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(*b*. Bologna, Italy, 1625: *d*. Bologna, 1686)

mathematics.

Mengoli’s name appears in the register of the University of Bologna for the years between 1648 and 1686. He studied with Cavalieri, whom he succeeded in the chair of mathematics, and also took a degree in Philosophy in 1650 and another in both civil and canon law in 1653. He was in addition ordained to the priesthood and from 1660 until his death served the parish of Santa Maria Maddalena, also in Bologna.

Mengoli’s mathematics were superficially conservative. He did not subscribe to the innovations of Torricelli, and his own discoveries were set out in an abstruse Latin that made his works laborious to read. His books were nevertheless widely distributed in the seventeenth century, and were known to Collins, Wallis, and Leibniz; they were then almost forgotten, so that Mengoli’s work has been studied again only recently. His significance to the history of science lies in the transitional position of his mathematics, midway between Cavalieri’s method of indivisibles and Newton’s fluxions and Leibniz’ differentials.

In *Novae quadraturae arithmeticae* (Bologna, 1650), Mengoli took up Cataldi’s work on infinite algorithms. As Eneström (1912) and Vacca (1915) have pointed out, he was the first to sum infinite series that were not geometric progressions and to demonstrate the existence of a series which, although its general term tends to zero, has a sum that can be greater than any number. In particular, he showed the divergence of the harmonic series

preceding Jakob Bernoulli’s demonstration of it by nearly forty years (it was known to Oresme in the fourteenth century). From this, Mengoli made the general deduction that any series formed from the reciprocals of the terms of an arithmetic progression must diverge.

Mengoli also considered the series of the reciprocals of the triangular numbers

and said that the sum is 1, because the sum of the first *n* terms is n/(n+2), which (for suitably large *n*) differs from 1 by less than any given quantity. He then demonstrated the convergence of the series of the reciprocals of the numbers (n + r) to the result that

and summed the reciprocals of the solid numbers,

In the *Geometriae speciosae elementa* (1659), Mengoli set out a logical arrangement of the concepts of limit and definite integral that anticipated the work of nineteenth-century mathematicians. In establishing a rigorous theory of limits, he considered a variable quantity as a ratio of magnitudes and hence needed to consider only positive limits. He then made the following definitions: a variable quantity that can be greater than any assignable number is called “quasiinfinite”; a variable quantity that can be smaller than any positive number is “quasi-nil”; and a variable quantity that can be both smaller than any number larger than a given positive number *a* and greater than any number smaller than *a* is “quasi-*a*.”

Using these precise concepts of the infinite, the infinitesimal, and the limit, and working from simple inequalities valid between numerical ratios, he demonstrated (as Agostini recognized by translating his obscure exposition into modern symbols and terminology) the properties of the limit of the sum and the product, and showed that the properties of proportions are conserved also at the limit. The proofs obtain when such limits are neither 0 nor ∞ for this case Mengoli set out the properties of the infinitesimal calculus and the calculus of infinites some thirty years before Newton published them in his *Principia*.

Mengoli’s predecessors (among them Archimedes, Kepler, Valerio, and Cavalieri) had assumed as intuitively evident that a plane figure has an area. By contrast, he proved the existence of the area by dividing an interval of the continuous figure $f(x)$ into $n$parts and considering, alongside the figure to be squared (which he called the “form”), the figures formed by paralletograms constructed on each segment of the interval and having the areas (in modern notation):

where l_i and L_i denote, respectively, the minimum and maximum of $f(x)$ on the interval $(x_i\ x_{i+1})$. Drawing upon the theory of limits that had worked so well in the study of series, Mengoli demonstrated that the sequences of the s_n and S_n tend to the same limit to which the sequences of the σ_n and σa_n compressed between them, also tend. Hence, since the figure to be squared is always compressed between the s_n and the S_n, it follows that this common limit is the area of the figure itself.

Mengoli also used this method to integrate the binomial differentials $Z^s(a-x)^r$ dx with whole and positive exponents. (He had, preceding Wallis, already integrated these some time before by the method of indivisibles.) Before publishing his results, however, he wished to give a rigorous basis to the method of indivisibles or to develop in its stead another method that would be immune to criticism. He therefore set out a purely arithmetic theory of logarithms; having given a definition of the logarithmic ratio similar to Euclid's definition of ratio between magnitudes, he then extended Euclid's book V to encompass his own logarithmic ratio. Mengoli also did significant work in logarithmic series (thirteen years before N. Mercator published his *Logarithmotecnia*).

In a short work of 1672, entitled *Circolo* Mengoli calculated the integrals of the form

finding for *n*/2 the same infinite product that had already been given by Wallis. Mengoli published other, minor mathematical writings; in addition he was interested in astronomy, and wrote a short vernacular book on music, published in 1670.

BIBLIOGRAPHY

I. Original Works. Mengoli's writings include *Novae quadraturae arithmeticaticae* (Bologna, 1650); *Geometriae speciosae elementa* (Bologna, 1659); *Speculazioni di musica* (Bologna, 1670); and *Circolo* (Bologna, 1672).

II. Secondary Literature. On Mengoli and his work, see A. Agostini, "La teoria dei limiti in Pietro Mengoli," in *Periodico di matematiche*, 4th ser., **5** (1925), 18–30; "II concetto di integrale definito in Pietro Mengoli," *ibid*., 137–146; and "Pietro Mengoli," in *Encictopedia italiana*, XXII (Milan, 1934), 585; E. Bortolotti, *La storia della matematica nella universitá di Bologna* (Bologna, 1947), 98–401, 137–138; G. Bneström, "Zur Geschichte der unendlichen Reihen in die Mitte des siebzehnten Jahrhunderte," in *Bibliotheca mathematica* (1912), 135–148; and G. Vacca, "Sulle scoperte di Pietro Mengoli," in *Atti dell' Accademia nazionale dei Lincei*. *Rendiconti* (Dee. 1915), 512.

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