Riccati, Jacopo Francesco | Encyclopedia.com

(*b*. Venice, Italy, 28 May 1676; *d*. Treviso, Italy, 15April 1754)

mathematics.

Riccati was the son of a noble family who held land near Venice. He received his early education at the school for the nobility in Brescia then entered the University of Padua where, to please his father, he began to study law. He took the degree on 7 June 1696. At Padua he became a friend of Stefano degli Angeli, who encouraged him in the pursuit of mathematics; Riccati's detailed study of recent methods of mathematical analysis enabled him to solve, in 1710, a difficult problem posed in the *Giornale de' letternati d'Italia*. He soon embarked on the extensive series of mathematical publications that brought him contemporary fame. His renown was such that Peter the Great invited him to come to Russia as president of the St. Petersburg Academy of Sciences; he was also asked to Vienna as an imperial councillor and offered a professorship as the University of Padua. He declined all these offers, since he preferred to stay in Italy and devote himself to his studies privately, in his own family circle. He was often consulted by the senate of Venice, particularly on the construction of dikes along rivers and canals, and his expertise was deferred to on this and other topics. In addition to his works in mathematics and enthusiastic advocacy of Newton's ideas.

Riccati's mathematical work dealt chiefly with analysis and, in particular, with differential equations. He achieved notable results in lowering the order of equations and in the separation of variables. In 1722–1723 he was engaged to teach infinitesimal calculus to two young noblemen, Lodovico Riva and Giuseppe Suzzi, and his lectures to them, subsequently published, demonstrate the technique he employed. In expounding the know nethods of integration of first-order differential equations, Riccati studied those that equations that may be integrated with appropriate agebraic transformation before considering those that require a change of varriable. He then discussed certain devices suggested by Johann I Bernoulli and expounded the method used by Gabreiele manfredi to integrate homogeneous equtions. He further pointed out that in order to determine a curve endowed with an assinged property, it may at times be useful to relate it to some of coordinates other than the usual one.

Riccati then discussed, with many examples, the integration methods that he himself had devised. Of these, one involves the reduction of the quation to a homogeneous one, while another, more interesting method is that of "halved separation," as Riccati called it. The technique of halved separation comprises three obrations. In the first, the entire equation is multiplied or divided by an appropriate function of the unknown so that it becomes integrable; second, after this integration has been carried out, the ersult is considered to be equal to a new unknown, and one of the original variables is thus eliminated; and finally, the first two procedures are applied to the result until a new and desired result is attained. Riccati communicated this method to Bernardino Zendrini, a mathematician who was also superintendent of waterworks for the Venetian state; Zendrini passed it no to Leibniz, who considered it hirgly ingenious and wrote Riccati an encouraging letter. Riccati first published it as "Contrarispostaalle annotazioni del sig. Nicclò Bernoulli" in *Giornale de' letterati d' italia* in 1715.

At a later point in his lectures, Riccati also dealt with higher-order equations, indicating how some of the techniques implicit in them may be further applied. He also took up the methods used by other mathematicians, of which he is occasionally critical.

In an earlier work, published in the *Giornale de' letterati d'ITalia* in 1712, Riccari had already given the solutions to a number of problems related to plane curves determined by curvature properties, for which the integration of second-order differential equations is required. His results were widely known and used by other mathematicians (indeed, they were sometimes republished without mention of his name), and he himself repeated them in his lectures. The earlier memoir also contains Riccati's; important statment that the method he had used will lead to the integration of all differential equations of the type

which he was the first to consider in their generality.

Riccati also provided, in a memoir communicated to Zendrini in 1715 (and intended for publication in the *Giornale*, although it was not actually published until 1747, when it appeared in the Commentarii of the Bolgna Academy of Sciences), the integration of an equation of the type r = f(s), in which *r* is the radius of curvature and *s* the length of the arc. His result is significant because it bears upon the search for the Cartesian equation of a curve determined by its intrinsic equation. Riccati had already solved a general problem of the same type, in determining a v\curve of which the expression for the radius of curvature is known at any point whatever as a function of the radius vector("Soluzione generate del problema inverso intorno ai raggi osculatori," published in the *Giornale* in 1712).

In his “Animadversiones in aequations differentiales secundi gradus,” published in *Acta eruditorum* in 1724, Riccati suggested the study of cases of integrability of the equation

which is how known by his name. In response to this suggestion Niklaus II Bernoulli wrote an important treatise on the equation and Daniel Bernoulli presented, in his *Exercitationes quaedam mathematicae*, the conditions under which it may be integrated by the method of separation of the variables. Euler also integrated it.

Riccati also drew upon the integration of differential equations in the context of his work in Newtonian mechanics. Thus, he studied the motion of cycloidal pendulums under the hypothesis that the force of resistance varies as the square of the velocity, and, in the results that he communicated to his former student Suzi (on 5 March 1732), included the integration, by means of halved separation, of an equation in two variables, namely the arc s and the velocity u. In a memoir on the laws of resistance governing the retardation of the motion of bodies by a fluid medium he integrated, using a procedure different from that used by Mandredi, the homogeneous equation $ydy + 2\ budy = udu$.

In differntial geometry, Riccati demonstrated that the segment lying between an arc and a tangent at point P of the ordinate at the point of the curve following P is an infinitesimal of the second order. In addition, he also studied a problem that had interested Descartes and Fermat—how to determine the algebraic curves of the minimum degree that must be employed to solve a geometric problem of a given order.

Riccati carried on an extensive correspondence with mathematicians all over Europe. His works were collected and published, four years after his death, by his sons, of whom two, Vincenzo and Giordano, were themselves eminent mathematicians.

BIBLIOGRAPHY

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II. Secondary Literature. On Riccati and his work, see A. Agostini, “Riccati,” in *Enciclopedia italiana*, XXIX (1936), 241; L. Berzolari, G. Vivanti, and D. Gigli, eds., *Enciclopedia delle matematiche elementari*, I, pt. 2 (Milan, 1932), 527; A. Fabroni, *Vitae italorum doctrina excellentium*, XVI (Pisa, 1795), 376 ff.; and G. Loria, *Curve piane speciali algebriche e trascendeniti*, II (Milan, 1930), 168, 170; and *Storia delle matematiche*, 2nd ed.l (Milan, 1950), 630, 631, 659 ff., 667, 701.

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