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(b. Warsaw, Poland, 25 September 1988; *d*. Grodzisk Mazowiecki, near Warsaw, 19 June 1945)

mathematics.

Mazurkiewicz was, with Zygmunt Janiszewski and Waclaw Sierpiński, a founder of the contemporary Polish mathematical school and, in 1920, of its journal *Fundamenta mathematicae*, which is devoted to set theory and to related fields, including topology and foundations of mathematics.

The son of a noted lawyer, Mazurkiewicz received his secondary education at the Iyceum in Warsaw. He passed his baccalaureate in 1907, studied mathematics at the universities of Cracow, Lvov, Munich, and Göttingen, and was awarded a Ph.D. in 1913 by the University of Lvov for his thesis, done under Sierpiński, on curves filling the square ("O krzywych wypelniajacych kwadrat"). Named professor of mathematics in 1915 at the University of Warsaw, he held this chair until his death. He was several times elected dean of the Faculty of Mathematical and Natural Sciences and, in 1937, protector of the University of Warsaw. He was a member of the Polish Academy of Sciences and Letters; of the Warsaw Society of Sciences and Letters, which elected him its secretary-general in 1935; of the Polish Mathematical Society, which elected him its president for the years 1933–1935; and member of the editorial boards of *Fundamenta mathematicae* and the *Monografie matematyozne* from their beginnings. His book on the theory of probability was written in Warsaw during the German occupation of Poland. The manuscript was destroyed in 1944 when the Germans burned and destroyed Warsaw before their retreat; it was partly rewritten by Mazurkiewicz and published in Polish eleven years after his death. Gravely ill, Mazurkiewicz shared the lot of the people of Warsaw. He died in the outskirts of the city during an operation for gastric ulcer.

Mazurkiewicz' scientific activity was in two principal areas: topology with its applications to the theory of functions, and the theory of probability. The topology seminar given by him and Janiszewski, beginning in 1916, was probably the world's first in this discipline. He exerted a great influence on the scientific work of his students and collaborators by the range of the ideas and problems in which he was interested, by the inventive spirit with which he treated them, and by the diversity of the methods that he applied to them.

As early as 1913 Mazurkiewicz gave to topology an ingenious characterization of the continuous images of the segment of the straight line, known today as locally connected continua. He based it on the notions of the oscillation of a continuum at a point and on that of relative distance; the latter concept, which he introduced, was shown to be valuable for other purposes. This characterization therefore differs from those established at about the same time by Hans Hahn and by Sierpiéski, which were based on other ideas. It is also this characterization that is linked with the Mazurkiewicz-Moore theorem on the arcwise connectedness of continua.

Mazurkiewicz' theorems, according to which every continuous function that transforms a compact linear set into a plane set with interior points takes the same value in at least three distinct points (a theorem established independently by Hahn), while every compact plane set that is devoid of interior point is a binary continuous image, enabled him to define the notion of dimension of compact sets as follows: the dimension of such a set *C* is at most *n* when *n* is the smallest whole number for which there exists a continuous function transforming onto *C* a nondense compact linear set and taking the same value in at most n + 1 distinct points of this set. This definition preceded by more than seven years that of Karl Menger and Pavel Uryson, to which it is equivalent for compact sets.

In a series of later publications Mazurkiewicz contributed considerably to the development of topology by means of solutions to several fundamental problems posed by Sierpiń; ski, Karl Menger, Paul Alexandroff, Pavel Uryson, and others, through which he singularly deepened our knowledge, especially of the topological structure of the Euclidean plane. In solving the problem published by Sierpiński (in *Fundamenta mathematicae*, **2** [1921], 286), he constructed on the plane a closed connected set which is the sum of a denumerable infinity of disjoint closed sets (1924) and which, in addition, has the property that all these summands except one are connected; at the same time he showed (independently of R. L. Moore) that on the plane the connectedness of all the summands in question is impossible, although, according to a result of Sierpiński's, it ought to be possible in space. Mazurkiewicz also solved, affirmatively, Alexandroff's problem (1935) on the existence of an indecomposable continuum (that is, one which is the sum of not fewer than 2^{80} subcontinua different from itself) in every continuum of more than one dimension; that of Menger (1929) on the existence, for every positive integer n, of weakly n-dimensional sets; and that of Uryson (1927) on the existence, for every integer n>1, of separable complete n-dimensional spaces devoid of connected subsets containing more than one point. He also showed (1929) that if R is a region in n-dimensional Euclidean space and E is a set of n—2 dimensions, then the difference R—E is always connected and is even a semicontinuum.

Mazurkiewicz also contributed important results concerning the topological structure of curves, in particular concerning that of indecomposable continua, as well as an ingenious demonstration, by use of the Baire category method, that the family of hereditarily indecomposable continua of the plane, and therefore that the continua of less paradoxical structure occur in it only exceptionally (1930).

By applying the same method to the problems of the theory of functions, Mazurkiewicz showed (1931) that the set of periodic continuous functions f, for which the integral diverges everywhere, is of the second Baire category in the space of all continuous real functions, and that the same is true with the set of continuous functions which are nowhere differentiable. In addition he provided the quite remarkable result that the set of continuous functions transforming the segment of the straight line into plane sets which contains Sierpiński's universal plane curve (universality here designating the presence of homeomorphic images of every plane curve) is also of the second Baire category. Among Mazurkiewiez' other results on functions are those concerning functional spaces and the sets in those spaces that are called projective (1936, 1937), as well as those regarding the set of singular points of an analytic function and the classical theorems of Eugène Roché, Julius Pál and Michael Fekete.

In the theory of probability, Mazurkiewicz formulated and demonstrated, in a work published in Polish (1922), the strong law of large numbers (independently of Francesco Cantelli); established several axiom systems of this theory (1933, 1934); and constructed a universal separable space of random variables by suitably enlarging that of the random variables of the game of heads or tails to a complete space (1935). These results and many others were included and developed in his book on the theory of probability.

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