Nichomachus Of Gerasa | Encyclopedia.com

(*fl. ca.* A.D. 100)

mathematics, harmonics.

That Nicomachus was from Gerasa probably the city in Palestine, is known from Lucian (*Philopatris,* 12), from scholia to his commentator Philoponus, and from some manuscripts that contain Nicomachus' works. The period of his activity is determined by inference. In his *Manual of Harmonics* Nicomachus mentions Thrasyllus, who died in A.D. 36; Apuleius, born about A.D. 125, is said to have translated the *Introduction to Arithmetic* into Latin; and a character in Lucian's *Philopatris* says, "You calculate like Nicomachus," Which shows that Lucian, born about A.D. 120, considered Nicomachus a famous man. [1] Porphyry mentions him, together with Moderatus and others, as a prominent member of the Pythagorean school, and this connection may also be seen in his writings. [2] Only two of his works are extant, *Manual of Harmonics* and *Introduction to Arithmetic*. He also wrote a *Thelogumena arithmeticae,* dealing with the mystic properties of numbers, and a larger work on music, some extracts of numbers, and a larger work on music, some extracts of which have survived.[3] Other works are ascribed to him, but it is not certain that he wrote any of them.[4]

In the *Manual of Harmonics,* after an introductory chapter, Nicomachus deals with the musical note in chapters 2–4 and devotes the next five chapters to the octave. Chapter 10 deals with tuning principles based on the stretched string; chapter 11, with the extension of the octave to the two-octave range of the Greater Perfect System in the diatonic genus; and the work ends with a chapter in which, after restating the definitions of note, interval, and system, Nicomachus gives a survey of the Immutable System in the three genera: diatonic, chromatic, and enharmonic. He deals with notes, intervals, systems, and genera, the first four of the seven subdivisions of harmonics recognized by the ancients, but not with keys, modulation, or melodic composition. The treatise exhibits characteristics of both the Aristoxenian and the Pythagorean schools of music. To the influence of the latter must be ascribed Nicomachus' assignment of number and numerical ratios to notes and intervals, his recognition of the indivisibility of the octave and the whole tone, and his notion that the musical consonances are in either multiple or superparticular ratios. But unlike Euclid, who attempts to prove musical propositions through mathematical theorems, Nicomachus seeks to show their validity by measurement of the lengths of strings. Hence his treatment of consonances and of musical genera, as well as his definition of the note, are Aristoxenian.

The *Introduction to Arithmetic* is in two books. After six preliminary chapters devoted to the philosophical importance of mathematics, Nicomachus deals with number per se, relative number, plane and solid numbers, and proportions. He enunciates several definitions of number and then discusses its division into even and odd. He states the theorem that any integer is equal to half the sum of the two integers on each side of it and proceeds to give the classification of even numbers (even times even, odd times even, and even times odd), followed by that of odd numbers (prime, composite, and relative prime).[5] The fundamental relations of number are equality and inequality, and the latter is divided into the greater and the less. The ratios of the greater are multiples, super-particulars, superpartients, multiple super particulars, and multiple superpartients; those of the less are the reciprocal ratios of these. Book I concludes with a general principle whereby all forms of inequality of ratio may be generated from a series of three equal terms.[6] At the beginning of the second book the reverse principle is given. It is followed by detailed treatments of squares, cubes, and polygonal numbers. Nicomachus divides proportions into disjunct and continuous, and describes ten types. He presents no abstract proofs (as are found in Euclid's *Elements*, VII-IX), and he limits himself for the most part to the enunciation of principles followed by examples with specific numbers.[7] On one occasion this method leads to a serious mistake,[8] but there are many other mistakes which are independent of the method of exposition—for example, his inclusion of composite numbers, a class which belongs to all numbers, as a species of the odd. Yet despite its notorious shortcomings, the treatise was influential until the sixteenth century and gave its author the undeserved reputation of being a great mathematician.

NOTES

1. For references to modern discussions, see Tarán, *Asclepius on Nicomachus*, p. 5, n. 3. J. M. Dillon, "A Date for the Death of Nicomachus of Gerasa?" in *Classical Review*, n.s. **19** (1969), 274–275, conjectures that Nicomachus died in A.D.196, because Proclus, who was born in A.D. 412, is said by Marinus, *Vita Procli* 28, to have believed that he was a reincarnation of Nicomachus, and because some Pythagoreans believed that reincarnations occur at intervals of 216 years. But Dillon fails to cite any passage in which Proclus would attach particular importance to the number 216 and, significantly enough, this number is not mentioned in Proclus' commentary on the creation of the soul in Plato's *Timaeus*, a passage where one would have expected this number to occur had Dillon's conjecture been a probable one.

2. In Eusebius of Caesarea, *Historia ecclesiastica*, VI, xix, 8.

3. Some of the contents of the *Theologumena* can be recovered from the summary of it given by Photius, *Bibliotheca* codex 187, and from the quotations from it in the extant *Theologumena arithmeticae* ascribed to lamblichus.

In his *Manual of Harmonics*, I, 2, Nicomachus promises to write a longer and complete work on the subject; and the extracts in some MSS, published by Jan in *Musici scriptores Gracci*, pp. 266–282, probably are from this work. They can hardly belong to a second book of the *Manual*, because Nicomachus' words at the end of this work indicate that it concluded with chapter 12. Eutocius seems to refer to the first book of the larger work on music; see *Eurocii Commeniarii in libros De sphaera et cylindro in Archimedis Opera omnia,* J. L. Heiberg, ed., Ill (Leipzig, 1915), 120, II. 20–21.

4. In his *Introduction to Arithmetic*, II, 6, I, Nicomachus refers to an *Introduction to Geometry*. Some scholars attribute to him a *Life of Pythagoras* on the grounds that Nicomachus is quoted by both Porphyry and lamblichus in their biographies of Pythagoras. It is also conjectured that he wrote a work on astronomy because Simplicius, *In Aristotelis De caelo* Heiberg ed., p. 507. II. 12–14, says that Nicomachus, followed by lamblichus, attributed the hypothesis of eccentric circles to the Pythagoreans. A work by Nicomachus with the title *On Egyptian Festivals* is cited by Athenaeus and by Lydus, but the identity of this Nicomachus with Nicomachus of Gerasa is not established. Finally, the "Nicomachus the Elder" said by Apollinaris Sidonius to have written a life of Apollonius of Tyana in which he drew from that of Philo-stratus cannot be the author of the *Manual* since Philostratus was born ca. A.D. 170.

5. Nicomachus considers prime numbers a class of the odd, because for him 1 and 2 are not really numbers. For a criticism of this and of Nicomachus' classifications of even and odd numbers, see Heath, *A History of Greek Mathematics*, I, 70–74. In I, 13, Nicomachus describes Eratosthenes' "sieve," a device for finding prime numbers.

6. This principle is designed to show that equality is the root and mother of all forms of inequality.

7. Euclid represents numbers by lines with letters attached, a system that makes it possible for him to deal with numbers in general, whereas Nicomachus represents numbers by letters having specific values.

8. See *Introduction to Arithmetic*, II, 28, 3, where he infers a characteristic of the subcontrary proportion from what is true only of the particular example (3, 5, 6) that he chose to illustrate this proportion. See Tarán, *Asclepius on Nicomachus*, p. 81 with references.

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For an exposition of the mathematical contents of Ntcomachus' treatise and a criticism of it, see T. Heath, *A History of Greek Mathematics,* I (Oxford, 1921), 97–112.

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