Menelaus Of Alexandria | Encyclopedia.com

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(*fl*. Alexandria and Rome, a.d. 100)

geometry, trigonometry, astronomy.

Ptolemy records that Menelaus made two astronomical observations at Rome in the first year of the reign of Trajan, that is, a.d. 98.[1] This dating accords with Plutarch’s choice of him as a character in a dialogue supposed to have taken place at or near Rome some lime after a.d.75.[2] He is called “Menelaus of Alexandria” by Pappus and Proclus.[3] Nothing more is known of his life.

The first of the observations that Ptolemy records was the occulation of the star Spica by the moon at the tenth hour in the night (that is, 4 a.m. in seasonal hours or 5 a.m. in standard hours) of the fifteenthsixteenth of the Egyptian month Mechir and its emergence at the eleventh hour.[4] In the second observation Menelaus noticed that at the eleventh hour in the night of 18–19 Mechir the southern horn of the moon appeared to fall in line with the middle and southern stars in the brow of Scorpio, while its center fell to the east of this straight line and was as distant from the star in the middle of Scorpio as the middle star was from the southern, and the northern star of the brow was occulted.[5] Both these observations took place in year 845 of the era of Nabonassar (reigned 747–734 b.c.). By comparing the position of the stars as observed by Timocharis in year 454 of the era of Nabonassar, Ptolemy (and presumably Menelaus before him) concluded that the stars had advanced to the east by 3°55’ in 391 years, from which he confirmed the discovery originally made by Hipparchus that the equinox was moving westward at the rate of 1° a century. (The true figure is 1° in about seventy-two years.) It was by comparing the position of Spica in his day with that recorded by Timocharis that Hipparchus had been led to postulate the precession of the equinoxes.

A list of works attributed to Menelaus is given in the register of mathematicians in the *Fihrist*(“Index”) of Ibn al-Nadim (second half of tenth century). His entry reads:[6]

He lived before Ptolemy, since the latter makes mention of him. He composed: *The Book on Spherical Propositions*. *On the Knowledge of the Weights and Distribution of Different Bodies*, composed at the commission of Oomilian.[7] Three books on the *Elements of Geometry*, edited by Thābit ibn Qurra. The *Book on the Triangle*. Some of these have been translated into Arabic.

From references by the Arabic writers al-Battānī (d. 929), al-Ṣūfi (d. 986), and Ḥajjī-Khalīfa it has been deduced that Menelaus composed a catalog of the fixed stars, but there is some uncertainty whether the observations that he undoubtedly made were part of a full catatog.[8]

According to Pappus, Menelaus wrote a treatise on the settings of the signs of the zodiac.[9] Hipparchus had shown “by numbers” that the signs of the zodiac take unequal times to rise, but he had not dealt with their settings. Menelaus appears to have remedied the omission.[10] The work has not survived, nor did Pappus redeem his promise to examine it later, not at least in any surviving writings.

The problem can be solved rigorously only by the use of trigonometry,[11] and it is on his contributions to trigonometry that the fame of Menelaus chiefly rests. The on of Alexandria noted that Hipparchus had treated chords in a circle in twelve books and Menelaus in six.[12] Almost certainly this means that Menelaus, like Hipparchus before him, compiled a table of sines similar to that found in Ptolemy. For the Greeks, if *AB* is a chord of a circle, sin *AB* is half the chord subtended by double of the arc *AB* and a table of chords is, in effect, a table of sines.

Menelaus’ work has not survive*d*. Menelaus’ major contribution to the rising science of trigonometry was contained in his *Sphaerica*, in three books. It is this work which entitles him to be regarded as the founder of spherical trigonometry and the first to have disengaged trigonometry from spherics and astronomy and to have made it a separate science. The work has not survived in Greek; but it was translated into Arabic, probably through a lost Syriac rendering, and from Arabic into Latin and Hebrew. There have been three printed Latin versions; and although it is debatable how much of them is due to Menelaus and how much to their editors, a modern study in German by A. A. Björnho and a critical edition of the Arabic text with German translation by Max Krause make the content of Menelaus’ work tolerably clear.[13]

Book I opens with the definition “A spherical triangle is the space included by arcs of great circles on the surface of a sphere,” subject to the limitation that “these arcs are always less than a semicircle.” This is the earliest known mention of a spherical triangle. Since the Arabic tradition makes Menelaus address a prince with the words, “O prince, I have discovered a splendid form of demonstrative reasoning,” it would appear that he was claiming originality. This is, indeed, implied in a reference by

Pappus, who, after describing how a spherical triangle is drawn, says, "Menelaus in his *Sphaerica* calls such a figure a *tripleuron*. [τρπλευρον]."[14] Euclid (in *Elements* I, defs. 19, 20) had used τρίπλευρον for plane rectilinear figures having three sides—that is, triangles —but in the body of his work, beginning with proposition 1, he regularly employed the term τργωνρον, "triangle." Menelaus' deliberate choice of *tripleuron* for a spherical triangle shows a consciousness of innovation.

In book I Menelaus appears to make it his aim to prove for a spherical triangle propositions analogous to those of Euclid for a plane triangle in *Elements* I. In proposition 11 it is proved that the three angles of a spherical triangle are together greater than two right angles. Menelaus did not always use Euclid's form of proof even where it can be adapted to the sphere, and he avoided the use of indirect proofs by *redactio ad absurdum*. Sometimes his treatment, as of the "ambiguous case" in the congruence of triangles (prop. 13), is more complete than Euclid's.

Book I is an exercise in spherics in the old sense of that term—the geometry of the surface of the sphere— and book II consists only of generalizations or extensions of Theodosius' *Sphaerica* needed in asironomy; the proofs, however, are quite different from those of Theodosius. It is in book III that spherical trigonometry is developed. It opens (prop. 1) with the proposition long since known as "Menelaus' theorem." This is best known from the proof in Ptolemy's *Syntaxis mathematica*, along with preliminary lemmas, but it is not there attributed by name to Menelaus.[15] According to the Arabic of Mansur ibn 'Iraq as contained in a Leiden manuscript, the proof runs:[16]

Between two arcs of great circles *ADB* and *AEC* let two other arcs of great circles intersect in *Z*. All four arcs are less than a semicircle. It is required to prove

Let *H* be the center of the circle and let *HZ HB, HE* be drawn. *AD* and *BH* lie in a plane and, if they are not parallel, let *AD* meet *BH* in the direction of *D* at *T*. Draw the straight lines *AKC, DLC*, meeting *HE* in *K* and *HZ* in *L*, respectively. Because the arc *EZB* is in one plane and the triangle *ACD* is in another plane, the points *K, L, T* lie on the straight line which is the line of their intersection. (More clearly, because *HB, HZ, HE*, which are in one plane, respectively intersect the straight lines *AD, DC, CA*, which are also in one plane, in the points *T, L, K*, these three points of intersection must lie on the straight line in which the two planes intersect.) Therefore, by what has become known as Menelaus' theorem in plane geometry (which is proved by Ptolemy, although not here).

But, as Ptolemy also shows.

, ,

and the conclusion follows.

Menelaus proceeds to prove the theorem for the cases where *AD* meets *HB* in the direction of *A* and where *AD* is parallel to *HB*. He also proves that

Björnbo observed that Menelaus proved the theorem in its most general and most concise form; Ptolemy proved only what he needed, and Theon loaded his pages with superfluous cases. But A. Rome challenged this view.[17] He considered that Ptolemy really covered all cases, that the completeness of Menelaus' treatment may have been due to subsequent amplification, and that Theon's prolixity was justified by the fact that he was lecturing to beginners.

In Ptolemy's *Syntaxis*, Menelaus' theorem is fundamental. For Menelaus himself it led to several interesting propositions, of which the most important is book III, proposition 5; it is important not so much in itself as in what it assumes. The proposition

is that if in two spherical triangles *ABG, DEZ*, the angles A, D are both right, and the arcs *AG, DZ* are each less than a quarter of the circumference,

from which may be deduced the modern formula

or

tab b= tan a cos C.

In the proof Menelaus casually assumes (to use modern lettering) that if four great circles drawn through any point *O* on a sphere are intersected in

A, B, C, D and *A', B', C', D'* by two other great circles (transversals), then

This is the anharmonic property, the property that the cross ratio or double ratio of the range (*A, D : B, C*) is unaltered by projection on to another great circle. There is, of course, a corresponding property for four concurrent lines in a plane cut by a transversal.

It is possible that Menelaus did not prove this property and the preliminary lemmas needed for book III, proposition 1, because he had done so in another work; but the balance of probability is that they were well known in his time and had been discovered by some earlier mathematician. The fact that Menelaus' theorem is proved, not as a proposition about a spherical triangle, but as a proposition about four arcs of great circles, suggests that this also was taken over from someone else. It would not be the first time that credit has been given to the publicist of a discovery rather than to the discoverer. If this is so, it is tempting to think that both Menelaus' theorem and the anharmonic property go back to Hipparchus. This conjecture is reinforced by the fact that the corresponding plane theorems were included by Pappus as lemmas to Euclid's *Porisms* and therefore presumably were assumed by Euclid as known.[18]

When Ptolemy in the former of his two references to Menelaus called him "Menelaus the geometer," [19] he may have had his trigonometrical work in mind, but Menelaus also contributed to geometry in the narrower sense. According to the *Fihrist*, he composed an *Elements of Geometry* which was edited by Thābit ibn Qurra (*d*. 901) and a *Book on the Triangle*. None of the former has survived, even in Arabic, and only a small part of the latter in Arabic;[20] but it was probably in one of these works that Menelaus gave the elegant alternative proof of Euclid, book I, proposition 25, which is preserved by Proclus.[21]

Euclid's enunciation is as follows: 'If two triangles have the two sides equal to two sides respectively, but have [one] base greater than the base [of the other], they will also have [one of] the angle[s] contained by the equal straight lines greater [than the other]." He proved the theorem by *redactio ad absurdum*. Menelaus' proof was direct and is perhaps further evidence of his distaste for indirect proofs already manifested in the *Sphaerica*. Let the two triangles be *ABC, DEF*, with *AB = DE, AC = DF*, and *BC EF*. From *BC* cut off *BG* equal to *EF*. At *B* make the angle GBH on the side of *BC* remote from *A* equal to angle *DEF*. Draw *BH* equal to *DE*. Join *HG* and produce *HG* to meet *AC* at *K*. Then the triangles *BGH, DEF* are congruent and *HG = DF = AC*. Now *HK* is greater than *HG* or *AC*, and therefore greater than *AK*. Thus angle *KAH* is greater than angle *KHA*. And since *AB = BH*, angle *BAH* = angle *BHA*. Therefore, by addition, angle *BAC* is greater than angle *BHG*, that is, greater than angle *EDF*

The *Liber trium fratrum de geometria*, written by Muḥammad, Aḥmad, and al-Ḥasan, the three sons of Mūsā ibn Shākir (Barū Mūsā) in the first half of the ninth century,[22] states that Menelaus' *Elements of Geometry* contained a solution of the problem of doubling the cube, which turns out to be Archytas' solution.

This bears on a statement by Pappus that Menelaus invented a curve which he called "the paradoxical curve" (γϱαμμή παϱάςοξος)[23]. Pappus, writing of the so-called "surface loci," says that many even more complicated curves having very remarkable properties were discovered by Demetrius of Alexandria in his *Notes on Curves* and by Philo of Tyana as a result of weaving together plektoids.[24] and other surfaces of all kinds. Several of the curves, he continues, were considered by more recent writers to be worthy of a longer treatment, in particular the curve called "paradoxical" by Menelaus.

If Menelaus really did reproduce Archytas' solution, which relies on the intersection of a tore and a cylinder, this lends support to a conjecture by Paul Tannery that the curve was none other than Viviani's curve of double curvature.[25] In 1692 Viviani set the learned men of Europe the problem "how to construct in a hemispherical cupola four equal windows such that when these areas are taken away, the remaining part of the curved surface shall be exactly capable of being geometrically squared." His own solution was to take through *O*, the center of the sphere, a diameter *BC* and to erect at *O* a perpendicular *OA* to the plane *BDCO*. In the plane *BACO* semicircles are described on the radii *BO*, *CO*, and on each a right half-cylinder is described. Each half-cylinder will, of course, touch the sphere internally; and the two half-cylinders will cut out of the hemispherical surface the openings *BDE*, *CDF* with corresponding openings on the other side. The curve in which the half-cylinders

intersect the hemisphere is classified as a curve of the fourth order and first species, and it is a particular case of the *hippopede* used by Eudoxus to describe the motion of a planet. The portion left on the hemispherical surface is equal to the square on the diameter of the hemisphere, and Tannery conjectures that the property of this area being squarable was considered at that time, when the squaring of the circle was much in the air, to be a paradox. It is an attractive conjecture but incapable of proof on present evidence.

According to several Arabic sources[26] Menelaus wrote a book on mechanics, the title of which was something like *On the Nature of Mixed Bodies*.[27] This is presumably to be identified with the unnamed work by Menelaus on which al-Kĥziñ draws in his Kiẗb m̃z̃n al-hikma ("Book of the Balance of Wisdom," 1121/1122), The fourth chapter of the first book quotes theorems by Menelaus respecting weight and lightness; the first chapter of the fourth book describes Archimedes' balance on the evidence of Menelaus; and the second and third chapters of the same book describe the balance devised by Menelaus himself and his use of it to analyze alloys, with a summary of the values he found for specific gravities.[28]

NOTES

1.*Syntaxis mathematica*, VII, 3, in *Claudii Ptolemaei opera quae exstant omnia*, J. L, Heiberg, ed., **I** , pt. 2 (Leipzig, 1903), pp. 30.18–19, 33.3–4.

2. Plutarch, *De facie quae in orbe lunae apparet*, 17, 930A, H. Cherniss and William C. Helmbold, eds., in *Moralia*, Loeh Classical Library, XII (London-Cambridge, Mass., 1957), 106.7–15. Lucius is the speaker and says, "In your presence, my dear Menelaus, I am ashamed to confute a mathematical proposition, the foundation, as it were, on which rests the subject of

catoptrics. Yet it must be said that the proposition, “All reflection occurs at equal angles,” is neither self-evident nor an admitted fact.” Menelaus is not allowed by Plutarch to speak for himself, and it would be rash to assume from this reference that he made any contribution to optics. Cherniss thinks that “the conversation was meant to have taken place in or about Rome some time—and perhaps quite a long time—after a.d. 75” (p. 12).

3. Pappus, *Collectio* VI.110, F. Hultsch, ed., II (Berlin, 1877), p. 102.1; Proclus, *In primum Euclidis*, G. Friedlein, ed. (Leipzig, 1873; repr. Hildesheim, 1967), 345.14; English trans., G. R. Morrow (Princeton, 1970).

4. Ptolemy, *op. cit*., 30.18–32.3.

5.*Ibid*., 33.3–34.8.

6. Heinrich Suter, “Das Mathematiker Verzeichniss im *Fihrist* des Ibn Ab̂ Ja′k̂b an-Nadim (Muhammad Ibn Ish̄k),” in *Abhandlungen zur Geschichte der Mathematik*, no. 6 (Leipzig, 1892), 19.

7. This is unlikely to be correct and is probably an embroidering of the reference to Trajan in Ptolemy.

8. A. A. Björnbo, “Hat Menelaus einen Fixsternkatalog verfasst?” in *Bibliotheca mathematica*, 3rd ser., **2** (1901), 196 – 212.

9. Pappus, *op. cit*., VI.110, vol. II, 600.25–602.1.

10. This at least is what the text of Pappus as we have it implies, but there is some reason to doubt whether the text can be correct. See Hultsch’s note at the point.

11. The inequality of the times was already known to Euclid, *Phaenomena, Euclidis opera omnia*, J. L. Heiberg and H. Menge, eds., VIII (Leipzig, 1916), props. 9, 12, 13, pp, 44, 62, 78: and Hypsicles (q.v.) attempted to calculate the times by an arithmetical progression. When Hipparchus is said to have solved the problem “by numbers,” it presumably means that he was the first to have given a correct solution by trigonometrical methods.

12.*Comaentary on the Syntaxis mathematica of Ptolemy*, A. Rome, ed., in the series Studi e Testi, LXXII (Vatican City, 1936), I.10, p. 451.4–5. For further discussion see A. Rome, “Premiers essais de trigonoḿtric rectiligne chez les Grecs,” in *L’intiquité classique*, **2** , fasc. I (1933), 177 192; and a brief earlier note by the same author with the same title in *Annales de la Société scientifique de Bruxelles*, ser. A, **52** , pt. 1 (1932), 271–274.

13. The trans. and eds. are summarized by George Sarton, *Introduction to the History of Science*, 1 (Baltimore, 1927; repr. 1968), 253–254; and are more fully examined by A. A. Björnbo, *Studien über Menelaos’ Sphärik* (Leipzig, 1902), 10–22, and Max Krause, *Die Sphärik von Menelaos aus Alexandrien* (Berlin, 1936), 1–116. See also the bibliography at the end of this article.

14. Pappus, *op. cit*., VI.1, p. 476.16–17. This is part of the evidence for the genuineness of the definitions even though thes do not appear in Gerard’s Latin trans.

15. Ptolemy, *Syntaxis mathematica*, I.13, J. L. Heiberg, ed., I, pt. I (Leipzig, 1898), pp. 68,14 76–9. See also the comaentary of Theon of Alexandria with the valuable notes of A. Rome, ed., *Comaentaires de Pappus et de Th́on d’ Alexndrie sur l’Almageste*, II, *Th́on d’ Alexandrie*, which is Studi e Testi, LXXII (Vatican City, 1936), 535–570.

16. A. A. Björnbo, Studien, 88–92. Menelaus omits a general enunciation (πϱότασιζ) and goes straight to the particular enunciation (∊̓ϰθ∊σιζ). Björnbo (p, 92) regards this as partial evidence that the proposition was taken from some other work; but Rome, “Les explications de Th́on d’Alexandrie sur le th́orème de Menelas,” in *Annales de la Socit́ scientifique de Bruxelles*, ser. A, **53** , pt. 1 (1933), 45, justly says that the length and complexity of a general enunciation, as given by Theon writing for his pupils, is a sufficient reason for the omission.

17. Björnbo, *Studien*, **92** . A. Rome, “Les explications de Th́on d’Alexandrie sur la th́orḿe de Ḿńlas” (see n. 16), 39–50; and *Comaentaires de Pappus et de Th́on d’ Alexandrie sur l’ Almageste*, II, 554, n. 1 (L’on est tenť de conclure que le compĺment de preuvc ̓tablissant le th́orème de Ḿńias pour tous les cas, a ̓t́ invenť a line date sitúe entre Th́on et les auteurs arabes qui nous font connaitre les Sph́riques.”)

18. Pappus, *op. cit*., VII, 3–19, props. 129, 136, 137, 140, 142, 145, Hultsch ed., vol. II, pp. 870.3–872.22, 880.13–882.16, 882.17–884.9, 886.23–888.8, 890.3–892.2, 894.14–28. M. Chasles, “Aperu̧ historique sur l’origine et le d́vetoppement des methodes en ǵometric,” in *Ḿmoires couronńs par l’Acad́mie royale des sciences et des belles-lettres de Bruxelles*, **2** (1837), 33, 39; and *Les trois livres de Porismes d’Euclide* (Paris, 1860), 11, 75–77, was the first to recognize the anharmonic property in the lemmas of Pappus and to see that “les propositions d’Euclide ̓taient de celles auxquelles conduisent naturellement les d́vetoppements et les applications de la notion du rapport anharmonique. devenu fondamentale dans la ǵoḿtrie moderne.” Actually, in prop. 129 Pappus does not use four concurrent lines cut by two transversals but three concurrent lines cut by two

transversals issuing from the same point. (The generality is not a fleeted.) Props. 136 and 142 are the converse; prop. 137 is a particular case and prop. 140 its converse; prop. 145 is another case of prop. 129.

19. Ptolemy, *op. cit*., 30.18.

20. M. Steinschneider, *Die arabischen Uebersetzungen aus dem Griechischen*, **2** . Abschnitt, Mathematik §111–112, in *Zeih schrift der Deutschen morgenländischen Gesellschaft*, **50** (1896), 199.

21. Proclus, *op. cit*., 345.9–346.13.

22. M. Curtze first edited Gerard of Cremona's trans, in *Nova acta Academiae Caesareae Leopoldino Carolinae germanicae naturae curiosorum*, **49** (1885), 105–167. This is now superseded by the later and better ed. of M. Clagett, *Archimedes in the Middle Ages*, I (Madison, Wis., 1964), 223–367, see particularly 334–341, 365–366.

23. Pappus, *op. cit*., **IV** .36, vol. I, p. 270.17–26.

24. A plektoid (πλϵκτοϵιδήζ ϵ̓πιΦάνϵια) is a twisted surface; the only other example of the word, also in Pappus, suggests that it may mean a conoi*d*.

25. Paul Tannery, "Pour l'histoire des lignes et surfaces courbes dans l'antiquit̓ in *Bulletin des sciences mathm̓atiques*, 2nd ser., **7** (1883), 289–291, repr. in his *Mm̓oires scienti fiques*, **II** (Toutouse-Paris, 1912), 16–18. On Vivian's curve see *Acta eruditorum* (Leipzig, 1692), "Aenigma geometricum de miro opificio testudinis quadrabilis hemispherica a D. Pio Lisci Posilto geomctra propositum die 4 April. A. 1692," pp. 274–275, also pp. 275–279, 370–371; Meritz Cantor, *Vorlesungen üher Geschichte der Mathematik*, III (Leipzig, 1898), 205.

26. Among them the *Fihrist*, see n. 7.

27. In *Codex Escurialensis* 905 the title is given as *Liber de quantitate et distinctione corporum mixtorum and in Codex Escurialensis* 955 as *De corporum mistorum quantitate et pondere*; but J. G. Wenrich, *De auctorum graecorum versionibus et comaentarüs Syriacis, Arabicis, Persicisque* (Leipzig, 1842), 211, gives *De cognitione quantitatis discretae corporum permixtorum*.

28. N. Khanikoff, "Analysis and Extracts of the Book of the Balance of Wisdom," in *Journal of the American Oriental Society*, **6** (1859), 1–128, especially pp. 34, 85. Unfortunately Khanikoff does not translate the passage referring to Menelaus, but the whole Arabic text has since been published—*Kitab mizan al-hikma* (Hyderabad, 1940). For further information see *Dictionary of Scientific Biography*, VII.

BIBLIOGRAPHY

I. Original Works. Menelaus wrote a work on spherics (the geometry of the surface of a sphere) in three books (the third treating spherical trigonometry); a work on chords in the circle, which would have included what is now called a table of sines; an elements of geometry, probably in three books; a book on the triangle, which may or may not have been a publication separate from the lastmentioned one; possibly a work on transcendental curves, including one called "paradoxical" that he discovered himself; a work on hydrostatics, dealing probably with the specific gravities of mixtures; a treatise on the setting of the signs of the zodiac; and a series of astronomical observations which may or may not have amounted to a catatog of the fixed stars.

None of these has survived in Greek, but after earlier efforts the *Sphaerica* was translated into Arabic by Isḥ̄q ibn Ḥunayn (*d*. 910/911), or possibly by his Hather, Ḥunayn ibn Isḥ̄q (*d*. 877), and the translation was revised by several eds., notably by Manṣūr ibn 'Irāq (1007/1008), whose redaction survives in the University library at Leiden as *Codex Leidensis* 930, and by Nasir al Din al-Ṭūsī(1265), whose work exists in many manuscripts. From Arabic the work was translated into Latin by Gerard of Cremona (d. 1187), and his trans, survives to varying extents in some 17 MSS; in many of them the author is called Mileus. 'The work was rendered into Hebrew by Jacob ben Mḥir ibn Tibbon (ca. 1273). The first printed ed. was a Latin version by Maurolico (Messina, 1558) from the Arabic; based on a poor MS, it is replete with interpolations. Nor was the Latin version of Mersenne (Paris, 1644) much better, Halley produced a Latin version which was published posthumously (Oxford, 1758) with a preface by G. Costard. Halley made some use of Arabic MSS, but in the main he has given a free rendering of the Hebrew version, with some mathematical treatment of his own. It held the held until Axel Anthon Björnbo produced his "Studien über Menelaos' Sphärik. Beiträur Geschichte der Sphärik und Trigonometric der Griechen," in *Abhandlungen zur Geschichte der Mathematischen Wissenschaften*, **40** (1902), 1–154, After the introductory matter this amounts to a free German rendering of the *Sphaerica* based mainly on Halley's ed. and *Codex Leidensis* 930. It was the best work on Menelaus that existed for many years, but as a doctoral thesis, the work of a young man who had to rely on secondhand information, it had many deficiencies. The need for a satisfactory ed. of the Arabic text with a German trans, and notes on the history of the text was finally met when Max krause, basing his work on the same Leiden MS, published "Die Sphärik son Menelaos aus Alexandrien in der verbesserung von Abu Nasr Mansur b. ’Ali b. ’Iraq mit Untersuchungen zur Geschichte des Textes bei den islamischen Malhematikern,” in *Ablandlungen der Gesellschaft der Wissenschaften zu Göttingen*, Phil-Hist, Klasse, 3rd ser., no. 17 (1936).

None of Menelaus’ other works survives even in trans, except for a small part of his *Book on the Triangle* (if this is different from his *Elements of Geometry*), For notes on the Arabic translations, see M. Steinschneider, “Die arabischen Uebersetzungen aus dem Griechischen, 2. Abschnitt, Mathematik 111–112,” in *Zeit- schrift der Deutschen Morgenländischen Gesellschaft*, **50** (1896), 196–199.

It is possible that the proof of Menelaus, theorem given by Ptolemy, *Syntaxis mathematica, in Claudii Ptolemaei opera quae exstant omnia*, J. L, Heiberg, ed., I, pt. 1 (Leipzig, 1898), 74,9–76.9, reproduces, at least to some extent, the language of Menelaus; but in the absence of direct attribution there can he no certainty.

II. Secondary Literature. The various references to Menelaus by Plutarch, Pappus, Proclus, and Arabic authors are given in the notes above. The chief modern literature is A. A. Björnbo, “Studien über Menelaos” Sphärik,” mentioned above; and his “Hat Menelaos einen Fixsternkatatog verfasst?” in *Bibliotheca mathematica*, 3rd ser., 2 (1901), 196–212; Thomas Heath, *A History of Greek Mathematics*, II (Oxford, 1921), 260–273; A. Rome, “Premiers essais de trigonomt́ric rectiligne chez les Grecs,” in *Annales de la Socit́ scientifique de Bruxelles*, ser. A, **52** , pt. 2 (1932), 271–274; an expanded version with the same title is in *L’antiquit́ classique*, II, lase. 1 (Touvain, 1933), 177–192; “Les explications de Th́on d’Alexandrie sur le th́orème de Ḿńlas,” in *Annales de la Socit́ scientifique de Bruxelles*, ser. A, **53** , pt. 1 (1933), 39–50; and *Commentaires de Pappus et de Th́on d’ Alexandrie sur l’Almayesie*, **II** , *Th́on d’ Alexondrie*, Studi e Testi, LXXII (Vatican City, 1936), 535–570; and Max Krause, Die Sphärik von Menelaos aus Alexandrien (mentioned above).

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