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(*b*. Sansepolcro, Italy, *ca*. 1445; *d*. Sansepolcro, 1571), *mathematics, bookkeeping*.

Luca Pacioli (Lucas de Burgo), son of Bartolomeo Pacioli, belonged to a modest family of Sansepolcro, a small commercial town in the Tiber valley about forty miles north of Perugia. All we know of his early life is that he was brought up by the Befolci family of Sansepolcro. It has been suggested that he may have received part of his early education in the atelier of his older compatriot Piero della Francesca (1410–1492). As a young man he entered the service of Antonio Rompiansi, a Venetian merchant who lived in the fashionable Giudecca district. Pacioli lived in Rompiansi's house and helped to educate his three sons. While doing so he studied mathematics under Domenico Bragadino, who held classes in Venice, probably at the school that the republic had established near the Church of San Giovanni di Rialto for those who did not want to go to Padua. The experience Pacioli gained in Rompiansi's business and the knowledge he gathered at Bragadino's school prompted him to write his works on arithmetic, the first of which he dedicated to the Rompiansi brothers in 1470. Their father was dead by then Pacioli's employment probably had ended. He then stayed for several months in Rome as the guest of the architect Leone Battista Alberti.

Sometime between 1470 and 1477 Pacioli was ordained as a friar in the Franciscan order in fulfillment of a vow. After completing his theological studies he began a life of peregrination, teaching mathematics in various cities of Italy. From 1477 to 1480 he gave lessons in arithmetic at the University of Perugia and wrote a treatise on arithmetic for the benefit of his students (1478). In 1481 he was in Zara (now Zadar, Yugoslavia), then under Venetian rule, where he wrote another work on arithmetic. After teaching mathematics successively at the universities of Perugia, Naples, and Rome in 1487–1489, Pacioli returned to Sansepolcro. In 1494 his major work, *Summa de arithmetica, geomertia, Proportioni et proportionalita*, was ready for the publisher and he went to Venice to supervise the printing. He dedicated the book to the young duke of Urbino, Guidobaldo da Montefeltro (1472–1508), who, it is believed, was his pupil. The dedicatory letter suggests that Pacioli had been closely associated with the court of Urbino. This is confirmed by the altarpiece painted by Piero della Francesca for the Church of San Bernardino in Urbino (now in Milan), in which the figure of St. Peter the Martyr is portrayed by Pacioli. The painting shows Duke Federigo (Guidobaldo's father) praying before the Virgin and Child surrounded by angels and saints. A painting by Jacopo de' Barbari in the Naples Museum shows Pacioli demonstrating a lesson in geometry to Guidobaldo.

In 1497 Pacioli was invited to the court of Ludovico Sforza, duke of Milan, to teach mathematics. Here he met Leonardo da Vinci, who was already in Sforza's employment. That Leonardo consulted Pacioli on matters relating to mathematics is evident from entries in Leonardo's notebooks. The first part of Pacioli's *Divina proportione* was composed at Milan during 1496–1497, and it was Leonardo who drew the figures of the solid bodies for it. Their stay in Milan ended in 1499 with the entry of the French army and the consequent capture of Sforza. Journeying through Mantua and Venice, they arrived in Florence, where they shared quarters. Leonardo's stay in Florence, which lasted until the middle of 1506, was interrupted by a short period in the service of Cesare Borgia.

In 1500 Pacioli was appointed to teach Euclid's *Elements* at the University of Pisa, which had been transferred to Florence because of the revolt of Pisa in 1494. The appointment was renewed annually until 1506. In 1504 he made a set of geometrical figures for the Signoria of Florence, for which he was paid 52.9 lire. He was elected superior of his order for the province of Romagna and shortly afterward (1505) was accepted as a member of the monastery of Santa Croce in Florence. During his stay in Florence, Pacioli also held an appointment at the University of Bologna as *lector ad mathematicam* (1501–1502). At this time the University of Bologna had several *lectores ad arithmeticam*, one of whom was Scipione dal Ferro, who was to become famous for solving the cubic equation. It has been suggested that Pacioli's presence in Bologna may have encouraged Scipione to seek a solution of the cubic equation, but there is no evidence to support this apart from Pacioli's statement in the *Summa* that the cubic equation could not be solved algebraically.

Since his arrival in Florence, Pacioli had been preparing a Latin edition of Euclid's *Elements* and an Italian translation. He had also written a book on chess and had prepared a collection of recreational problems. On 11 August 1508 Pacioli was in Venice, where he read to a large gathering in the Church of San Bartolomeo in the Rialto an introduction to book V of Euclid's *Elements*. A few months later, on a supplication made by him to the doge of Venice, he was granted the privilege that no one but he could publish his works within the republic for fifteen years. The works listed were the fifteen books of Euclid. *Divina proportione*, "De viribus quantitatis, " "De ludo scachorum," and *Summa de arithmetica*. The Latin edition of Euclid and the *Divina proportione* were published in 1509. Pacioli was called once more to lecture in Perugia in 1510 and in Rome in 1514.

On several occasions Pacioli came into conflict with the brethren of his order in Sansepolcro. In 1491, on a complaint made to the general of the order, he was prohibited from teaching the young men of the town; but this did not prevent his being called to preach the Lenten sermons there in 1493. It is likely that certain minor privileges granted to him by the Pope had aroused

enmity or jealousy. Although a petition had been sent to the general of the order in 1509, he was shortly afterward elected commissioner of his convent in Sansepolcro. A few years later Pacioli renounced these privileges and in 1517, shortly before his death, his fellow townsmen petitioned that he be appointed minister of the order for the province of Assisi.

The commercial activity of Italy in the late Middle Ages had led to the composition of a large number of treatises on practical arithmetic to meet the needs of merchant apprentices. Evidence of this is found in the extant works of the *maestri d'abbaco* of central and northern Italy. Some of them even contained chapters devoted to the rules of algebra and their application, no doubt influenced by the *Liber abbaci* of Leonardo Fibonacci. The first printed commercial arithmetic was an anonymous work that appeared at Treviso in 1478. By the end of the sixteenth century about 200 such works had been published in Italy. Pacioli wrote three such treatises: one at Venice (1470), one at Perugia (1478), and one at Zara (1481). None of them was published and only the second has been preserved.

Pacioli's *Summa de arithmetica*… (1494) was more comprehensive. Unlike the practical arithmetics, it was not addressed to a particular section of the community. An encyclopedic work (600 pages of close print, in folio) written in Italian, it contains a general treatise on theoretical and practical arithmetic; the elements of algebra; a table of moneys, weights, and measures used in the various Italian states; a treatise on doubleentry bookkeeping; and a summary of Euclid's geometry. He admitted to having borrowed freely from Euclid, Boethius, Sacrobosco, Leonardo Fibonacci, Prosdocimo de' Beldamandi, and others.

Although it lacked originality, the *Summa* was widely circulated and studied by the mathematicians of the sixteenth century. Cardano, while devoting a chapter of his *Practica arithmetice* (1539) to correcting the errors in the *Summa*, acknowledged his debt to Pacioli. Tartaglia's *General trattato de' numeri et misure* (1556–1560) was styled on Pacioli's *Summa*. In the introduction to his *Algebra*, Bombelli says that pacioli was the first mathematician after Leonardo Fibonacci to have thrown light on the science of algebra— "primo fu che luce diede a quella scientia, "[1] This statement, however, does not mean that algebra had been neglected in Italy for 300 years. Another edition of Pacioli's *Summa* was published in 1523.

Pacioli's treatise on bookkeeping, "De computis et scripturis," contained in the *Summa*, was the first printed work setting out the "mehod of Venice," that is, double-entry bookkeeping. Brown has said. "The history of bookkeeping during the next century the *De computis* through the various countries of Europe." [2]

The *Divina proportione*, written in Italian and published in 1509, was dedicated to Piero Soderini, perpetual gonfalonier of Florence. It comprised three books: "Compendio de divina proportione," " Tractato de l'architectura," and "Libellus in tres partiales tractatus divisus quinque corporum regularium," The first book, completed at Milan in 1497, is dedicated to Ludovico Sforza. Its subject is the golden section or divine proportion, as Pacioli called it, the ratio obtained by dividing a line in extreme and mean ratio. It contains a summary of Euclid's propositions (including those in Campanus' version) relating to the golden section, a study of the properties of regular polyhedrons, and a description of semiregular polyhedrons obtained by truncation or stellation of regular polyhedrons. Book 2 is a treatise on architecture, based on Vitruvius, dedicated to Pacioli's pupils at Sansepolcro. To this he added a treatise on the right proportions of roman lettering. The third book is an Italian translation, dedicated to Soderini, of Piero della Francesca's *De corporibus regularibus*.

Also in 1509 Pacioli published his Latin translation of Euclid's *Elements*. The first printed edition of Euclid (a Latin translation made in the thirteenth century by Campanus of Novara from an Arabic text) had appeared at Venice in 1482. It was severely criticized by Bartolomeo Zamberti in 1505 when he was publishing a Latin translation from the Greek. Pacioli's edition is based on Campanus but contains his own emendations and annotations. It was published in order to vindicate Campanus, apparently at the expense of Ratdolt, the publisher of Campanus' translation.

Among the works that Pacioli had intended to publish is "De viribus quantitatis," a copy of which in the hand of an amanuensis, is in the University Library of Bologna.[3] The name of the person to whom the work was dedicated has been left blank. It is an extensive work (309 folios) divided into three parts: the first is a collection of eighty-one mathematical recreational problems, a collection larger than those published a century later by Bachet de Méziriac and others; the second is a collection of geometrical problems and games; the third is a collection of proverbs and verses. No originality attaches to this work, for the problems are found scattered among earlier arithmetics and, in fact, a collection is attributed to Alcuin of York. Pacioli himself called the work a compendium. Some of the problems are found in the notebooks of Leonardo da Vinci, and the work itself contains frequent allusions to him.

Pacioli's Italian translation of Euclid's *Elements* and his work on chess, "De ludo scachorum," dedicated to the marquis of Mantua, Francesco Gonzaga, and his wife, Isabella d'Este, were not published and there is no trace of the manuscripts.

Vasari, in writing the biography of Piero della Francesca, accused Pacioli of having plagiarized the work of his compatriot on perspective, arithmetic, and geometry.[4] The accusations relate to three works by Piero— *De prospectiva pingendi*, "Libellus de quinque corporibus regularibus," and *Trattato d'abaco*, all of which have been published only since the turn of the twentieth century.[5] In 1908 Pittarelli came to the defense of Pacioli, pointing out that any accusation of plagiarism in regard to *De prospectiva* was unjust, since Pacioli had acknowledged Piero's work in both the *Summa* and the *Divina proportione*.[6] As for the *Libellus*, it has been established by Mancini that Pacioli's work is a translation of it that lacks the clarity of the original.[7] In the case of the *Trattato*, although Piero can claim no originality for it, it has been possible to find in it at least 105 problems of the *Summa*.[8]

The writings of Pacioli have provided historians of the Renaissance with important source material for the study of Leonardo da Vinci. The numerous editions and translations of the *De computis et scripturis* are evidence of the worldwide esteem in which Pacioli is held by the accounting profession. Pacioli made no original contribution to mathematics; but his *Summa*, written in the vulgar tongue, provided his countrymen, especially those not schooled in Latin, with an encyclopedia of the existing knowledge of the subject and enabled them to contribute to the advancement of algebra in the sixteenth century.

NOTES

1. Rafael Bombelli, *Algebra* (Bologna, 1572), d 2v.

2. Brown, *History of Accounting*, p. 119.

3. An ed. of the MS by Paul Lawrence Rose of New York University is in press.

4. Vasari, *Vite*, pp. 360, 361, 365.

5. Codex Palat. Parma, Published by C. Winterberg (1899); Codex Vat. Urb. lat. 632, published in 1915 by Mancini; Codex Ash. 280, published in 1971 by Arrighi.

6. Pittarelli, "Luca Pacioli…."

7. Mancini, "L'opera 'De corporibus regularibus'…."

8. Jayawardene, "The *Trattato d'abaco* of Piero della Francesca."

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II. Secondary Literature. Studies of Pacioli's life and work are listed by G. Masotti Biggiogero in "Luca Pacioli e la sua 'Divina proportione,'" in *Rendiconti dell' Instituto Iombardo di scienze, ser*. A, **94** (1960), 3–30.

The earliest biographical sketch, written by Bernardino Baldi in 1589, was not published. Baldassare Boncompagni made a critical study of it with the help of archival documents: "Intorno alle vite inedite di tre matematici … scritte da Bernadino Baldi," in Bullettino di bibliografia e di storia del le Scienze mathematiche e fisiche, **12** (1879), 352–438, 863–872. Other archival documents were published by D. Ivano Ricci in *Luca Pacioli, l'uomo e lo scienziato* (Sansepolcro, 1940). R. E. Taylor, *No Royal Road:* Luca pacioli and His Times (Chapel Hill, N. C., (1942), is a lively narrative but unreliable as a biography. Pacioli's work is discussed by L. Olschki in Geschichte *der neusprachlichen wissenschaftlichen Literatur*, I (Leipzig, 1919), 151–239.

Stanley Morison, *Fra Luca Pacioli of Borgo San Sepolcro* (New York, 1933), contains a study of that part of the Divina Proportione dealing with roman lettering. The history of bookkeeping is discussed by Richard Brown in *A History of Accounting and Accountants* (London, 1905), 108–131.

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