John Napier | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
21-26 minutes

(*b*. Edinburgh, Scotland, 1550; *d*. Edinburgh, 4 April 1617)

mathematics.

The eighth laird of Merehiston, John Napier was the son of Sir Archibald Napier by his first wife, Janet Bothwell, daughter of an Edinburgh burgess. At the age of thirteen he went to St. Salvator's College, St. Andrews, where he lodged with John Rutherford, the college principal. Little is known of his life at this time save that he gained some impetus toward theological studies during the brief period at St. Andrews. His mother's brother, Adam Bothwell, bishop of Orkney, recommended that he continue his studies abroad and it seems likely that he did so, although no explicit evidence exists as to his domicile, or the nature of his studies. At all events, by 1571 he had returned to Scotland and, in 1572, he married Elizabeth, daughter of Sir James Stirling, and took up residence in a castle at Gartnes (completed in 1574). On the death of his father in 1608, he moved to Merchiston Castle, near Edinburgh, where he lived for the rest of his life. In 1579 his wife died and he subsequently married Agnes Chisholm of Cromlix, Perthshire. There were two children by the first marriage, a son, Archibald, who in 1627 was raised to the peerage by the title of Lord Napier, and a daughter, Joanne. By the second marriage there were ten children; the best known of these is the second son, Robert, his father's literary executor.

Napier lived the full and energetic life of a sixteenth- century Scottish landowner, participating vigorously in local and national affairs. He embraced with great fervor the opinions of the Protestant party, and the political activities of his papist father-in-law, Sir James Chisholm, involved him in continuous em- barrassment. There were quarrels with his half brothers over the inheritance and disputes with tenants and neighboring landlords over land tenure and rights. In all these matters, Napier seems to have shown himself forthright and determined in the pursuit of his aims, but nonetheless just and reasonable in his demands and willing to accept a fair settlement. As a landowner, Napier gave more than the usual attention to agriculture and to the improvement of his crops and his cattle. He seems to have experimented with the use of manures and to have discovered the value of common salt for this purpose, a monopoly for this mode of tillage being granted to his eldest son, Archibald, in 1698. A monopoly was granted to Napier also for the invention of a hydraulic screw and revolving axle to keep the level of water down in coal pits (1597). In 1599 Sir John Skene mentioned that he had consulted Napier, "a gentleman of singular judgement and learning, especially in mathematic sciences,"with reference to the proper methods to be used in measuring lands.

In sixteenth-century Scotland, intellectual interest centered on religion, theology, and politics rather than on science and mathematics and Napier's first literary work arose out of the fears entertained in Scotland of an invasion by Philip II of Spain. *A Plaine Discovery of the Whole Revelation of Saint John* occupied him for about five years before its publication in 1593. In this tract Napier urged the Scottish king, James VI (the future James I of England), to see that "justice be done against the enemies of Gods church"and implored him to "purge his house, family and court of all Papists, Atheists and Newtrals."Through this publication, Napier gained a considerable reputation as a scholar and theologian and it was translated into Dutch, French, and German, going through several editions in each language. It is possible that, in later life, his authority as a divine saved him from persecu- tion as a warlock, for there are many stories told suggesting that, locally, he was suspected of being in league with the powers of darkness. Not content with opposing popery by the pen, Napier also invented various engines of war for the defense of his faith and his country. In a document preserved in the Bacon Collection at Lambeth Palace, Napier outlines four inventions, two varieties of burning mirrors for setting fire to enemy ships at a distance, a piece of artillery for destroying everything round the arc of a circle, and an armored chariot so constructed that its occupants could fire in all directions. It is not know whether any of these machines were ever constructed.

Although documentary evidence exists to sub- stantiate the active part Napier plaved in public affairs in this tumultuous age, it is more difficult to trace the development of his mathematical work, which seems to have begun in early life and persisted, through solitary and indefatigable labors, to the very end, when he made contact with Henry Briggs. Some material was, apparently, assembled soon after his first marriage in 1572 and may have been prompted by knowledge he had gleaned during his travels abroad. This treatise, dealing mainly with arithmetic and algebra, survived in manuscript form and was transcribed, after Napier's death, by his son Robert for the benefit of Briggs. It was published in 1839 by a descendant, Mark Napier, who gave to it the title *De arte logistica*. From this work, it appears that Napier had investigated imaginary roots of equations, a subject he refers to as a great algebraic secret.

There is evidence that Napier began to work on logarithms about 1590; the work culminated in the publication of two Latin treatises, known respectively as the *Descriptio* (1614) and the *Constructio* (1619). The *Descriptio* bears evidence of having been written all at one time and contains, besides the tables, a brief general account of their nature and use. An English translation of this work was made by Edward Wright but was published only after Wright's death by his son, Samuel Wright

(1616). Napier approved the transla- tion, both in substance and in form. The *Constructio* was brought out by Robert Napier, after the death of his father, and consists of material which Napier had written many years before. The object of the *Constructio* was to explain fully the way in which the tables had been calculated and the reasoning on which they were based. In the *Constructio* the phrase “artificial numbers”is used instead of “logarithms,”the word “logarithm”being apparently of later invention. Napier offered no explanation for the choice but Briggs, in the *Arithmetica logarithmica* (1624), explains that the name came from their inventor because they exhibit numbers which preserve always the same ratio to one another.

Although it is as the inventor of logarithms that Napier is known in the history of mathematics, the two works mentioned above contain other material of lesser importance but nonetheless noteworthy. In the course of illustrating the use and application of logarithms Napier made frequent use of trigonometric theorems and the contribution he made to the develop- ment and systematization of spherical trigonometry has been rated highly. Napier’s rules (called the Napier analogies) for the right-angled spherical triangle were published in the *Descriptio* (Bk. II, Ch. 4). He expressed them in logarithmic form and exhibited their character in relation to the star pentagon with five right angles. Another achievement was the effective use he made of decimal notation (which he had learnt of from Stevin) in conjunction with the decimal point. Although he was not the first to use a decimal separatrix in this way, the publicity thai he gave to it and to the new notation helped to establish its use as standard practice. In 1617 Napier’s intense concern for the practicalities of computation led him to publish another book, the *Rabdologiae*, which contains a number of elementary calculating devices, including the rods known as “Napier’s bones.”These rods, which in essence constitute a mechanical multiplication table, had a considerable vogue for many years after his death. Each rod is engraved with a table of multiples of a particular digit, the tens and units being separated by an oblique stroke. To obtain the product 267 × 8, the rods 2, 6, 7 are assembled and the result is read off from the entries in the eighth row; thus gives 2,136. Book II is a practical treatment of mensuration formulas. Book III, the method of the promptuary, deals with a more complicated system of multiplication by engraved rods and strips, which has been called the first attempt at the invention of a calculating machine. The concluding section deals with a mechanical method of multiplication that was based on an “areal abacus”consisting of a checkerboard with counters, in which numbers were expressed in the binary scale.

Until recently the historical background of the invention of logarithms has remained something of an enigma. At the Napier tercentenary celebrations. Lord Moulton referred to Napier’s invention as a “bolt from the blue”and suggested that nothing had led up to it, foreshadowed it, or heralded its arrival. Notwithstanding, Joost Bürgi, a maker of watches and astronomical instruments, had turned his attention to the problem about the same time and developed a system of logarithms entirely independently. Many Continental historians have accorded him priority in the actual invention, although he certainly did not have it in the publication of his *Arithmetische unit geometrischie Progress-Tabulen* (1620).

After the revival of learning in western lurope some of the first advances made were in trigonometry, which was developed as an independent field of study, largely in the interests of astronomy but also for surveying, mapmaking, and navigation. Much time was spent in calculating extensive tables of sines and tangents. Trigonometric tables were appearing in all parts of Europe, and stress was laid on the develop- ment of formulas, analogous to

which could, by converting the product of sines into sums and differences, reduce the computational difficulties. This conversion process was known as prosthaphaeresis. Formulas generated in this way were much used in astronomical calculations and were linked with the names of Longomontanus and Wittich, who both worked as assistants to Tycho Brahe. It is said that word of these developments came to Napier through a fellow countryman, John Craig, who accompanied James VI to Norway in 1590 to meet his bride, Anne of Denmark. The party landed near Tycho Brahe’s observatory at Hven and was entertained by the astronomer. Although the con- struction of Napier’s logarithms clearly owes nothing to prosthaphaeresis, the aim—that of substituting addition and subtraction for multiplication and division in trigonometrical calculations—was the same, and if Napier was already working on the problem, he may well have been stimulated to further efforts by the information he received through Craig. There is evidence in a letter written by Kepler in 1624 that he had received an intimation of Napier’s work as early as 1594. This information presumably came through Tycho Brahe and Craig.

Napier’s own account of his purpose in undertaking the work is printed in the author’s preface to the *Descriptio* and is reprinted with slight modification in Wright’s translation. Napier says that there is nothing more troublesome to mathematical practice than the “multiplications, divisions, square and cubical ex- tractions of great numbers”and that these operations involve a tedious expenditure of time, as well as being subject to “slippery errors.”By means of the tables all these operations could be replaced by simple addition and subtraction.

As presented, Napier’s canon is specifically as- sociated with trigonometric usage, in the sense that it gives logarithms of natural sines (from the tables of Erasmus Reinhold). The sine of an arc was not, at that time, given as a ratio but as the length of the semichord of a circle of given radius, subtending a specified angle at the center. In tabulating such sines, it was customary to choose a large number for the radius of the circle (or whole sine); Napier’s choice of 10^7 gave him seven significant figures before introducing fractions.

The theory of arithmetic and geometric progressions, which played a central role in Napier’s constructions, was of course available from ancient times (Napier quotes Euclid). The correspondence between the terms of an arithmetic and a geometric progression had been explored in detail by many sixteenth-century mathematicians; and Stifel in *Arithmetica integra* (1544) had enunciated clearly the basic laws—but without the index notation—corresponding to $a^m a^n = a^{m+n}$, $(am)^n = a^{mn}$.

But, in all this work, only the relation between discrete sets of numbers was implied. In Napier's geometric model the correspondence between the terms of an arithmetic and a geometric progression was founded on the idea of continuously moving points and involved concepts of time, motion, and instantaneous speed. Although such notions had played a prominent part in the discussions of the fourteenth-century philosophers of the Merton school (most notably Swineshead in his *Liber calculationum*), there is nothing to suggest that any of this work directly influenced Napier.

Most historical accounts of Napier's logarithms have suffered considerably through translation into modern symbolism. Napier himself used virtually no notation, and his explanatory detail is almost wholly verbal. Without any of the tools of modern analysis for handling continuous functions, his propositions inevitably remained on an intuitive basis. He had, nonetheless, a remarkably clear idea of a functional relation between two continuous variables.

Briefly, two points move along parallel straight lines, the first moving arithmetically through equal distances in equal times and the second moving geometrically toward a fixed point, cutting off proportional parts of the whole line and then of subsequent remainders, also in equal times.

If the first point moves through the spaces $T' A_1, A_1A_2, A_2A_3\cdots$, in equal times, then $T' A_1 = A_1A_2 = A_2A_3 = \cdots$.

If the second point moves toward a fixed point S and is at $T, G_1, G_2, G_3, \cdots$, when the first point is at T', $A_l, A_2, A_3\cdots$, then the spaces $TG_1, G_1, G_1G_2, G_2G_3$ are also covered in equal times. But since the second point moves geometrically, $TG_1 = G_1/G_1S = G_2/G_3/G_2$,

It follows that the velocity of the second point is everywhere proportional to its distance from S.

The definition of the logarithm follows: Two points start from T' and *T* respectively, at the same instant and with the same velocities, the first point moving uniformly and the second point moving so that its velocity is everywhere proportional to its distance from S; if the points reach L and G respec- tively, at the same instant, the number that measures the line T L is defined as the logarithm of GS (GS is the sine and TS, the whole sine, or radius).

From the definition, it follows that the logarithm of the whole sine (10^7) is 0 and that the logarithm of n, where $n > 10^7$, is less than 0. In modern notation, if $T'L = y$, $y_0 = 0$, $GS = x$, $TS = x_0 = r\ 10^7$, $dx/dt = -kx$, $dy/dt = kr$, $dy/dx = -r/x$, $\log_e(x/r) = -y/r$, or $\log_{1/e}(x/r) = y/r$. It remained to apply this structure in the calculation of the canon. Without any machinery for handling continuous functions it was necessary for Napier to calculate bounds, between which the logarithm must lie. His entire method depends upon these bounds, together with the corresponding bounds for the difference of the logarithms of two sines.

If the point O lies on ST produced such that OS/TS= TS/SG, then the spaces OT and TG are covered in equal times. But, since OS > TS > GS, the velocity at O > the velocity at T > the velocity at G. It follows that OT > T'L > TG, and OS - TS > log SG > TS - GS. If TS = r, GS = x, we have

the corresponding bounds for the difference between two logarithms are given by

Napier then calculates in a series of tables the values of

$n = 0, 1, 2, 3, \ldots ,100;$

$n = 0, 1, 2, 3, \ldots ,50;$

and finally,

$n = 0, 1, 2, \ldots, 20;$

$m = 0, 1, 2, \ldots, 68;$

The terms in each progression were obtained by successive subtraction, the last figure in the first table giving the starting point for the second. The final figure in the last table gave a value very little less than $10^7/2$, so that Napier had available a very large number of geometric means distributed over the interval $10^7, 10^7/2$. Using his inequalities, he was able to derive bounds for the logarithms of these numbers and, by taking an arithmetic mean between the bounds, to obtain an accuracy of seven significant figures. By interpolation, he tabulated the values of the logarithms of the sines (and tangents) of angles, taken at one-minute intervals, extending the tables to cover angles between 0 and 90 degrees.

Napier did not think in terms of a base, in the modern sense of the word, although since is very nearly it is clear that we have virtually a system of logarithms to base 1/e. In Napier's system, the familiar rules for the logarithms of products, quotients, and exponents did not hold because of the choice of the whole sine (10^7), rather than 1, as the logarithm whose number was zero. Napier's tables were also awkward to use in working with ordinary numbers, rather than sines or tangents.

The calculation of the canon was a tremendous task and occupied Napier personally for over twenty years. Although not entirely free from error the calculations were essentially sound and formed the basis for all subsequent logarithm tables for nearly a century. The publication in 1614 received immediate recognition. Henry Briggs, then Gresham professor of geometry in the City of London, was enthusiastic and visited Napier at Merchiston in the summers of 1615 and 1616. During discussions that took place there

the idea emerged of changing the system so that 0 should become the logarithm of unity and 10^{10} that of the whole sine. Briggs in the preface to *Arithmetica logarithmica* (1624) clearly attributes this suggestion to Napier and apparently believed that Napier had become convinced of the desirability of making this change, even before the publication of the *Descriptio*. Because of failing health, however, Napier did not have the energy to embark on this task, and it was left to Briggs to recalculate the tables, adapting them to use with a decimal base. The first 1,000 logarithms of the new canon were published after Napier's death by Briggs, without place or date (but at London before 6 December 1617), as *Logarithmorum chilias prima*. The earliest publication of Napier's logarithms on the Continent was in 1618, when Benjamin Ursinus included an excerpt from the canon, shortened by two places, in his *Cursus mathematici practici*. Through this work Kepler became aware of the importance of Napier's discovery and expressed his enthusiasm in a letter to Napier dated 28 July 1619, printed in the dedication of his *Ephemerides* (1620).

In matters of priority in the invention of logarithms the only serious claims have been made on behalf of Joost Bürgi. Many German historians have accorded him priority in the actual invention on the grounds that his tables had been computed about 1600, although they were not published until 1620. Since Napier's own work extended over a long period of time, both must be accorded full credit as independent inventors. The tables were quite differently conceived, and neither author owed anything to the other. Napier enjoyed the right of priority in publication.

BIBLIOGRAPHY

I. Original Works. Napier's works are *A Plaine Discovery of the Whole Revelation of Saint John* (Edinburgh, 1593); *Mirifici logarithmorum canonis descriptio, ejusque usus*, … (Edinburgh, 1614); *Rabdologiae, seu numerationis per virgulas libri duo* (Edinburgh, 1617); *Mirifici logarithmorum canonis constructio; et eorum ad naturales ipsorum numeros habitudines* (Edinburgh, 1619); *De arte logistica*, Mark Napier, ed. (Edinburgh, 1839); *A Description of the Admirable Table of Logarithmes*: …, translated by Edward Wright, published by Samuel Wright (London, 1616). *The Construction of the Wonderful Canon of Logarithms* (Edinburgh, 1889), W. R. Macdonald's trans. of the *Constructio*, contains an excellent catalog of all the editions of Napier's works and their translations into French, Dutch, Italian, and German. Details are also included of the location of these works at that date. Further details and descriptions are included in R. A. Sampson, ed., "Bibliography of Books Exhibited at the Napier Tercentenary Celebrations, July 1914,"in C. G. Knott, ed., *Napier Tercentenary Memorial Volume* (London, 1915).

II. Secondary Literature. Such information as is available about Napier's life and work has been fairly well documented by his descendants. Mark Napier, *Memoirs of John Napier of Merchiston; His Lineage, Life and Times* (Edinburgh, 1834), based on careful research of the private papers of the Napier family, is the source of most modern accounts. The tercentenary of the publication of the *Descriptio* was celebrated by an international congress, organized by the Royal Society of Edinburgh. The papers communicated to this congress were published in the *Napier Tercentenary Memorial Volume* (see above) and supply much detail on the historical background to Napier's work. E. M. Horsburgh, ed., *Modern Instruments and Methods of Calculation: A Handbook of the Napier Tercentenary Exhibition* (London, 1914), is also useful. Of the various reconstructions of Napier's work, Lord Moulton's, in the *Tercentenary Memorial Volume*, pp. 1–24, is the most imaginative; E. W. Hobson, *John Napier and the Invention of Logarithms* (Cambridge, 1914), is the most useful.

Still valuable on the early history of logarithms are J. W. L. Glaisher's articles, "Logarithms,"in *Encyclopaedia Britannica*, 11th ed. (1910), XVI, 868–877; and "On Early Tables of Logarithms and Early History of Logarithms,"in *Quarterly Journal of Pure and Applied Mathematics*, **48** (1920), 151–192. Florian Cajori, "History of the Exponential and Logarithmic Concepts,"in *American Mathematical Monthly*, **20** (1913), 5–14, 35–47, 75–84, 107–117, 148–151, 173–182, 205–210, is also useful. A more recent discussion of the development of the concept of logarithm is that of D. T. Whiteside, "Patterns of Mathematical Thought in the Later Seventeenth Century,"in *Archive for History of Exact Sciences*, **1** (1961), 214–231.

Margaret E. Baron