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33-42 minutes

(*fl*. Athens and Cyzicus, middle of fourth century b.c.)

mathematics.

In the summary of the history of Greek geometry given by Proclus, derived at this point from Eudemus, it is stated that "Amyclas of Heraclea, one of the friends of Plato, and Menaechmus, a pupil of Eudoxus and associate of Plato, and his brother Dinostratus made the whole of geometry still more perfect".[1] There is no reason to doubt that this Menaechmus is to be identified with the Manaechmus who is described in the *Suda Lexicon* as "a Platonic philosopher of Alopeconnesus, or according to some of Proconnesus, who wrote works of philosophy and three books on plato's *Republic*".[2] Alopeconnesus was in the Thracian Chersonese, and Proconnesus (the Island of Marmara) was in the Propontis (the Sea of Marmara), no great distance from it; and both were near Cyzicus (Kapidăi Yarimadasi, Turkey), where Eudoxus look up his abode and where Helicon, another pupil, was born.[3] This dating of Menaechmus, about the middle of the fourth century b.c., accords with an agreeable anecdote reproduced by Stobaeus from the grammarian Serenus; when Alexander the Great requested Menaechmus to teach him geometry by an easy method, Menaechmus replied: "O king, for traveling through the country there are private roads and royal roads, but in geometry there is one road for all."[4] A similar story is told of Euclid and Ptolemy I;[5] but it would be natural to transfer it to the more famous geometer, and the attribution to Menaechmus is to be preferred. If true, it would suggest that Menaechmus was the mathematical tutor of Alexander. He could have been introduced to Alexander by Aristotle, who had close relations with the mathematicians of Cyzicus.[6] A phrase used by Proclus in two places—οἱ περἰ Μέναιχμον μαθηματικοί—implies that Menaechmus had a school;[7] and Allman has argued cogently that this was the mathematical school of Cyzicus, of which Eudoxus and Helicon (probably) were heads before him and Polemarchus and Callippus after him.[8]

According to Proclus, Menaechmus differentiated between two senses in which the word, στοιχεῖον, "element," is used.[9] In one sense it means any proposition leading to another proposition, as Euclid I.1 is an element in the proof of I.2, or I.4 is in that of I.5; and in this sense propositions may be said to be elements of each other if they can be established reciprocally—for example, the relation between the sum of the interior angles of a rectilineal figure and the sum of the exterior angles. In the second sense an element is a simple constituent of a composite entity, and in this sense not every proposition is an element but only those having a primordial relation to the conclusion, as the postulates have to the theorems. As Proclus notes, this is the sense in which "element" is used by Euclid, and Menaechmus may have helped to fix this terminology.

In another passage Proclus shows that many so-called conversions of propositions are false and are not properly called conversions, that is, not every converse of a proposition is true,[10] As an example he notes that every hexagonal number is triangular but not every triangular number is hexagonal, and he adds that these matters have not escaped the notice of the mathematicians in the circle of Menaechmus and Amphinomus.

In yet another passage Proclus discusses the division of propositions into problems and theorems.[11] While the followers of Speusippus and Amphinomus held that all propositions were theorems, the school of Menaechmus maintained that they were all problems but that there were two types of problems: at one time the aim is to find the thing sought, at another to see what some definite thing is, or to what kind it belongs, or what change it has undergone, or what relation it has to something else. Proclus considers that both schools were right; it might be argued with equal justice that both were wrong and that the distinction between theorem and problem is valid.

It is clear from these references that Menaechmus gave much attention to the philosophy and technology of mathematics. He must also have applied himself to mathematical astronomy, for Theon of Smyrna records that Menaechmus and Callippus introduced the system of "deferent" and "counteracting" spheres into the explanation of the movements of the heavenly bodies (οἵ τάς μέν φερονσας, τάς δέ ἀνελιττονσας εἰσηγήσαντο).[12] The terms mean that one of the spheres bears the heavenly body; the other corrects its motion so as to account for the apparent irregularities of their paths. Eudoxus was the first to devise a mathematical model to explain the motions of the sun and planets, and he did so by a highly ingenious system of concentric spheres, the common center being the center of the earth. The sun, moon, and planets were each regarded as fixed on the equator of a moving sphere; the poles of that sphere were themselves borne round on a larger concentric sphere moving about two different poles with a different speed; and so on. For the sun and moon Eudoxus postulated three spheres; for the planets, four. The modifications in this system made by Callippus are known in some detail. For example, he added one sphere for each planet except Jupiter and Saturn and two spheres for the sun and the moon—five in all. Nothing more is known of Menaechmus' contribution than what Theon relates, but he would appear to have been working on the same lines as Callippus. T. H. Martin conjectured that Menaechmus made his contribution in his commentary on Plato's *Republic* when dealing with the passage on the distaff of the Fates.[13]It is not, however, on these achievements but on the discovery of the conic sections that the fame of Menaechmus chiefly rests. Democritus had speculated on plane sections of a cone parallel to the base and very near to each other, [14] and other geometers must have cut the cone (and cylinder) by sections not parallel to the base; but Menaechmus is the first who is known to have identified the resulting sections as curves with definite properties.

The discovery was a by-product of the search for a method of duplicating the cube. Hippocrates had shown that this could be reduced to the problem of finding two mean proportionals between two lines, and Menaechmus showed that the two means could be obtained by the intersection of a parabola and a hyperbola. His solution is given in a collection of such solutions preserved by Eutocius in his commentary on Archimedes' *On the Sphere and the Cylinder*.[15] Another of the solutions, by Eratosthenes, is introduced by a letter purporting to be from Eratosthenes to Ptolemy Euergetes.[16] The letter is spurious, but it quotes a genuine epigram by Eratosthenes written on a votive pillar to which was attached a device for effecting the solution mechanically. The epigram included the lines:[17]

Try not to do the difficult task of the cylinders of Archytas, or to cut the cones in the triads of Menaechmus or to draw such a pattern of lines as is described by the god-fearing Eudoxus.

Proclus, in a passage derived from Geminus, also attributes the discovery of the conic sections to Menaechmus and cites a line from the verses of Eratosthenes in the form Μή δέ Μεναιχμίονς κωνοτμεῖν τριάδας.[18] He notes again in his commentary on Plato's *Timaeus* that Menaechmus solved the problem of finding two means by "conic lines" but says that he prefers to transcribe Archytas' solution.[19]

Eratosthenes' epigram implies not only that Menaechmus was aware of the conic sections but that he was aware of all three types and saw them as sections of a cone—that is, not as plane curves that he later identified with sections of a cone. The proof itself shows also that he knew the properties of the asymptotes of a hyperbola,[20] at least of a rectangular hyperbola, which is astonishing when it is remembered that Apollonius does not introduce the asymptotes until his second book, after the properties of the diameter and ordinates have been proved. There are no signs of any knowledge of the conic sections before Menaechmus, but with him it suddenly blossomed forth into full flower.[21]

The proof as we have it cannot reproduce the words of Menaechmus himself and no doubt has been recast by Eutocius in his own language, or by someone earlier.[22] It uses the terms παραβολή and ύπερβολή although these words were first coined by Apollonius; and we have the evidence of Geminus, as transmitted by Eutocius, that "the ancients" (οί παλαιοί) used the names "section of a right-angled cone" for the parabola, "section of an obtuse-angled cone" for the hyperbola, and "section of an acute-angled cone" for the ellipse.[23] This is undeniable evidence that at the time of "the ancients" the three curves were conceived as sections of three types of cone. But how ancient were "the ancients" Pappus gives a similar account to that of Geminus but says these names were given by Aristaeus;[24] and there is some reason to believe that the name used by Menaechmus for the ellipse was θνρεός, because its oval shape resembled a shield.[25] The question of name is not so important as the question behind it: whether Menaechmus discovered his curves as sections of cones or whether he investigated them as plane curves, which were only later (by Aristaeus?) identified with the curves obtained by plane sections of a cone. It will be necessary to return to this question later.

The term άσύμπτωτοι employed by Eutocius, would also not have been used by Menaechmus, who probably used the expression αί ἔγγιστα ενθεῖαι τῆς τον̂ ἀμβλνγωνίον κώνον τομῆς or simply αί ἔγγιστα which is found in Archimedes, who also employed the old names for the sections. Other terms that Menaechmus would not have used are α̉ξων, "axis," and όρθία πλενρά or *latus rectum*.

By way of introduction to Menaechmus' proof it may be pointed out that if x, y are two mean proportionals between a, b, so that

$$a : x = x : y = y : b,$$

then

$$x^2 = ay$$

and

$$xy = ab.$$

These are easily recognized today as the equations of a parabola referred to a diameter and a tangent at its extremity as axes and the equation of a hyperbola referred to its asymptotes as axes; the means may therefore be obtained as the intercepts on the axes of a point of intersection of a parabola and hyperbola, but Menaechmus had to discover *ab initio* that there were such curves and to ascertain their properties.

He proceeded by way of analysis and synthesis.

Suppose the problem solved. Let a, b be the given straight'lines and x, y the mean proportionals—where the letters both indicate the lines and are a measure of

their length—so that a : x = x : y = y : b. On a straight line *OY* given in position and terminating at *O,* let *ON* = y be cut off, and let there be drawn perpendicular to it at *N* the straight line *PN* = x. Because a : x = x : y, it follows that $ay = x^2$ or $a \cdot ON = x^2$ that is, $a \cdot ON = PN^2$, so that *P* lies on a parabola through *O*. Let the parallels *PM, OM* be drawn. Since *xy* is given, being equal to *ab*, $PM \cdot PN$ is also given; and therefore *P* lies on a hyperbola in the asymptotes *OM, ON*. *P* is therefore determined as the intersection of the parabola and hyperbola.

In the synthesis the straight lines *a, b* are given, and *OY* is given in position with *O* as an end point. Through *O* let there be drawn a parabola having *OY* as its axis and *latus rectum a*. Then the squares on the ordinates drawn at right angles to *OY* are equal to the rectangle contained by the *latus rectum* and the abscissa. Let *OP* be the parabola, let *OM* be drawn perpendicular to *OY*, and in the asymptotes *OM, OY* let there be drawn a hyperbola such that the rectangle contained byz the straight lines drawn parallel to *OM, ON* is equal to the rectangle contained by *a, b* (that is, PM. PN = ab). Let it cut the parabola at *P*.

and let the perpendiculars *PM, PN* be drawn. Then by the property of the parabola

$PN^2 = a \cdot ON.$

that is,

$a : PN = PN : ON,$

and by the property of the hyperbola

$ab = PN. PM = PN. ON.$

Therefore

$a : PN = ON : b,$

and

$a : PN = PN : ON = ON : b.$

Let a straight line *x* be drawn equal to *PN* and a straight line *y* equal to *ON*. Then *a, x, y, b* are in continuous proportion.

This solution is followed in the manuscripts of Eutocius by another solution introduced with the word "AλλωS, "Otherwise," in which the two means are obtained by the intersection of two parabolas.

In the figure, *AO, BO* are the two given straight lines, the two parabolas through *O* intersect at *P,* and it is easily shown that

$BO : ON = ON : OM = OM : OA,$

or

$a : x = x : y = y : b.$

The proof is established by analysis and synthesis as in the first proof, and it corresponds to the equations

$x^2 = ay$

$y^2 = bx.$

It has hitherto been assumed by all writers on the subject that this second proof is also by Menaechmus, but G. J. Toomer has discovered as proposition 10 of the Arabic text of Diocles' *On Burning Mirrors* a solution of the problem of two mean proportionals by the intersection of two parabolas with axes at right angles to each other, and with *latera recta* equal to the two extremes, which looks remarkably like the second solution;[26] and it is followed as proposition 11 by another solution which is identical in its mathematical content with that attributed to Diocles by Eutocius. Toomer believes that the second solution should therefore be attributed to Diocles, not to Menaechmus. A final judgment must await publication of his edition of Diocles, but it may at once be noted that there are differences as well as resemblances. In particular, in the Arabic text Diocles starts from the focus-directrix property of the parabola—of which Menaechmus shows no awareness—and in order to get his means deduces from it the property that the ordinate at any point is a mean proportional between the abscissa and the *latus* *rectum*. It could be that Diocles found his solution independently, or he may have made a conscious adaptation of Menaechmus' solution in order to start from the focus-directrix property.

C. A. Bretschneider first showed how Menaechmus could have investigated the curves, and his suggestion has been generally followed.[27] In a semicircle the perpendicular from any point on the circumference to the diameter is a mean proportional between the segments of the diameter. This property would have been familiar before Menaechmus, and Bretschneider thinks it probable that he would have sought some similar property for the conic sections. We know from Geminus, as transmitted by Eutocius, that "the ancients" generated the conic sections by a plane section at right angles to one side (generator) of the cone, getting different curves according to whether the cone was right-angled, obtuse-angled, or acuteangled.[28] If *ABC* is a right-angled cone and *DEF* is a plane section at right angles at *D* the generator *AC*, the resulting curve where the plane intersects the cone is a parabola. Let *J* be any point in *DE*, and through *J* let there be drawn a plane parallel to the base of the cone. It will cut the cone in a circle. Let it meet the parabola at *K*. The planes *DEF* and *HKG* are both perpendicular to the plane *BAC*, and their line of

intersection *JK* is thus perpendicular to the diameter *HG*. Therefore.

$JK^2 = HJ.\, JG = LD.\, JG = DJ.\, DM,$

because *JDG* and *DLM* are similar triangles. That is to say, the square on the ordinate of the parabola is equal to the rectangle contained by the abscissa and a given straight line (*latus rectum*), which is the fundamental property of the curve. Bretschneider demonstrates in similar manner the corresponding properties for the ellipse and hyperbola.

Despite Eratosthenes' epigram, the clear statement of Geminus, and the evidence of the early names, it has been doubted whether Menaechmus first obtained the curves as sections of a cone. Charles Taylor suggests that they were discovered as plane loci in investigations of the problem of doubling the cube.[29] In support he argues that Menaechmus used a machine for drawing conics, that in his solutions he uses only the parabola and hyperbola, and that the ellipse—the most obvious of the sections of a cone—is treated last by Apollonius; but he agrees that the conception of a conic as a plane locus was immediately lost. If it be the case that such names as "section of a right-angled cone" were introduced by Aristaeus after the time of Menaechmus, this raises a slight presumption that Menaechmus did not obtain the curves as sections of a cone; but it can hardly outweigh the evidence of Eratosthenes and Geminus.[30]

Allman believes that Menaechmus was led to his discovery by a study of Archytas' solution of the problem of doubling the cube. "In the solution of Archytas the same conceptions are made use of and the same course of reasoning is pursued, which, in the hands of his successor and contemporary Menaechmus, led to the discovery of the three conic sections."[31] This is more than likely. The brilliant solution of Archytas must have made a tremendous splash in the mathematical pool of ancient Greece.

If it be granted that Menaechmus knew how to obtain a hyperbola by a section of an obtuse-angled cone perpendicular to a generator, how did he obtain the rectangular hyperbola required for his proof? H. G. Zeuthen showed how this could be done.[32] In Figure 4, *TKC* is a plane section through the axis

of an obtuse-angled cone, *AP* is a perpendicular to the generator *TK* and a plane section through A parallel to the base meets *TC* at I. If P is the foot of an ordinate to the hyperbola with value y, then

$Y^2 = GP \cdot PH$

$= AP \cdot PQ$

where AP = x and A′P = x′.

The hyperbola will be rectangular if *A′A* = 2AL. The problem is therefore as follows: Given a straight line *A′A*, and *AL* along *A′A* produced equal to A′A | 2 to find a cone such that *L* is on its axis and the section through *AL* perpendicular to the generator through *A* is a rectangular hyperbola with *A′A* as transverse axis. That is to say, the problem is to find a point *T* on the straight line through *A* perpendicular to *A′A* such that *TL* bisects the angle that is the supplement of *A′TA*. Suppose that *T* has been found. The circle circumscribing the triangle *A′AT* will meet *LT* produced in some point *S*; and because the angle *A′AT* is right, *A′T* is its diameter. Therefore *A′SL* is right and *S* lies on the circle having *A′L* as its diameter. But

whence it follows that the segments *AS, A′S* are equal and *S* lies on the perpendicular to the midpoint of *A′A*. Therefore *S* is determined as the intersection of the perpendicular to the midpoint of *A′A* with the circle drawn on *A′L* as diameter; and if *SL* is drawn, *T*, the vertex of the cone, is obtained as the intersection of *SL* with the perpendicular to *A′A* at A.

Some writers, such as Allman, have doubted whether Menaechmus could have been aware of the asymptotes of a hyperbola;[33] but unless it is held that Eutocius rewrote Menaechmus' proof so completely that it really ceased to be Menaechmus, the evidence is compelling. It is easy to see (again following

Zeuthen) how in the case of a rectangular hyperbola Menaechmus could have deduced the asymptote property from the axial property without difficulty.[34]

Let *AA′* be the transverse axis of a rectangular hyperbola, and *CE*, *CE'* its asymptotes meeting at right angles at *C*. Let *P* be any point on the curve and N' the foot of the perpendicular to *AA′* (the principal ordinate). Let *PF*, *PF'* be drawn perpendicular to the asymptotes. Then

$CA^2 = CN^2$, by the axial property

$= CN \cdot NE - PN \cdot ND$

$= 2(\Delta CNE - \Delta PND)$

$= 2$ quadrilateral CDPE

$= 2$ rectangle CF′PF, since $\Delta PEF = \Delta CDF'$,

$= 2PF.\ PF'$

$PF.\ PF' =$ which is the asymptote property.

Alternatively,

$CA^2 = CN^2 - PN^2$

$= EN^2 - PN^2$

$= (EN - PN)(EN + PN)$

$= (EN - PN)(PN + NE')$

$= EP \cdot PE'$

, because is 45°.

The letter of the pseudo-Eratosthenes to Ptolemy Euergetes says that certain Delians, having been commanded by an oracle to double one of their altars, sent a mission to the geometers with Plato in the Academy. Archytas solved the problem by means of half-cylinders, and Eudoxus by means of the so-called curved lines. Although they were able to solve the problem theoretically, none of them except Menaechmus was able to apply his solution in practice—and Menaechmus only to a small extent and with difficulty.[35] According to Plutarch, Plato censured Eudoxus, Archytas, Menaechmus, and their circle for trying to reduce the doubling of the cube to mechanical devices, for in this way geometry was made to slip back from the incorporeal world to the things of sense.[36]

Despite this emphatic evidence, Bretschneider considers it doubtful whether Menaechmus had an instrument for drawing his curves.[37] He notes that it is possible to find a series of points on each curve but agrees that this is a troublesome method of obtaining a curve without some mechanical device. Allman develops this hint and believes that by the familiar Pythagorean process of the "application" Παϱαβολ**Σ** of areas, which later gave its name to the parabola, Menaechmus could have found as many points as he pleased—"with the greatest facility"—on the parabola $y^2 = px$; that, having solved the Delian problem by the intersection of two parabolas, he later found it easier to employ one parabola and the hyperbola $xy = a^2$, "the construction of which by points is even easier than that of the parabola"; and that this was the way by which in practice he drew the curves.[38] He also implies that this was what the pseudo-Eratosthenes and Plutarch had in mind. The evidence, however, seems inescapable that Menaechmus attempted to find some mechanical device for tracing the curves. Bretschneider's objection that no trace of any such instrument has survived is not substantial. Centuries later, Isidorus of Miletus is said to have invented a compass, διαβήτης for drawing the parabola in Menaechmus' first solution.[39] Every schoolchild knows, of course, how to draw the conic sections with a ruler, string, and pins;'[40] but this easy method was not open to Menaechmus, since it depends upon the focus-directrix property.

There is a possible solution to this dilemma, so simple that apparently it has not hitherto been propounded, although Health came near to doing so. In Eutocius' collection of solutions to the problem of doubling the cube is a mechanical solution attributed to Plato.[41] It is now universally agreed that it cannot be by Plato because of his censure of mechanical solutions, which fits in with his whole philosophy. M. Cantor, however, thought it possible that he worked it out in a spirit of contempt, just to show how easy such things were in comparison with the real business of the philosopher.[42] The lines between which it is desired to find two means are placed at right angles, as *AB, BC*. The dotted figure *FGLK* is an instrument in which a ruler *KL* moves in slots in the two vertical sides so as to be always parallel to the base *FG*. The instrument is moved so that *FG* is made to pass through *C*, and *f* lies on *AB* produced. The ruler is then moved so that *KL* passes through *A*.

If *K* does not then lie on *CB* produced, the instrument is manipulated until the four following conditions are all fulfilled: FG passes through *C; F* lies on *AB* produced; *KL* passes through *A; K* lies on *CB* produced. The conditions can be satisfied with difficulty— δυσχερῶς, as the pseudo-Eratosthenes says—and when it is done.

AB : BE = BE : BD = BD : BC,

so that *EB, BD* are the required means.

The arrangement of the extremes and the means in Figure 6 is exactly the same as in the second solution attributed to Menaechmus. "Hence," says Heath, "it seems probable that someone who had Menaechmus' second solution before him worked to show how the same representation of the four straight lines could be got by a mechanical construction as an alternative to the use of conics."[43] But why not Menaechmus himself? If he was the author, it would be easy for the tradition to refer it to his master, Plato. This cannot be proved or disproved, but it would be the simplest explanation of all the facts.

NOTES

1. Proclus, *In primum Euclidis*, G. Friedlein, ed. (Leipzig, 1873; repr. Hildesheim, 1967), 67.8–12. An English trans. is Glenn R. Morrow, *Proclus: A Commentary on the First Book of Euclid's Elements* (Princeton, 1970), 55–56. For Dinostratus see *Dictionary of Scientific Biography*, IV, 103–105.

2.*Suda Lexicon*, A. Adler, ed., M. No. 140, I, pt. 3 (Leipzig, 1933), 317–318. It is entirely in character that the *Suda* not only misspells Menaechmus' name but omits his most important achievement. The *Suda* is followed by Eudocia, *Violarium*, No. 665, H. Flach, ed. (Leipzig, 1880), p. 494.3–5.

3. Also Athenaeus of Cyzicus, if that is the correct interpretation of the name in Proclus, *op. cit.*, 67.16, as seems probable, but it could possibly be understood as Cyzicinus of Athens.

4. Stohaeus, *Anthologium*, C. Wachsmuth, ed., II (Leipzig, 1884), 228.30–33.

5. Proelus, *op, cit*., 68.13–17.

6. G. J. Allman, *Greek Geometry From Thales to Euclid* (Dublin-London, 1889), 154, n. 2, 179 and n. 42.

7.*Op. cit*., 78.9, 254.4.

8.*Op. cit*., 171–172.

9.*Op. cit*., 72.23–73.14. This passage is subjected to an elaborate analysis by Malcolm Brown in "A Pre-Aristotelian Mathematician on Deductive Order," in *Philosophy and Humanism: Essays in Honor of Paul Oskar Kristeller* (New York, in press). Brown sees Menaechmus as the champion of the relativity of mathematical principles and Aristotle as the champion of their absolute character.

10.*Ibid*., 253.16–254.5.

11.*Ibid*., 77.6–79.2.

12.*Liber de astronomia*, T. H. Martin, ed. (Paris, 1849; repr. Groningen. 1971), 330.19–332.3; *Expositio rerum mathematicarum ad legendum Platonem utilium*, E. Hiller, ed. (Leipzig, 1878), 201.22–202.2.

13.*Liber de astronomia*, "Dissertatio," 59–60; *Republic*, X, 616–617.

14. Plutarch, *De communibus nolitiis contra Stoicos* 39.3, 1079E, M. Pohlenz and R. Westman, eds., in *Plutarchi Moralia*, VI, fasc. 2 (Leipzig, 1959), 72.3–11. Plutarch writes on the authority of Chrysippus.

15.*Commenatarii in libros De sphaera et cylindro, in Archimedis opera omnia*, J. L. Heiberg, ed., 2nd ed., III (Leipzig, 1915; repr. Stuttgart, 1972), 54.26–106.24.

16. For Eratosthenes there is now available P. M. Fraser, *Eratosthenes of Cyrene* (London, 1971), which was the 1970 “Lecture on a Master Mind” of the British Academy.

17. Eutocius, *op. cit*., 96.16–19; διζήση0323; is the conjecture of Ulrich von Wilumowitz-Moellendorff for the solecism of the MS.

18.*Op. cit*., 111.20–23.

19.*In Platonis Timaeum ad* 32A, B, E. Diehl, ed. (Leipzig, 1914), pp. 33.29–34.4. The promise to transcribe Archytas is not redeemed in this work.

20. Allman, *op. cti*., is skeptical on this point. “Menaechmus may have discovered the asymptotes; but, in my judgement, we are not justified in making this assertion, on account of the fact, which is undoubted, that the solutions of Menaechmus have not come down to us in his own words” (p. 170). “There is no evidence, however, for the inference that Menaechmus … knew of the existence of the asymptotes of the hyperbola, and its equation in relation to them” (p. 177).

21. The first historian of Greek mathematics, J. E. Montucla, was deeply impressed by this fact. Writing of the proof by means of the parabola and hyperbola between asymptotes he notes, “Cette derniére montre même qu’on avoit fait à cette époque quelque chose de plus que les premiers pas dans cette théorie” *(Histoire des mathématiques*, I [Paris, 1758], 178). And again, “On ne peut y méconnoitre une théorie déjà assez scavante de ces courbes” (p. 183).

22. This appears to have been first recognized by N. T. Reimer, *Historia problematis de cubi duplicatione* (Göttingen, 1798), 68, n.

23.*Commentaria in Conica, in Apollonii Pergaei quae … exstant …*, J. L. Heiberg, ed., II (Leipzig, 1893), 168.17–170.27.

24.*Collectio*, F. Hultsch, ed., II (Berlin, 1877), VII.30–31, pp. 672.20–24, 674.16–19; Hultsch attributes the second passage to an interpolator.

25. In the Following passages of Greek authors θυϱεός and ἐ̈λλει 홍 ις are used interchangeably: Eutocius, *Commentaria in Conica*, Heiberg, ed., II, 176.6; Proelus, *In primum Euclidis*, Friedlein, ed., 103.6, 9, 10, 111.6 (citing Geminus), 126.19, 20–21, 22. The name appears also to have been familiar to Euclid, for in the preface to the *Phaenomena*, in *Euclidis opera omnia*, J. L. Heiberg and H. Menge, eds., VIII (Leipzig, 1916), 6.5–7, he says; “If a cone or cylinder be cut by a plane not parallel to the base, the section is a section of an acute-angled cone which is like a shield (θυϱεός).” From such passages Heiberg concluded that θυϱεός was the term used for the ellipse by Menaechmus (“Nogle Bidrag til de graeske Mathematikeres Terminologi,” in *Phitologisk-historiske Samfunds Mindeskrift*, XXVI [Copenhagen, 1879], 7; *Litterärgeschichtliche Studien über Euklid* [Leipzig, 1882], 88). The primary meaning of θυϱεός is “stone put against a door” (to keep it shut)—so H. G. Liddell and R. Scott, *A Greek-English Lexicon*, new ed., H. Stuart Jones (Oxford, 1940)—whence it comes to mean “oblong shield” (shaped like a door).

26. Dublin, Chester Beatty Library, Chester Beatty MS Arabic no. 5255, fols. 1–26.

27.*Die Geometric und die Geometer vor Euklides* (Leipzig, 1870), 157–158.

28.*Commentaria in conica*, Heiberg, ed., II, 168.17–170.18.

29.*Introduction to the Ancient and Modern Geometry of Conics* (Cambridge, 1881), xxxi, xxxiii, xliii.

30. There is a similar uncertainty about the term “solid loci” (στεϱεοι0333; τόποι)According to Pappus *(Collectio*, VII.30, Hultsch, ed., II, 672.21), Aristaeus wrote five books of *Solid loci* connected with (or continuous with) the *Conics*. This implies that “solid loci” were conics; and the name suggests that when it was given, the curves were regarded as sections of a solid, in contrast with “plane loci” such as straight lines and “linear loci,” which were higher curves. But there can be no certainty that the name is older than Aristaeus. T. L. Heath, *A History of Greek Mathematics*, II (Oxford, 1921), 117–118, gives an alternative explanation, deriving plane, solid, and linear loci from plane, solid, and linear problems; but he concedes that it would be natural to speak of the conic sections as solid loci, “especially as they were in fact produced from sections of a solid figure, the cone.”

31.*Op. cit*., 115. In detail he writes:

In each investigation two planes are perpendicular to an underlying plane; and the intersection of the two planes is a common ordinate to two curves lying one in each plane. In one of the intersecting planes the curve is in each case a semi-circle, and the common ordinate is, therefore, a mean proportional between the segments of its diameter. So far the investigation is the same for all. Now, from the consideration of the figure in the underlying plane—which is different in each case—it follows that:—in the first case—the solution of Archytas—the ordinate in the second intersecting plane is a mean proportional between the segments of its base, whence it is inferred that the extremity of the ordinate in this plane also lies on a semi-circle; in the second case—the section of the right-angled cone—the ordinate is a mean proportional between a given straight line and the abscissa; and, lastly, in the third case—the section of an acute-angled cone—the ordinate is proportional to the geometric mean between the segments of the base [p. 169].

32.*Die Lehre von den Kegelschnitten im Altertum*, R. von Fischer-Benzon, ed. (Copenhagen, 1886), repr. with foreword and index by J. E. Hofmann (Hildesheim, l966), 464–465. T. L. Heath, who followed Zeuthen's method in *Apollonius of Perga*, (Cambridge, 1896), xxvi-xxviii, gives a different method in *A History of Greek Mathematics*, 113–114. for determining *T*. He shows that *T* is on the circle which is the locus of all points such that their distances from A, A are in the ratio 3:1, and T is determined as the intersection of the perpendicular to A′A at A with this circle.

33. See n. 20.

34.*Op. cit*., 463–464.

35. Eutocius, *Commentarii in libros De sphaera et cylindro*, in *Archimedis opera omnia*, J. L. Heiberg, ed., III, 88.23–90.11. There are similar accounts of the Delian mission in other authors. Plutarch, *De genio Socratis*, 7, 579A-D, P. H. De Lacy and B. Einarson, eds., Loeb Classical Library (London—Cambridge, Mass., 1959), 396.17–398.22, says that Plato referred the Delians to Eudoxus of Cnidus and Helicon of Cyzicus; John Philoponus, *Commentary on the Posterior Analytics of Aristotle*, l.vii, 75bl2, M. Wallies, ed., *Commentaria in Aristotelem Graeca*, XIII. pt. 3 (Berlin, 1909), p. 102.7–18, is in general agreement but omits the references to the geometers. Theon of Smyrna, *Expositio*, E. Hiller, ed., 2.3–12, quoting a lost work of Eratosthenes entitled *Platonicus*, says the god gave this oracle to the Delians, not because he wanted his altar doubled but because he wished to reproach the Greeks for their neglect of mathematics and contempt for geometry. Plutarch also in another work, *De E apud Delphos*, c. 6, 386E, F. C. Babbitt, ed., Loeb Classical Library, Plutarch's *Moralia*, V (London-Cambridge, Mass., 1936), p. 210.6–11 agrees that the god was trying to get the Greeks to pursue geometry rather than to have his altar doubled.

36. Plutarch, *Quaestiones conviviales*, viii.2.1, 718F-F, E. L. Minar, W. C. Helmbold, and F. H. Sandbach, eds., Loeb Classical Library, *Plutarch's Moralia*, IX, trans. as *Table Talk* (London-Cambridge, Mass., 1961), pp. 120.20–122,7. The same censure of Eudoxus and Archytas is repeated in Plutarch, "Vita Marcelli," xiv.5–6, *Plutarch's Lives*, B. Perrin, ed., V, Loeb Classical Library (London-Cambridge, Mass., 1917; repr. 1961), pp. 470.17–472.6, but here there is no mention of Menaechmus.

37.*Op. cit*., 162.

38.*Op. cit*., 176–177.

39. [Eutocius], *Commentarii in libros De sphaera et cylindro*, in *Archimedis opera omnia*, Heiberg, ed., III, 84.7–11. The words are bracketed by Heiberg and are no doubt an interpolation made by one of the pupils of Isidorus, who revised Eudocius' text.

40. Charles Smith, *Geometrical Conics* (London, 1894). 32, 84, 125.

41. Eutoeius, *Commentarii in libros De sphaera et cylindro*, in *Archimedis opera omnia*, Heiberg, ed., III, 56.13–58.14.

42.*Vorlesungen über Geschichte der Mathematik*, 3rd ed., I (Leipzig, 1907), 234.

43. T. L. Heath, *A History of Greek Mathematics*, I (Oxford, 1921), 256.

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Menaechmus is known to have written a commentary on Plato's *Republic* in three books and other philosophical works, and he must have written at least one work in which he described his discovery of the conic sections. (Whether he wrote a separate book on the subject has been doubted, since Pappus, *Collectio*, F. Hultsch, ed., II [Berlin, 1877], VII 30, p. 672, does not mention any treatise on conics before those of Euclid and Aristaeus.) None of his works has survived. The fragments relating to his life and work are collected in Max C. P. Schmidt, "Die Fragmente des Mathematikers Menaechmus," in *Philologus*, **42** (1884), 77–81. Malcolm Brown (see below) believes that a passage in Proclus, *op. cit*., 72.23–73.9, may be a quotation from Menaechmus.

The most complete account of Menaechmus is still that of G. J. Allman, *Greek Geometry From Thales to Euclid* (Dublin–London, 1889), 153–179, reproducing an article which appeared in *Hermathena*, no. 12 (July 1886), 105–130. Other accounts to which reference may profitably be made are C. A. Bretschneider, *Die Geometric und die Geometer vor Eukleides* (Leipzig, 1870), 155–163; H. G. Zeuthen, *Keglesnitslaeren i Oldtiden* (Copenhagen, 1885), German trans. *Die Lehre von den Kegelschnitten im Alterturn*, R. von Fischer-Benzon, ed, (Copenhagen, 1886), repr. with foreword and index by J. E. Hofmann

(Hildesheim, 1966), 457–467; T. L. Heath, *Apollonius of Perga* (Cambridge, 1896), xvii-xxx and *A History of Greek Mathematics* (Oxford, 1921), I, 251–255, II, 110–116; J. L. Coolidge, *A History of the Conic Sections and QuadricSurfaces* (Oxford, 1945), 1–5; Malcolm Brown, "A Pre-Aristotelian Mathematician on Deductive Order," in *Philosophy and Humanism: Essays in Honor of Paul Oskar Kristeller* (New York, in press).

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