Ostrogradsky, Mikhail Vasilievich | Encyclopedia.com

Complete Dictionary of Scientific Biography COPYRIGHT 2008 Charles Scribner's Sons
21-27 minutes

(*b*. Pashennaya [now in Poltava oblast], Russia, 24 September 1801; *d*. Poltava [now Ukrainian S.S.R.], 1 January 1862), *mathematics, mechanics*.

Ostrogradsky was born on the estate of his father, Vasily Ivanovich Ostrogradsky, a landowner of modest means; his mother was Irina Andreevna Sakhno-Ustimovich. After he had spent several years at the Poltava Gymnasium, the question of his future arose. Ostrogradsky hoped to become a soldier; but the life of an officer was expensive, the salary alone would not support him, and the family had little money to spare. It was decided to prepare him for the civil service and to give him a university education, without which his career would be limited. In 1816 Ostrogradsky enrolled in the physics and mathematics department of Kharkov University, where he received a good mathematical education under A. F. Pavlovsky and T. F. Osipovsky. He was especially influenced by the latter, an outstanding teacher and author of the three-volume *Kurs matematiki* (l8O1–1823), which was well known in its time, and also of philosophical papers in which he criticized Kan's apriorism from the materialistic point of view. In 1820 Ostrogradsky passed the examinations for the candidate's degree, and the university council voted to award it to him. But the minister of religious affairs and national education refused to confirm the council's decision and proposed that Ostrogradsky take the examinations again if he wished to receive his degree. Ostrogradsky rejected this proposal, and therefore did not obtain a university diploma.

The true reason for the arbitrary reversal of the council's decision was the government's struggle with the nonconformist and revolutionary attitudes prevalent among the Russian intelligentsia. The national educational system was headed by conservative bureaucrats who encouraged a combination of piety and mysticism at the universities. In the autumn of 1820 Osipovsky was suspended after having been rector of Kharkov University for a number of years. The animosity felt toward him was extended to Ostrogradsky, his best and favorite pupil, who, according to his own account later, was at that time a complete materialist and atheist. The ground for the refusal to grant him a diploma was that, under the influence of Osipovsky, he and the other students of mathematics did not attend lectures on philosophy and theology.

Ostrogradsky continued his mathematical studies in Paris, where Laplace and Fourier, Legendre and Poisson, Binet and Cauchy worked, and where outstanding courses were offered at the École Polytechnique and other educational institutions. Ostrogradsky's rapid progress gained him the friendship and respect of the senior French mathematicians and of his contemporaries, including Sturm. The Paris period of his life (1822–1827) was for Ostrogradsky not only "years of traveling and apprenticeship" but also a period of intense creative work. Between 1824 and 1827 he presented to the Paris Academy several papers containing important new discoveries in mathematical physics and integral calculus. Most of these discoveries were incorporated in his later papers; a memoir on hydrodynamics was published by the Paris Academy in 1832, and individual results in residue theory appeared, with his approval, in the works of Cauchy.

In the spring of 1828 Ostrogradsky arrived in St. Petersburg. There, over a period of several months, he presented three papers to the Academy of Sciences. In the first, on potential theory, he gave a new, more exact derivation of Poisson's equation for the case of a point lying within or on the surface of an attracting mass. The second was on heat theory, and the third on the theory of double integrals. All three appeared in *Mémoires de l'Académie impériale des sciences de St. Péterbourg*, 6th ser., **1** (1831). On 29 December 1828 Ostrogradsky was elected a junior academician in the section of applied mathematics. In 1830 he was elected an associate and in 1832 a full academician. His work at the Academy of Sciences restored to it the brilliance in mathematics that it had won in the eighteenth century but had lost in the first quarter of the nineteenth.

Ostrogradsky's activity at the Academy was manifold. He contributed some eighty-odd reports in mathematics and mechanics, delivered public lectures, wrote detailed reviews of papers submitted to the Academy, and participated in the work of commissions on the introduction of the Gregorian calendar and the decimal system of measurement. At the behest of the government he also investigated exterior ballistics problems. Ostrogradsky also devoted a great deal of time to teaching and did much to improve mathematical instruction in Russia. From 1828 he lectured at the Naval Corps (later the Naval Academy); from 1830, at the Institute of Means of Communication; and from 1832, at the General Pedagogical Institute. Later he also lectured at the General Engineering College and at the General Artillery College.

From 1847 Ostrogradsky accomplished a great deal as chief inspector for the teaching of the mathematical sciences in military schools. His textbooks on elementary and higher mathematics include a very interesting course on algebra and an exposition of the theory of numbers. Ostrogradsky's educational views were ahead of their time in many respects, particularly his program for the education of children between the ages of seven and twelve, which is expounded in *Considérations sur l'enseignement* (St. Petersburg-Paris, 1860), written with I. A. Blum.

It was mainly Ostrogradsky who established the conditions for the rise of the St. Petersburg mathematical school organized by Chebyshev, and who was the founder of the Russian school of theoretical mechanics. His direct disciples included I. A. Vyshnegradsky, the creator of the theory of automatic regulation, and N. P. Petrov, the author of the hydrodynamic theory of lubricants. Ostrogradsky's services were greatly appreciated by his contemporaries. He was elected a member of the American Academy of Arts and Sciences in 1834, the Turin Academy of Sciences in 1841, and the Rome Academy of Sciences in 1853; in 1856 he was elected a corresponding member of the Paris Academy of Sciences.

Ostrogradsky's scientific work closely bordered upon the developments originating in the École Polytechnique in applied mathematics and in directly related areas of analysis. In mathematical physics he sought a grandiose synthesis that would embrace hydromechanics, the theory of elasticity, the theory of heat, and the theory of electricity by means of a unique homogeneous method. The realization of this plan was beyond the capacity of one man and beyond the resources of the nineteenth century; it remains uncompleted to date.

Ostrogradsky contributed significantly to the development of the method of separating variables that was so successfully applied by Fourier in his work on the conduction of heat (1822). In "Note sur la théorie de la chalcur," presented in 1828 and published in 1831 (see his *Polnoe sobranie trudov,* I, 62–69), Ostrogradsky was the first to formulate a general schema of the method of solving boundary-value problems, which Fourier and Poisson had applied to the solution of individual problems.

For linear partial differential equations with constant coefficients Ostrogradsky established the orthogonality of the corresponding system of proper functions (eigenfunctions). Auxiliary means of calculation in this determination were Ostrogradsky's theorem for the reduction of certain volume integrals to surface integrals and the general formula for arbitrary conjugate linear differential operators with constant coefficients for a three-dimensional space, generally called Green's theorem. In terms of modern vector analysis Ostrogradsky's theorem states that the volume integral of the divergence of a vector field A taken over any volume v is equal to the surface integral of A taken over the closed surface s surrounding the volume v:

(Ostrogradsky himself expressed this proposition in terms of ordinary integral calculus.) This theorem is also called Gauss's theorem, Green's theorem, or Riemann's theorem.

Ostrogradsky next applied his general results to the theory of heat, deriving formulas for the coefficients a_k in the expansion of an arbitrary function $f(x, y, z)$ into a series of eigenfunctions $u_k(x, y, z, \theta_k)$ of the corresponding boundary-value problem—a generalized Fourier series. He noted the difficulty connected with investigating the convergence of this type of series expansion and only touched on the problem of the existence of eigenvalues of θ_k; satisfactory solutions to these questions were not found until the turn of the twentieth century, by Poincaré and V. A. Steklov, among others.

A large part of these discoveries was contained in two memoirs presented by Ostrogradsky to the Paris Academy of Sciences in 1826–1827. In the second of these he solved the problem of the conduction of heat in a right prism with an isosceles right triangle as a base; Fourier and Poisson had previously examined the cases of a sphere, a cylinder, and a right rectangular parallelepiped. Lamé mentioned this solution, which was not published during Ostrogradsky's lifetime, in an 1833 paper. General results in the theory of heat analogous to Ostrogradsky's (but without his integral theorem) were also obtained by Lamé and Duhamel, who presented their papers to the Paris Academy of Sciences in 1829 (published in 1833).

At first Ostrogradsky investigated heat conduction in a solid body surrounded by a medium having a constant temperature. In "Deuxième note stir la théorie de la chaleur," presented in 1829 and published in 1831 (see *Polnoe sobranie trudov*, I, 70–72), he reduced this problem to the case when the temperature of the surrounding medium is a given function of the coordinates of space and time. Finally, in "Sur l'equation relative a la propagation de la chaleur dans l'interieur des liquides," presented in 1836 and published in 1838 (*ibid*., pp. 75–79), he derived the corresponding differential equation for an uncompressed moving liquid free of internal friction, thereby confirming Fourier's results by more thorough analysis.

At the same time Ostrogradsky studied the theory of elasticity; in this field his work meshed with Poisson's parallel investigations. Starting from the work of Poisson, who was the first to establish precisely the necessary condition of the extremum of a double integral with variable limits (1833), Ostrogradsky obtained important results in the calculus of variations. In "Mémoire sur le calcul des variations des intégrates multiples," presented in 1834 and published in 1838 (*ibid*., Ill, 45–64), he derived equations containing the necessary conditions of the extremum of an integral of any multiplicity. To accomplish this he had to develop substantially the theory of multiple integrals. He generalized the integral theorem which he had found earlier, that is, reduced an n-tuple integral from an expression of the divergent type taken over any hypervolume to an $(n - 1)$-tuple integral taken over the corresponding boundary hypersurface; derived a formula for the substitution of new variables in an n-tuple integral (independently of Jacobi, who published it in 1834); and described in detail the general method for computing an n-tuple integral by means of n consecutive integrations with respect to each variable.

In "Sur la transformation des variables dans les intégrales multiples," presented in 1836 and published in 1838 (*ibid*., pp. 109–114), Ostrogradsky was the first to derive in a very modern manner (with a geometrical interpretation) the rule of the substitution of new variables in a double integral; he later extended this method to triple integrals. His work in the calculus of variations was directly related to his work in mechanics.

Ostrogradsky made two important discoveries in the theory of ordinary differential equations. In “Note sur la méthode des approximations successives,” presented in 1835 and published in 1838 (*ibid*., pp. 71–75), he proposed a method of solving nonlinear equations by expanding the unknown quantity into a power series in α, where α is a small parameter, in order to avoid “secular terms” containing the independent variable outside the sign of trigonometric functions. This important idea received further development in the investigations of H. Gylden (1881), Anders Lindstedt (1883), Poincaré, and Lyapunov. In “Note sur les équations différentielles linéaires,” presented in 1838 and published in 1839 (*ibid*., pp. 124–126), Ostrogradsky derived, simultaneously with Liouville,a well-known expression for Wronski’s determinant, one of the basic formulas in the theory of differential linear equations.

Ostrogradsky also wrote several papers on the theory of algebraic functions and their integrals (*ibid*, pp. 13–44, 175–179). The foundation of this theory was laid in 1826 by Abel, whom Ostrogradsky may have met in Paris. From Ostrogradsky’s general results there follows the transcendency of a logarithmic function and of the arc tangent. His investigations were parallel to Liouville’s work in the same area; they were continued in Russia by Chebyshev and his pupils. In De 1’intégration des fractions rationnelles,” presented in 1844 and published in 1845 (*ibid*., pp. 180–214), Ostrogradsky proposed a method for finding the algebraic part of an integral of a rational function without preliminary expansion of the integrand into the sum of partial fractions. This algebraic (and rational) part is calculated with the aid of rational operations and differentiations. Hermite rediscovered this method in 1872 and included it in his textbook on analysis (1873). It is sometimes called Hermite’s method.

In “Mémoire sur les quadratures définies,” written in 1839 and published in 1841 (*ibid*., pp. 127–153), which grew out of his work in ballistics, Ostrogradsky gave a new derivation of the Euler-Maclaurin summation formula with a remainder term in the form in which it is now often presented (Jacobi published an equivalent result in 1834) and applied the general formulas to the approximation calculus of definite integrals. Several articles are devoted to probability theory—for example, one on the sample control of production, presented in 1846 and published in 1848 (*ibid*., pp. 215–237), and to algebra. In general, however, as a mathematician Ostrogradsky was always an analyst.

Ostrogradsky’s memoirs in mechanics can be divided into three areas: the principle of virtual displacements; dynamic differential equations; and the solution of specific problems.

Ostrogradsky’s most important investigations in mechanics deal with generalizations of its basic principles and methods. He made a substantial contribution to the development of variational principles. The fundamental “Mémoire sur les équations différentielles relatives au problème des isopérimètres,” presented in 1848 and published in 1850 (*ibid*.. II, 139–233). belongs in equal measure to mechanics and the calculus of variations. Because of his mathematical approach Ostrogradsky’s investigations significantly deepened the understanding of variational principles.

In the paper just cited Ostrogradsky examined the variational problem in which the integrand depends on an arbitrary number of unknown functions of one independent variable and their derivatives of an arbitrary order and proved that the problem can be reduced to the integration of canonical Hamiltonian equations, which can be viewed as the form into which any equations arising in a variational problem can be transformed. This transformation requires no operation other than differentiation and algebraic operations. The credit for this interpretation of the dynamics problem belongs to Ostrogradsky. He also eased the restrictions on constraints, which had always been considered stationary, and thus significantly generalized the problem. Therefore the variational principle formulated by Hamilton in 1834–1835 might more accurately be called the Hamilton-Ostrogradsky principle. Jacobi also worked in the same direction, but his results were published later (1866).

At the same time Ostrogradsky prepared the important paper “Sur les intégrales des équations générales de la dynamique” also presented in 1848 and published in 1850 (*ibid*., III. 129–138). In it he showed that even in the more general case, when the constraints and the force function depend on time (this case was not considered by Hamilton and Jacobi). the equations of motion can be transformed into Hamiltonian form. Generally, the development of the classical theory of the integration of canonical equations was carried out by Hamilton, Jacobi, and Ostrogradsky.

Ostrogradsky’s results related to the development of the principle of virtual displacements are stated in “Considerations generales sur les moments des forces,” presented in 1834 and published in 1838 (*ibid*., 11, 13–28). This paper significantly broadened the sphere of application of the principle of virtual displacements, extending it to the relieving constraints.

In “Mémoire sur les déplacements instantanés des systémes assujettis à des conditions variables,” presented and published in 1838 (*ibid*., pp. 32–59), and “Sur le principe des vitesses virtuelles et sur la force d’inertie,” presented in 1841 and published in 1842 (*ibid*., pp. l04 109), Ostrogradsky gave a rigorous proof of the formula expressing the principle of virtual displacements for the case of nonstationary constraints.

“Mémoire sur la théorie générale de la percussion,” presented in 1854 and published in 1857 (*ibid*., pp. 234–266), presents Ostrogradsky’s investigations of the impact of systems. in which he assumed that the constraints arising at the moment of impact are preserved after the impact. The principle of virtual displacements is extended here to the phenomenon of inelastic impact, and the basic formula of the analytical theory of impact is derived.

Ostrogradsky also wrote papers containing solutions to particular problems of mechanics that had arisen in the technology of his time. A series of his papers on ballistics deserves special mention: “Note sur le mouvement des projectiles sphériques dans un milieu résistant" and "Mémoire sur le mouvement des projectiles spheriques dans l'air," both presented in 1840 and published in 1841; and "Tables pour faciliter le calcul de la trajectoire que decrit un mobile dans un milieu resistant," presented in 1839 and published in 1841 (*ibid*., pp. 70–94). In the first two papers Ostrogradsky investigated the motion of the center of gravity and the rotation of a spherical projectile the geometrical center of which does not coincide with the center of gravity; both topics were important for artillery at that time. The third paper contains tables, computed by Ostrogradsky, of the function $\Phi(\theta) = 2\int d\theta/\sin^3\theta$ used in ballistics. These papers stimulated the creation of the Russian school of ballistics in the second half of the nineteenth century.

BIBLIOGRAPHY

I. Original Works. Most of Ostrogradsky's papers appeared in French in publications of the St. Petersburg Academy of Sciences. The most complete bibliography of his works and of writings concerning him is by M. G. Novlyanskaya in Ostrogradsky's *Izbrannye trudy* ("Selected Works"), V. I. Smirnov, ed. (Moscow, 1958), 540–581. Other collections of Ostrogradsky's writings are *Polnoe sobranie sochineny* ("Complete Collected Works"), I, pt. 2, *Lektisii po analiticheskoy ,mekhanike*, 1834 ("Lectures on Analytic Mechanics"), and II, *Lektsii algebraicheskogo i transtsendenogo analiza*, 1837 ("Lectures on Algebraic and Transcendental Analysis" : MoscowLeningrad, 1940–1946), never completed; and *Polnoe sobranie trudov* ("Complete Collected Works"), I. Z. Shtokalo, ed., 3 vols. (Kiev, 1959–1961), which contains commentaries and articles by I. Z. Shtokalo, I. B. Pogrebyssky, E. Y. Remez, Y. D. Sokolov, S. M. Targ, and others but does not include the 1834 and 1837 works above or the two articles that fol1o and "Dokazatelstvo odnoy teoremy integralnogo ischislenia" ("Proof of One Theorem in the Integral Calculus") and "Memuar o rasprostranenii tepla vnutri tverdykh tel" ("Memoir on the Conduction of Heat Within Solid Bodies"), in *Istorikomatematicheskie issledovaniya*, **16** (1965), 49–96, Russian translations of two previously unpublished articles presented to the Paris Academy in 1826–1827, with an introduction by A. P. Youschkevitch.

II. Secondary Literatrure. Y. L. Geronimus, *Ocherki o rabotakh korifeev russkoy mekhaniki* ("Essays on the Work of the Leading Figures in Russian Mechanics"; Moscow, 1952), 13–57; B. V. Gnedenko and I. B. Pogrebyssky, *Mikhail Vasilievich Ostrogradsky (1801–1862). Zhizn i rabota. Nauchnoe i pedagogicheskoe nasledie* ("… Life and Work. Scientific and Pedagogical Heritage"; Moscow, 1963), the most complete work on his life and accomplishments; A. T. Grigorian, *Mikhail Vasilievich Ostrogradsky (1801–1862)* (Moscow, 1961); and *Ocherki istorii mekhaniki v Rossii* ("Essays on the History of Mechanics in Russia"; Moscow, 1961), see index; *Istoria otechestvennoy matematiki* ("History of Russian Mathematics"). I. Z. Shtokalo, ed.-in-chief, II (Kiev, 1967), see index; A. I. Kropotov and I. A. Maron, *M. V. Ostrogradsky i ego pedagogicheskoe nasledie* ("Ostrogradsky and His Pedagogical Heritage"; Moscow, 1961); *Mikhail Vasilievich Ostrogradsky. 1862–1962. Pedagogicheskoe nasledie. Dokumentry o zhizni i deyatelnosti* ("… Pedagogical Heritage. Documents on His Life and Activity"), I. B. Pogrebyssky and A. P. Youschkevitch, eds. (Moscow, 1961), a supp. to *Polnoe sobraine trudov* containing a Russian trans. of Ostrogradsky and Blum's *Considerations sur l'enseignement* (St. Peterburg-Paris, 1860), and Ostrogradsky's "Zapiski integralnogo ischislenia" ("Lectures on Integral Calculus"); E. Y. Remez, "O matematicheskikh rukopisyakh akademika M. V. Ostrogradskogo" ("On the Mathematical Manuscripts of Academician M. V. Ostrogradsky"), in *Istoriko-matematicheskie issledovaniya*, **4** (1951), 9–98; S.P. Timoshenko, *History of Strength of Materials* (New York-TorontoLondon, 1953); I. Todhunter, *A History of the Progress of the Calculus of Variations During the Nineteenth Century* (Cambridge, 1861); P. I. Tripolsky, ed., *Mikhail Vasilievich Ostrogradsky. Prazdnovanie stoletia dnya ego rozhdenia* ("… Celebration of the Centenary of His Birth"; Poltava, 1902), which contains short sketches on his life and scientific and educational activities-of special interest are an article by Lyapunov on his work in mechanics (pp. 115–118) and one by Steklov on Ostrogradsky's paper in mathematical physics (pp. 118–131); A. Youschkevitch, *Michel Ostrogradski et le progres de la science au XIX^e siecle* (Paris, 1967); and *Istoria matematiki v Rossii do 1917 goda* ("History of Mathematics in Russia to 1917"; Moscow, 1968), see index; and N.E. Zhukovsky, "Uchenye trudy M. V. Ostrogradskogo po mekhanike" ("Ostrogradsky's Scientific Works in Mechanics"), in Zhukovsky's *Polnoe sobranie sochineny* ("Complete Collected Works"), VII (Moscow-Leningrad, 1950), 229–246.

A. P. Youschkevitch