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(*b*. Ambert, Basse-Auvergne, France, 21 April 1652; *d*. Paris, France, 8 November 1719)

mathematics.

The son of a shopkeeper, Rolle received only a very elementary education. He worked first as a transcriber for a notary and then for various attorneys in his native region. At the age of twenty-three he moved to Paris, Married early and burdened with a family, he had difficulty earning sufficient money as master scribe and reckoner. But by independent study he learned algebra and Diophantine analysis. In the *Journal des sçavans* of 31 August 1682 Rolle gave an elegant solution to a difficult problem publicly posed by Ozanam: to find four numbers the difference of any two of which is a perfect square as well as the sum of the first three. Ozanam had stated that the smallest of the four numbers would have at least fifty figures. Rolle provided a solution in which the four numbers were expressed by homogeneous polynomials in two variables and of degree twenty. The smallest numbers found in this fashion each had only seven figures.

This brilliant exploit brought Rolle public recognition. Colbert took an interest in him and obtained for him a reward and, it was said, a pension. Rolle later enjoyed the patronage of the minister Louvois. He gave lessons in elementary mathematics to the latter's fourth son, Camille Le Tellier, abbé de Louvois (1675–1718). Rolle even received an administrative post in the ministry of war, from which he soon resigned.

Rolle entered the Académie des Sciences in 1685 with the title—rather disconcerting for us—of *élève astronome*. When the Académie was reorganized in 1699 he became *pensionnaire géomètre*, a post that assured him a regular salary. In 1708 he suffered an attack of apoplexy. He recovered, but a second attack in 1719 proved fatal.

Although it was his skill in Diophantine analysis that made Rolle's reputation, his favorite area was the algebra of equations, in which he published *Traité d'algèbre* (1690), his most famous work. In this book he designated, following Albert Girard (1629). the *n*th root of a number *a*, *a*, not as $\sqrt{a}$, as was usually done before him. His notation soon became generally accepted. He retained the Cartesian equality sign until 1691. when he adopted the equal sign (=), which originated with Robert Recorde (1557).

In 1691, Rolle adopted, in advance of many of his contemporaries and in opposition to Descartes, the present order relation for the set of the real numbers: "I take $-2a$ for a greater quantity than $-5a$."

Rolle's *Algèbre* contains interesting considerations on systems of affine equations. Following the techniques established by Bachet de Méziriac (1621), Rolle utilized the Euclidean algorithm for resolving Diophantine linear equations. He employed the same algorithm to find the greatest common divisor of two polynomials, and in 1691 he was able to eliminate one variable between two equations.

The *Traité d'algèbre*, the language of which is so special, has remained famous, thanks notably to the method of "cascades." Rolle used this method to separate the roots of an algebraic equation. He justified it by showing (1691) that if $P(x) = 0$ is the given equation, and if it admits two reals roots a and b. then $P'(b)= (b - a)\, Q(b)$, where Q is a polynomial. $P'(x)$, a polynomial derived from $P(x)$, is what Rolle called the "first cascade" of the polynomial $P(x)$. The second cascade is our second derivative, and so on.

Arranging the real roots in ascending order, Rolle showed that between two consecutive roots of $P(x)$ there exists a root of $P'(x)$. His methods of demonstration are elaborations of the method utilized by Jan Hudde in his search for extrema (1658).

In 1846 Giusto Bellavitis gave Rolle's name to the present theorem: if the function $f(x)$ is defined and continuous on the segment $[ab]$, if $f(a) = f(b)$, and if $f'(x)$ exists in the interior of the segment, then $f'(x)$ is equal to zero at least once in the segment.

In 1699 the three pensionary geometers of the Academy were the Abbé Jean Gallois, a partisan of Greek mathematics; Rolle, an autodidact but very well versed in Cartesian techniques; and Pierre Varignon, who favored the ideas of Leibniz. L'Hospital was an honorary academician; in 1696 he had published *Analyse des infiniment petits*. The Academy was very divided over infinitesimal analysis. Rolle—incited, it was said, by influential persons—vigorously attacked infinitesimal analysis and strove to demonstrate that it was not based on solid reasoning and led to errors. Among the examples he chose were the curves

Varignon defended the new methods and pointed out the paralogisms that Rolle displayed in discussing these examples. The latter, too plebeian to control himself, created an uproar. A commission established to resolve the matter was unable to come to a decision. The dispute lasted from 1700 to 1701, and then continued in the *Journal des sçavans* in the form of exchanges between Rolle and a newcomer, Joseph Saurin. The Academy again intervened, and in the fall of 1706 Rolle acknowledged to Varignon, Fontenelle, and Malebranche that he had given up and fully recognized the value of the new techniques.

Rolle also displayed a certain vigor in the field of Cartesian geometry. In 1693, in the *Journal des sçavans*, he offered a prize of sixty pistoles for the solution, without the use of his methods, of the following problem: construct the roots of an equation by utilizing a given arc of an algebraic curve. Before leaving Paris, Johann I Bernoulli had given a solution to this problem in Latin to L'Hospital and had requested him to submit a French translation to the *Journal*. The solution did not meet with Rolle's approval, and the resulting polemic lasted for five numbers of the *Journal*; the sixty pistoles remained in the donor's coffers.

Another of Rolle's achievements is an observation that, though initially paradoxical, was recognized as correct by Saurin: two arcs of algebraic curves the convexity of which is in the same direction can have a large number of common points.

Rolle was a skillful algebraist who broke with Cartesian techniques; and his opposition to infinitesimal methods, in the final analysis, was beneficial.

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