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(*b*. Philadelphia, Pennsylvania, 28 January 1888; *d*. Cambridge, England, 12 March 1972)

mathematics.

Mordell was the third child of Phineas Mordell, who later became a noted Hebrew scholar, and of Annie Feller Mordell. Both were poor Jewish immigrants from Lithuania. At a young age he went to England, where he spent the rest of his life, and became a British subject in 1929. In May 1916 he married Mabel Elizabeth Cambridge; they had a daughter and a son.

At the age of fourteen, when he entered Central High School in Philadelphia, Mordell was already fascinated with mathematics learned from used books he had bought for five or ten cents at a well-known Philadelphia bookstore. Many of the examples in these books were taken from Cambridge scholarship or tripos papers, a fact that gave Mordell the desire to attend Cambridge. Having demonstrated his mathematical abilities by completing the four-year high school mathematics course in two years and earning a very high grade on a test administered by a friend of his father's he scraped together the fare to Cambridge. In 1907 he placed first on the scholarship examination and received a scholarship to St. John's College. He took part I of the mathematical tripos in 1909 and was third wrangler (after P. J. Daniell and E. H. Neville). On completing the tripos he began research in the theory of numbers, in which there was then little interest in England; he regarded himself as self-taught in the subject.

Mordell's work was a systematic study of the integral solutions (x, y) of the Diophantine equation

This equation has a long history, going back to Pierre de Fermat (1601–1665) for some values of k, but Mordell decided the solubility for many new values of k and in some cases gave complete solutions. He also showed that the determination of the solutions was equivalent to solving the set of equations $f(u, v) = 1$, where f runs through representatives of the equivalence classes of integral cubic forms of discriminant $-4k$. This won him a Smith's Prize, but he failed to obtain a college fellowship. He went on to show that the determination of the integral solutions (x, y) of an equation

is equivalent to determining the solutions of $f(u, v) = 1$, where f now runs through representatives of a class of quartic forms with given invariants. Axel Thue had already shown that equations of the type $f(u, v) = 1$ have only finitely many solutions, which implies that this is also the case for (1) and (2): but Mordell learned of Thue's work only later, and at the time believed that some equations (1) or (2) could have infinitely many integral solutions.

From 1913 to 1920 Mordell was a lecturer at Birkbeck College, London. His main interest was in modular forms, and he made two important advances, both anticipating approaches that remain central in the theory. One of these concerned the tau function, introduced by Srinivasa Ramanujan, who had conjectured that it has the property of multiplicativity. This Mordell proved. The central argument (the Hecke operator) was rediscovered by Erich Hecke in 1937. The tau function is the set of coefficients of a certain modular form, and Hecke showed that Mordell's theorem is a special case of a general and important phenomenon. Mordell's other advance was to systematize the theory of the representation of integers as the sum of a fixed number n of squares of integers. Many special results were known. Mordell showed how to deduce those results and to obtain new ones by using the finite dimensionality of the space of modular forms of a given type. In the hands of Hecke and others, this idea was exploited with great effect in the study of representation by positive definite quadratic forms in general.

During the period from 1920 to 1922, when he was lecturer at Manchester College of Technology, Mordell worked out the result for which he is most widely known, his "finite basis theorem." Henri Poincaré had shown that the determination of all the rational points on a given curve of genus 1 defined over the rationals is equivalent, once one rational point has been given, to the determination of the rational points on a curve

where g_2, g_3 are given rational numbers (depending on the given curve). The rational points on (3) have a natural structure as an abelian group, and Poincaré conjectured that this group is finitely generated. In the course of an investigation with another aim. Mordell found a proof of this conjecture. The curve (3) is an abelian variety of dimension 1. The finite basis theorem was extended by André Weil to abelian varieties of any dimension and to any algebraic number field as ground field, and there are further generalizations. The theorem plays a key role in many aspects of Diophantine analysis, Mordell, however, played no part in these later developments.

Toward the end of the paper just discussed, Mordell conjectures that there are only finitely many rational points on any curve of genus greater than unity. This acquired notoriety as “Mordell’s conjecture” and was proved by Gerd Faltings in 1983.

In 1922 Mordell moved to the University of Manchester, where from 1923 to 1945 he was Fielden professor of pure mathematics and head of the mathematics department; in 1924 he was elected to the Royal Society—while still an American citizen. There was already a fine tradition of mathematics at Manchester, and during Mordell’s tenure it became a leading center. He gave great attention both to teaching and to research and built up a strong team that attracted many visitors. He was extremely active in assisting refugees from continental tyrannies, and for some of them he found temporary or even permanent positions. Mordell’s own research ranged widely within the theory of numbers. One problem (suggested by Harold Davenport) was the estimation of trigonometric sums and of the number of points on curves and other varieties defined over finite fields. Mordell devised an averaging argument that gave stronger results than those already known. The results were largely superseded by the Riemann hypothesis for function fields of Hasse and Weil, but Mordell’s argument suggested to Ivan Vinogradov his technique for estimating more general sums. In the late 1930’s with Harold Davenport and Kurt Mahler (both then at Manchester), Mordell initiated a period of great advances in the geometry of numbers.

In 1945 Mordell succeeded Godfrey Harold Hardy as Sadleirian professor of pure mathematics at Cambridge, and became a fellow of St. John’s. He rapidly built up a strong research school. After his retirement in 1953 he continued to live in Cambridge but traveled widely. He retained his passionate interest in mathematics and did much to foster that interest in others, particularly beginners. He published many papers; perhaps none were of the first rank, but some nevertheless display an extraordinary virtuosity with comparatively elementary techniques. Mordell was a problem solver, not a system builder. Even when his work revealed a system waiting to be built (as, for example, with modular forms or the finite basis theorem) he turned to other work after solving the problem of interest to him.

Mordell enjoyed robust health throughout his life. After a very brief period of ill health, he slipped into unconsciousness and died five days later.

BIBLIOGRAPHY

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II. Secondary Literature. There is an account of Mordell’s life and work by John W. S. Cassels in *Biographical Memoirs of Fellows of the Royal Society*, **19** (1973), 493–520, with a complete bibliography. A slightly different version of this account is in *Bulletin of the London Mathematical Society*, **6** (1974), 69–96, also with a complete list of his publications. A short biographical memoir by Harold Davenport is in *Acta arithmetica*, **9** (1964), 1–22, with a bibliography and a portrait.

John W. S. Cassels