number theory, Departing in this way from the most fruitful paths of mathematical research, Liouville saw his influence decline rapidly. Yet it was the abandonment of the editorship of his *Journal* in 1874 that signaled the real end of his activity. His publications, which had been appearing with decreasing frequency since 1867, stopped with decreasing frequency since 1867, stopped altogether at this time. Simultaneously he gave up his courses at the Sorbonne, where his *suppléants* were Darboux and then Tisserand. He still attended the sessions of the Académic des Sciences and of the Bureau des Longitudes and still lectured at the Collège de France; but he no longer really participated in French mathematical life.

Mathematical Works. Liouville published thirty-nine volumes of the *Journal de mathématiques*, republished a work by Monge, and published a treatise by Navier; but he never composed a general work of his own. He did write nearly 400 memoirs, articles, and notes on a great many aspects of pure and applied mathematics. Despite its great diversity, this literary output is marked by a limited number of major themes, the majority of which were evident in his first publications. Liouville also published numerous articles correcting, completing, or extending the results of others, especially of articles that had appeared in the *Journal de mathématiques*. Other articles by him were summaries of or extracts from courses he gave at the Collège de France.

The divisions used below tend to obscure one of the main characteristics of Liouville's works, their interdisciplinary nature. Yet they are indispensable in the presentation of such a diverse body of work. References generally will be limited to the year of publication; given that information, further details can rapidly be found by consulting the *Catalogue of Scientific Papers*.

Mathematical Analysis. It was in mathematical analysis that Liouville published the greatest number and the most varied of his works. Composed mainly from 1832 to 1857, they number about one hundred. It is thus possible to mention only the most important and most original of these contributions, including some that were virtually ignored by contemporaries. To grasp their significance fully, it must be noted that these apparently disordered investigations were actually guided—for lack of an overall plan—by a few governing ideas. It is also important to view each of them in the context of the studies carried out at the same time by Gauss, Jacobi, Cauchy, and Sturm. It should be remembered that in his courses at the Collège de France, Liouville treated important questions that do not appear among his writings and that he inspired many disciples.

Certain of Liouville's earliest investigations in mathematical analysis should be viewed as a continuation of the then most recent works of Abel and Jacobi. The most important are concerned with attempts to classify all algebraic functions and the simplest types of transcendental functions, with the theory of elliptic functions, and with certain types of integrals that can be expressed by and algebraic function.

Following the demonstration by Abel and Galois of the impossibility of algebraically solving general equations higher than the fourth degree, Liouville devoted several memoirs to determining the nature of the roots of algebraic equations of higher degree and of transcendental equations. One of the results he obtained was that according to which the number *e* cannot be the root of any second- or fourth-degree equation (1840). In 1844 he discovered a specific characteristic of the expansion as a continued fraction of every algebraic number, and showed that there are continued fractions that do not possess this characteristic. This discovery ("Liouville's transcendental numbers") is set forth in a memoir entitled "Sur des classes très-étendues de nombres

dom la valeur n'est ni algébrique, ni même réductible à des irrationnelles algébriques" (*Journal de mathématiques*, **16** [1851], 133-142). It was not until 1873 that G. Cantor demonstrated the existence of much more general transcendental numbers.

In 1844 Liouville took up the study of all the functions possessing—like the elliptic functions—the property of admitting two periods. He set forth his theory to Joachimsthal and Borchardt (1847) before presenting it at the Collège de France (1850-1851). Thus, although he published little on the subject, the theory itself rapidly became widely known through the publications of Borchardt and the *Théorie des foncttons doublement périodiques* (1859) of Briot and Bouquet. Liouville made important contributions to the theory of Eulerian functions. He also sought to develop the theory of differential equations and of partial differential equations. In addition to numerous memoirs treating particular types of differential equations, such as Riccati's equation, and various technical questions, Liouville also worked on general problems, such as the demonstration (1840) of the impossibility, in general, of reducing the solution of differential equations either to a finite sequence of algebraic operations and indefinite integrations or—independently of Cauchy—to a particular aspect of the method of successive approximations (1837-1838). His contributions in the theory of partial differential equations were also of considerable value, even though a large portion of his "discoveries" had been anticipated by Jacobi. Much of his research in this area was closely linked with rational mechanics, celestial mechanics, and mathematical physics.

Pursuing the pioneer investigations of Leibniz, Johann Bernoulli, and Euler, Liouville devoted a part of his early work (1832-1836) to an attempt to enlarge as for as possible the notions of the differential and the integral, and in particular to establish the theory of derivatives of arbitrary index. Assuming a function $f(x)$ to be representable as a series of exponentials

he defined its derivative of order s (s being an arbitrary number, rational or irrational, or even complex) by the series

This definition does in fact extend the ordinary differential calculus of integral indexes, but its generality is limited: not every function admits an expansion of the type proposed; and the convergence and uniqueness of the expansion are not guaranteed for all values of s. Despite its weaknesses and deficiencies, Liouville's endeavor, one of the many efforts that led to the establishment of functional calculus, shows his great virtuosity in handling the analysis of his time.

Liouville and Sturm published an important series of memoirs in the first two volumes of the *Journal de mathémaiiques* (1836-1837). Bourbaki (*Éléments d'histoire des mathématiques,* 2nd ed., pp. 260-262) shows that these investigations, which extended the numerous earlier works devoted to the equation of vibrating strings, permitted the elaboration of a general theory of oscillations for the case of one variable, He also shows that they are linked to the beginning of the theory of linear integral equations that contributed to the advent of modern ideas concerning functional analysis.

This constitutes only a broad outline of Liouville's chief contributions to analysis. The principal works mentioned below in connection with mathematical physics should for the most part be considered direct applications—or even important elements—of Liouville's analytical investigations.

Mathematical Physics. Under the influence of Ampère and of Navier, whose courses he had followed at the Collège de France and at the École des Ponts et Chaussées from 1828 to 1830, Liouville directed his first investigation to two areas of mathematical physics of current interest: the theory of electrodynamics and the theory of heat. After these first studies he never returned to the former topic. On the other hand he later (1836-1838, 1846-1848) devoted several memoirs to the theory of heat, but only for the purpose of employing new analytic methods, such as elliptic functions. Similarly, it was to demonstrate the power of analysis that he considered certain aspects of the theory of sound (1836, 1838) and the distribution of electricity on the surface of conductors (1846, 1857).

Celestial Mechanics. Liouville came in contact with celestial mechanics in 1834, through a hydrodynamical problem: finding the surface of equilibrium of a homogeneous fluid mass in rotation about an axis. He showed that with Laplace's formulas one can demonstrate Jacobi's theorem concerning the existence among these equilibrium figures of an ellipsoidal surface with unequal axes. Between 1839 and 1855 Liouville returned to this topic on several occasions, confirming the efficacy of the Laplacian methods while verifying the great power of the new procedures. From 1836 to 1842 he wrote a series of memoirs on classical celestial mechanics (perturbations, secular variations, the use of elliptic functions). Subsequently, although he was active in the Bureau des Longitudes, he devoted only a few notes to the problem of attraction (1845) and to particular cases of the three-body problem (1842, 1846).

Rational Mechanics, Rational mechanics held Liouville's interest for only two short periods. From 1846 to 1849, influenced by the work of Hamilton and Jacobi, he sought to erect a theory of point dynamics. He did not return to related subjects until 1855 and 1856, when he published, respectively, two studies of the differential equations of dynamics and an important note on the use of the principle of least action.

Algebra. Although Liouville wrote only a few memoirs dealing with algebraic questions, his contributions in this field merit discussion in some detail.

In 1836, with his friend Sturm, Liouville demonstrated Cauchy's theorem concerning the number of complex roots of an algebraic equation that are situated in the interior of a given contour, and in 1840 he gave an elegant demonstration of the fundamental theorem of algebra. From 1841 to 1847 he took up, on new bases, the problem of the elimination of an unknown

from two equations in two unknowns, and applied the principles brought to light in this case to the demonstration of various properties of infinitesimal geometry. In 1846 he presented a new method for decomposing rational functions, and in 1863 he generalized Rolle's theorem to the imaginary roots of equations.

Liouville's most significant contribution by far, however, was concerned with the theory of algebraic equations and group theory. He published very little on these questions, but from 1843 to 1846 he conducted a thorough study of Galois's manuscripts in order to prepare for publication, in the October and November 1846 issues of his *Journal de mathémaiiques,* the bulk of the work of the young mathematician who tragically had died in 1832. Liouville's brief "Avertissement" (pp. 381-384) paid fitting homage to the value of Galois's work, but the second part of the *Oeuvres*, announced for the following volume of the *Journal*, was not published until 1908 by J. Tannery. According to J. Bertrand, Liouville invited a few friends, including J. A. Serret, to attend a series of lectures on Galois's work.[4] Although he did not publish the text of these lectures, Liouville nevertheless contributed in large measure to making known a body of work whose extraordinary innovations were to become more and more influential. He thus participated, indirectly, in the elaboration of modern algebra and of group theory.

Geometry. Liouville's work in geometry, the subject of an excellent analysis by Chasles (*Rapport sur les progrès de la géométrie* [Paris, 1870], pp. 127-130), consists of about twenty works written over the period from 1832 to 1854. The majority of them are applications or extensions of other investigations and are more analytic than geometric in their inspiration.

In 1832 Liouville showed that his "calculus of differentials of arbitrary index" could facilitate the study of certain questions of mechanics and geometry. In 1841 and 1844 he demonstrated and extended, by employing elimination theory, numerous metric properties of curves and surfaces that were established geometrically by Chasles. In 1844 Liouville published a new method for determining the geodesic lines of an arbitrary ellipsoid, a problem that Jacobi had just reduced to one involving elliptic transcendentals. In 1846 he proposed a direct demonstration of the so-called Joachimsthal equation and generalized the study of polygons of maximal or minimal perimeter inscribed in or circumscribed about a plane or spherical conic section to geodesic polygons traced on an ellipsoid. Finally, in 1847, prompted by a note of W. Thomson concerning the distribution of elec-tricity on two conducting bodies, Liouville undertook an analytic study of inversive geometry in space, which he called "transformation by reciprocal vector rays." He showed that the inversion is the only conformal nonlinear spatial transformation and pointed out its applications to many questions of geometry and mathematical physics.

When Liouville published the fifth edition of Monge's *Application de l'analyse à la géométrie* (1850), he added to it Gauss's famous memoir *Disquisitiones generales circa superficies curvas*. He also appended to it seven important related notes of his own; these dealt with the theory of space curves, the introduction of the notions of relative curvature and geodesic curvature of curves situated on a surface, the integration of the equation of geodesic lines, the notion of total curvature, the study of the deformations of a surface of constant curvature, a particular type of representation of one surface on another, and the theory of vibrating strings.

Liouville returned to certain aspects of the general theory of surfaces in later works and in his lectures at the Collège de France—for instance, the determination of the surface the development of which is composed of two confocal quadrics (1851). Although they treat rather disparate topics, these works on infinitesimal geometry exerted a salutary influence on research in this field, confirming the fruitfulness of the analytic methods of Gauss's school.

Number Theory. Liouville entered the new field of number theory in 1840 with a demonstration that the impossibility of the equation $x^n + y^n = z^n$ entails that of the equation $x^{2n} + y^{2n} = z^{2n}$. He returned to this field only occasionally in the following years. publishing a comparison of two particular quadratic forms (1845) and a new demonstration of the law of quadratic reciprocity (1847). In 1856, however, he began an impressive and astonishing series of works in this area, abandoning—except for some brief notes on analysis—the important investigations he was carrying out in other branches of mathematics. At first he took an interest in quite varied questions, including the sum of the divisors of an integer; the impossibility, for integers, of the equation $(p - 1)! + 1 = p^m$, p being a prime number greater than 5; and the number $\phi(n)$ of integers less than n that are prime to n. From 1858 to 1865, in eighteen successive notes published in the *Journal de mathématiques* under the general title "Sur quelques formules générates qui peuvent être utiles dans la théorie des nombres," Liouville stated without demonstration a series of theorems—a list is given in volume II of L. E. Dickson, *History of the Theory of Numbers*—that constitute the foundations of "analytic" number theory.

But from 1859 to 1866 Liouville devoted the bulk of his publications—nearly 200 short notes in the *Journal de mathématiques*—to a monotonous series of particular problems in number theory. These problems are reducible to two principal types: the exposition of certain properties of prime numbers of a particular form and their products (properties equivalent to the existence of integral solutions of equations having the form $ax + by = c$, the numbers a, b, c being of a given form); and the determination of different representations of an arbitrary integer by a quadratic form of the type $ax^2 + by^2 + cz^2 + dt^2$, where $a, b, c,$ and d are given constants.

L. E. Dickson mentions the examples Liouville studied in this series and the demonstrations that were published later: Liouville's notes contain only statements and numerical proofs. One may well be astonished at his singular behavior in abandoning his other research in order to amass, without any real demonstration, detailed results concerning two specialized topics in number theory. He never explained this in his writings. Hence one may wonder, as did G. Loria, whether he may have justified his apparently mysterious choice of examples in the course of his lectures at the Collège de France. Bôcher, more severe, simply considers these last publications to be mediocre.

Four hundred memoirs, published mainly in the *Journal de maihématiques* and the *Comptes rendus hebdomadaires des séances de l'Académic des sciences,* and 340 manuscript notebooks[5] in the Bibliothèque de l'Institut de France, Paris—this is the mass of documents that must be carefully analyzed in order to provide an exhaustive description of Liouville's work. To complete this account, it would also be necessary to trace more exactly his activity as head of the *Journal de mathématiques,* to assemble the greatest amount of data possible on the courses he taught, above all at the Collège de France, and to ascertain his role in the training of his students and in their publications.

NOTES

1. This, Liouville's second publication—he was then twentyone—appeared in *Annales de mathématiques pares et appliquées* (**21,** nos. 5 and 6 [Nov. and Dee. 1830]). The second of these issues also contains two short notes by Evariste Galois, whose work Liouville published in *Journal de mathématiques pures et appliquées,***11** (1846). But this circumstance obviously is not sufficient to prove that the two knew each other. A note by Gergonne, following Liouville's memoir, severely criticizes its style and presentation. The editor of the *Annales* denied all hope of a future for the young man who, five years later, succeeded him by creating the *Journal de mathématiques*. See also Liouville's "Note sur l'électro-dynamique," in *Bulletin des sciences mathématiques* …,**15** (Jan. 1831), 29-31, which is omitted from all bibliographies.

2. In 1843 the election for the chair of mathematics left vacant by the death of S. F. Lacroix saw three opposing candidates, all members of the Académie des Sciences: Cauchy, Liouville, and Libri-Carrucci. At the time Liouville was engaged in a violent controversy with Libri-Carrucci, whose abilities he publicly disputed. He therefore resigned as soon as he learned that the Council of the Collège de France was recommending that the Assembly vote for Libri-Carrucci. In 1850, when this chair became vacant, following Libri-Carrueei's dismissal, Cauchy and Liouville again were candidates. The particular circumstances of the votes that resulted in Liouville's finally being chosen (18 Jan. 1851) gave rise to vigorous protests by Cauchy.

3. Thus the works on doubly periodic functions that Liouville had presented privately (1847) or in public lectures (1851) were used by Borchardt and Joachimsthal and also by Briot and Bouquet. Similarly, Lebesgue published in his *Exercices d'analyse numérique* (1859) Liouville's demonstration, unpublished until then, of Waring's theorem for the case of biquadratic numbers.

4. J. Bertrand (*Éloages académiques*. II [Paris, 1902], 342-343) wrote that it was out of deference to Liouville, who had taught him Galois's discoveries, that J. A. Serret mentioned these discoveries in neither the first (1849) nor the second edition (1859) of his *Cours d'algèbre supérieure,* but only in the third (1866).

5. A rapid examination of some of these notebooks shows that they consist mainly of outlines and final drafts of the published works, but the whole collection would have to be gone over very thoroughly before a definitive evaluation could be made.

BIBLIOGRAPHY

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Liouville published the first 39 vols, of the *Journal de mathématiques pures et appliquées:* 1st ser., **1** -**20** (1836-1855); 2nd ser., **1** -**19** (1856-1874). He also wrote approximately 400 papers, of which the majority are cited in the Royal Society's *Catalogue of Scientific Papers,* IV (1870), 39-49 (nos. 1-309 for the period 1829-1863, plus three articles written in collaboration with Sturm); VIII (1879), 239-241 (nos. 310-377 for the period 1864-1873); X (1894), 606 (nos. 378-380 for the period 1874-1880). See also Poggendorff, 1, cols. 1471-1475 and III, p. 818.

The list of Liouville's contributions to the *Journal de mathématiques* is given in the tables of contents inserted at the end of vol. **20** , 1st ser. (1855), and at the end of vol. **19** ., 2nd ser. (1874), (The articles in the *Journal de mathématiques* signed "Besge" are attributed to Liouville by H. Brocard, in *Intermédiaire des mathématiciens*, **9** [1902], 216.) The memoirs and notes that Liouville presented to the Académie des Sciences are cited from 1829 to July 1835 in the corresponding vols. of the *Procèsverbaux des séances de l'Académie des sciences* (see index) and after July 1835 in the vols. of *Tables des Comptes rendus hebdomadaires de l'Académie des sciences*, published every 15 years.

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Liouville's MSS are in the Bibliotheque de l'Institut de France, Paris. They consist of 340 notebooks and cartons of various materials, MS 3615-3640. See also the papers of Evariste Galois, MS 2108, folios 252-285.

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Interesting details and comments concerning various aspects of Liouville's work are in J. Bertrand, *Rapport sur les progrès les plus récents de l'analyse mathématique* (Paris, 1867), pp. 2-5, 32; M. Chasles, *Rapport sur les progrès de la géométric* (Paris, 1870), 127-140; N. Saltykow, "Sur le rapport des travaux de S. Lie à ceux de Liouville," in *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, **137** (1903), 403-405; M. Bôcher, "Mathématiques et mathématiciens français," in *Revue Internationale de l'enseignement*, **67** (1914), 30-31; F. Cajori, *History of Mathematics*, rev. ed. (New York, 1919), see index; L. E. Dickson, *History of the Theory of Numbers*, 3 vols. (Washington, D.C., 1919-1923), see index; and N. Bourbaki, *Éléments d'histoire des mathématiques*, 2nd ed. (Paris, 1964),260-262.

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