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(*b*. Lany, Bohemia [now Czechoslovakia], 29 May 1893;*d*. Stanford, California, 8 January 1968)

mathematics.

Loewner was the son of Sigmund and Jana Loewner. He studied mathematics with G. Pick at the German University of Prague and received the Ph.D. in 1917. From 1917 to 1922 he was an assistant at the German Technical University of Prague; from 1922 to 1928, assistant and *Privatdozent* at the University of Berlin; from 1928 to 1930, extraordinary professor at the University of Cologne; and from 1930 to 1939, full professor at Charles University in Prague, which he left when the Nazis occupied Czechoslovakia. From 1939 to 1944 he was lecturer and assistant professor at the University of Louisville, Kentucky; from 1945 to 1946, associate professor, and from 1946 to 1951, full professor at Syracuse University; and from 1951 until his retirement in 1963, full professor at Stanford University.

Loewner was married to Elizabeth Alexander, who died in 1955; they had one daughter. Of short stature, soft-spoken, modest, shy (but exceedingly kind to his acquaintances), he had a large number of research students, even after his retirement. His knowledge of mathematics was broad and profound, and included significant parts of mathematical physics. His originality was remarkable; he chose as his problems far from fashionable topics.

One idea pervades Loewner's work from his Ph.D. thesis: applying Lie theory concepts and methods to semigroups, and applying semigroups to unexpected mathematical situations. This led him in 1923 to a sensational result (4): the first significant contribution to the Bieberbach hypothesis. (A *schlicht* function $f(z) = \Sigma\, a_n z^n$ in the unit circle with $a_0 = 0$, $a_1 = 1$, has $| a_n | \leq n$—the case $n = 2$ was Bieberbach's and Loewner proved it for $n = 3$; in its totality the problem is still open.) In 1934 Loewner defined *n*th-order real monotonic functions by the property of staying monotonic if extended to *n*th-degree symmetric matrices (7) and characterized ∞th-order monotonic functions as functions which, analytically extended to the upper half-plane, map it into itself. The semigroups of first- and second-order monotonic mappings are infinitesimally generated; this property breaks down for orders greater than 2 (28, 32). The infinitesimally generated closed subsemigroup of monotonic mappings of infinite order is characterized by *schlicht* extensions to the upper half-plane (21). Loewner studied minimal semigroup extensions of Lie groups; for the group of the real projective line there are two: that of monotonic mappings of infinite order, and its inverse (21). In higher dimensions the question becomes significant under a suitable definition of monotony (30). Loewner also studied semigroups in a more geometrical context; deformation theorems for projective and Moebius translations (19), and infinitesimally generated semigroups invariant under the non-Euclidean or Moebius group (19), particularly if finite dimensionality and minimality are requested (22).

Among Loewner's other papers, many of which deal with physics, one should be mentioned explicitly: his non-Archimedean measure in Hilbert space (8), which despite its startling originality (or rather because of it) has drawn little attention outside the circle of those who know Loewner's work.

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