

# Löwenheim, Leopold | Encyclopedia.com

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(b. Krefeld, Germany, 26 June 1878; d. East Berlin, German Democratic Republic, 5 May 1957)

*mathematical logic.*

The son of Elise Röhn, a writer and Detmold Louis (Ludwig) Löwenheim, a mathematics teacher at the Krefeld Polytechnic until 1881 and a private scholar theater, Löwenheim received his secondary education in Berlin at the Königliche Luisen Gymnasium, from which he graduated in 1896. That same year he showed inclinations to philosophical and social thought by joining the Deutsche Gesellschaft für Ethische Kultur (to which his father had belonged). He studied mathematics and natural science from 1896 to 1900 at Friedrich Wilhelm University in Berlin and at the Technische Hochschule in Charlottenburg (then an autonomous neighboring city of Berlin). Having qualified in 1901 as a teacher of mathematics and physics in the upper grades, and of chemistry and mineralogy in the middle grades, Löwenheim was appointed *Oberlehrer* at the Jahn-Realgymnasium in Berlin in 1904, after a year of postgraduate training and a year as a probationary teacher. In 1919 he became *Studienrat*.

Although in subsequent years much of his time was taken up by teaching obligations, Löwenheim not only managed to revise and edit his father's unfinished work on Democritus (it was published in 1914) but also began his most fruitful period of study and research in mathematical logic, a field with which he had become acquainted through review articles and Ernst Scaröder's *Vorlesungen über die Algebra der Logik* (1891–1905). In 1906 Löwenheim joined the Berlin Mathematical Society, to which he read his first paper in 1907, and became a member of the reviewers' staff of the *Jahrbuch über die Fortschritte der Mathematik*. In spite of [World War I](#), during which Löwenheim served in France, Hungary, and Serbia from August 1915 to December 1916 (after which he returned to teaching in Berlin), it was between 1908 and 1919 that he published his most important papers on the algebra of logic, continuing and adding to the work of C. S. Peirce, Schröder and [Alfred North Whitehead](#).

In papers of 1908 and 1910, Löwenheim analyzed and improved upon the customary methods for solving equations in the calculus of classes or domains (that is, set theory in its Peirce-Schröderian setting) and proved what is now known as Löwenheim's general development theorem for functions of functions. The techniques employed admit of extension to the Peirce-Schröder calculus of relatives, a form of the logic of relations shown by Löwenheim in a paper of 1913, to be expandable into a calculus of  $m \times n$  "matrices of domains" similar to the now customary theory of matrices, especially regarding the representation of transformations by matrices. Löwenheim greatly simplified some of Whitehead's results on substitutions and proved that every theorem about transformations of any arbitrary object is valid if its validity can be shown for transformations of domains.

In his 1915 paper "Über Möglichkeiten im Realkalkül," Löwenheim reported three results considered classic today. He proved that in classic first order quantificational logic (with equality, but this is an unnecessary restriction), every formula without free individual variables that is satisfiable at all is already satisfiable in a denumerable (a finite or denumerably infinite) domain. The importance of this theorem of Löwenheim lies in its demonstration that one cannot "implicitly define" the domain of objects satisfying an axiom system formulated in first-order logic, or even the structure of such domains. For example, an axiom system for the real numbers (a nondenumerable domain, as George Cantor had shown in 1874) also has a denumerable model, and therefore at least two nonisomorphic models: the intended "standard" one consisting of the reals and the denumerable, "nonstandard" model shown to exist by Löwenheim's theorem.

This situation has been called the Löwenheim Skolem paradox, but Thoralf Skolem, who extended Löwenheim's theorem in 1920 and 1923, and developed general methods for constructing nonstandard models. Points out that the result was not paradoxical; rather, it indicated a limit to the characterizability of structure by formal systems and revealed the relativity of (for example, set-theoretical) concepts defined within them. Besides this result, the 1915 paper contained a decision procedure for monadic quantification logic (that is, with only one-place predicates) and a reduction of the decision problem for full quantification logic (that is of finding an algorithmic procedure that effectively decides between satisfiability or nonsatisfiability of an arbitrary formula of the theory) to the decision problem for the subtheory with one- and two-place predicates.

Löwenheim's later papers in mathematical logic have not so far proved to be of equal importance, and in fact his interests broadened to a more philosophical and existential outlook in which questions of mathematical education maintained a prominent place.

In 1931 Löwenheim married Johanna Rassmussen Teichert. His professional life came to an abrupt (but temporary) end in 1934, when he had to accept forced retirement as a 25 percent non-Aryan. He subsequently supported himself by teaching

eurythmy and geometry at the Anthroposophic School of Eurythmy in Berlin. A further blow was the loss of all his belongings, including 1,100 geometrical drawings and some geometrical models, and unpublished manuscripts on logic, geometry, music and the history of art, in the bombing of Berlin on 23 August 1943. Löwenheim managed to survive, however, and again taught mathematics at the Pestalozzi-Schule and the Franz-Mehring-Schule, both in Berlin-Lichtenberg, from 1946 to 1949. Professional logicians (among whom Paul Bernays, Heinrich Scholz, and [Alfred Tarski](#) had visited Löwenheim in the 1930's) were convinced that he had not survived the war.

It seems that Löwenheim did not know of the publication of his paper "On Making Indirect Proofs Direct" (translated from the German by Willard Van Orman Quine) in 1946. He left to his stepson, Johannes Teichert, some manuscripts to be used for instruction in schools of eurythmy and for the training of teachers in Waldorf schools, some autobiographical notes, and the sheep proofs of an unpublished sequel to his paper of 1915, written for publication in the 1939 volume of *Fundamenta mathemntticac*.

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