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(*b*. Mannheim, Germany, 24 September 1844; *d*. Erlangen Germany, 13 December 1921)

mathematics.

Max Noether was the third of the five children of Hermann Noether and Amalia Würzburger. Noether's father was a wholesaler in the hardware business—a family tradition until 1937 when it was "Aryanized" by the Nazis. Max attended schools in Mannheim until an attack of polio at age fourteen made him unable to walk for two years and left him with a permanent handicap. Instruction at home enabled him to complete the Gymnasium curriculum; then, unassisted, he studied university-level mathematics.

After a brief period at the Mannheim observatory, he went to Heidelberg University in 1865. There he earned the doctorate in 1868 and served as *Privatdozent* (1870–1874) and as associate professor (*extraordinarius*) from 1874 to 1875. Then he became affiliated with Erlangen as associate professor until 1888, as full professor (*ordinarius*) from 1888 to 1919, and as professor emeritus thereafter. In 1880 he married Ida Amalia Kaufmann of Cologne. She died in 1915. Three of their four children became scientists, including Emmy Noether, the mathematician.

Noether was one of the guiding spirits of nineteenth-century algebraic geometry. That subject was motivated in part by problems that arose in Abel's and Riemann' treatment of algebraic functions and their integrals. The purely geometric origins are to be found in the work of Plücker, Cayley, and Clebsch, all of whom developed the theory of algebraic curves—their multiple points, bitangents, and inflections. Cremona also influenced Noether, who in turn inspired to the great Italian geometers who followed him—Segre, Severi, Enriques, and Castelnuovo. In another direction Emmy Noether and her discipel B. L. van der Wareden made algebraic geometry rigorous and more general. Lefschetz, Weil, Zariski, and others later used topological and abstract algebraic concepts to provide further generalization.

In both the old and the new algebraic geometry, the central object of investigation is the algebraic variety, which, in n-dimensional space, is the set of all points $(x_1, x_2, \dots, x_n)$ satisfying a finite set of polynomial equations,
$f_i(x_1,x_2, \dots, x_n) = 0, i= 1, 2, \dots, r$
with $r \leq n$ and coefficients in the real (or complex) field or, in modern algebraic geometry, an arbitrary field. Thus, in the plane, the possible varieties are curves and finite sets of points; in space there are surfaces, curves, and finite point sets; for $n > 3$ there are hypersurfaces and their intersections.

Following Cremona, Noether studied the invariant properties of an algebraic variety subjected to birational transformations; that is, one-to-one rational transformations with rational inverses, those of lowest degree being the collineations or projective transformations. Next in order of degree come the quadratic transformations, for which Noether obtained a number of important theorems. For example, any irreducible plane algebraic curve with singularities can be transformed by a finite succession of standard quadratic transformations into a curve whose multiple points are all "ordinary" in the sense that the curve has multiple but distinct tangents at such points.

In 1873 Noether proved what came to be his most famous theorem: Given two algebraic curves
$\Phi(x, y) = 0, \psi(x, y) = 0$
which intersect in a finite number of isolated points, then the equation of an algebraic curve which passes through all those points of intersection can be expressed in the form $A\Phi + B\psi = 0$(where A and A are polynomials in x and y) if and only if certain conditions (today called "Noetherian conditions") are satisfied. If the intersections are nonsingular points of both curves, the desired from can readily be achieved. The essence of Noether's theorem, however, is that it provides necessary and sufficient conditions for the case where the curves have common multiple points with contact of any degree of complexity.

Although Noether asserted that his results could be extended to surfaces and hyper surfaces, it was not until 1903 that the Hungarian Julius König actually generalized the Noether theorem to n dimensions by providing necessary and sufficient conditions for the
$A_1f_1 + A_2f_2 + \dots + A_nf_n = 0$
from to be possible for the equation of the surface or hypersurface through the finite set of points of intersection of n surfaces (n=3), or n hypersurfaces (n > 3),
$f_1(x_1, x_2, \dots, x_n) = 0, \dots, f_n(x_1, x_2, \dots, x_n) = 0.$

Noether himself derived a theorem that gives conditions for the equation of a surface passing through the curve of intersection of the surfaces $\Phi(x,y,z)= 0$ and $\psi(x,y,z) = 0$ to have the from $A\Phi + B\psi$. Generalization turned out to be complicated and difficult, but Emanuel Lasker, the chess champion, saw that the issue could be simplified by the use of the theory of polynomial ideals which he and Emmy Noether had developed. Thus he was able to derive Noetherian conditions for the

$A_1f_1 + \ldots + A_rf_r = 0$

form to be possible for a hypersurface through the intersection of

$f_1(x_1, x_2, \ldots, x_n) = 0, \ldots f_r(x_1, x_2, \ldots, x_n) = 0$

with $n > 3$, $r < n$, in which case the intersection will, in general, be a curve or a surface or a hypersurface.

The Noether, König, and Lasker theorems all start with a set of polynomial equations that defines a variety the nature of which varies in the different propositions. The objective in every case is the same, namely to see under what conditions a polynomial that vanishes at all points of the given variety can be expressed as a linear combination of the polynomials originally given. Since those polynomials play a basic role, it is especially significant that the representation of a variety as the intersection of other varieties, that is, by a set of polynomial equations, is not unique. Thus a circle in space might be described as the intersection of two spheres, or as the intersection of a cylinder and a plane, or as the intersection of a cone and a plane, and so forth. Hence, in general, the only impartial way to represent a given variety

$f_i(x_1, x_2, \ldots, x_n) = 0, i = 1, 2, \ldots, r$

where $r \leq n$ and the f_i are polynomials with real or complex coefficients, is not by this one system of equations, but rather in terms of all polynomial equations which points on the variety satisfy. Now if $f(x_1, x_2, \ldots, x_n)$ and $g(x_1, x_2, \ldots, x_n)$ are any two polynomials that vanish at all points of the given variety, then the difference of the polynomials also vanishes at those points, as does the product of either polynomial by an arbitrary polynomial, $A(x_1, x_2, \ldots, x_n)$. By the definition of an ideal, these two facts are sufficient for the set of all polynomials that vanish at every point of the variety to be a real (complex) coefficients, and it is that ideal which is considered to represent the variety. The linear combinations $A_1f_1 + A_2f_2 + \ldots + A_rf_r$ obviously vanish at all points of the given variety and hence belong to the representative polynomial ideal. These are the linear combinations that were the subject of the special criteria developed in the Noether, König, and Lasker theorems. Other important results related to the representative polynomial ideal are contained in a famous proposition of Hilbert, namely his basis theorem.

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Edna E. Kramer