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(*b*.. Nizhniy Lomov, Penza guberniya [now oblast], Russia, 11 February 1891; *d*. Moscow, U.S.S.R., 13 July 1941)

mathematics.

Privalov, son of Ivan Andreevich Privalov, a merchant, and Eudokia Lvovna Privalova, graduated from the Gymnasium in Nizhniy Novgorod (now Gor'kiy) in 1909 and in the same year entered the department of physics and mathematics of Moscow University. He graduated in 1913 and remained at the university to prepare for an academic career. His scientific supervisor was D. F. Egorov, and his work was greatly influenced by N. N. Lusin. In 1916 he passed the examinations for the master's degree and began teaching at Moscow University as lecturer; in 1917 he became professor at Saratov University. Privalov returned to Moscow in 1922 and for the rest of his life was professor of the theory of functions of a complex variable; from 1923 he also taught at the Air Force Academy. He received his doctorate in physics and mathematics in 1935 without defending a dissertation. He was an active member of the Moscow Mathematical Society, of which he was vice-president from 1936. He was elected a corresponding member of the Soviet Academy of Sciences in 1939.

Privalov's first works, dealing with orthogonal series and integral equations, appeared in 1914; he then turned to the study of properties of Fourier series. His principal interests soon concentrated, however, upon boundary properties of analytic functions, that is, their properties in the vicinity of the set of their singular points; a considerable part of his seventy-nine published works is concerned with these problems. Privalov was closely preceded in this field by V. V. Golubev, another Moscow mathematician who taught at Saratov University and in 1916 published his master's degree thesis on analytic functions with a perfect set of singular points.

In 1917 Privalov and Lusin established a wide-ranging program of studies on the theory of analytical functions by means of the theory of measure and Lebesgue integrals, and began to put it into effect at once. In "Cauchy lntegral" (1918), which continued the works of Pierre Fatou (1906) and Golubev (1916) and was initially intended as a master's degree thesis, Privalov described many new discoveries in the theory of boundary properties of analytic functions defined in the domain bounded by one rectifiable curve. Thus it was proved that under conformal mapping of such domains the angles are preserved on the boundary almost everwhere. Privalov and Lusin established the invariance of a point set with a measure equal to zero on the boundary ; and Privalov solved many problems on the unicity of analytical functions, proved the existence almost everwhere of the Cauchy type of integral, established its boundary properties, and investigated in detail the problem of determining the analytical function with its values on the boundary by means of the Cauchy type of integral. Because "Cauchy Integral" appeared at a time when scientific contacts between Russia and other countries were almost nonexistent, it did not attract attention abroad. In 1924–1925 some of the results obtained in that work were reported by Privalov in two articles in French, the second of which was written with Lusin. These results were considerably supplemented here by the solution of a number of new and difficult problems of unicity of analytic functions determined by the set of their values on the boundary.

In 1934 Privalov began to study subharmonic functions, which had been introduced as early as 1906 and became the subject of Riesz's works in 1925–1930. In *Subgarmonicheskie funktsii* Privalov presented an original systematic construction of the general theory of this class of functions in close connection with the theory of harmonic functions. He also elaborated the ideas of his work on Cauchy's integral. Shortly before his death Privalov summarized many studies in *Granichnye svoystva odnoznachnykhanaliticheskikh funktsy*.

Some of Privalov's manuals, especially university-level courses on the theory of functions of complex variables and a manual of analytical geometry for technological colleges, became very widely used in the Soviet Union.

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II. Secondary Literature. For bibliographies of Privalov's works see *Matematika v SSSR za tridsat let* ("Mathematics in the U.S.R.S. During the [Last] Thirty Years"; Moscow-Leningrad, 1948); and V. Stepanov, "Ivan Privalov. 1891–1941", in *Izvestiya Akademii nauk* SSSR, ser. math., **6** (1941), 389–394.

A. P. Youschkevitch

A. T. Grigorian