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(*b*. Saloniki, Greece, 7 November 1898; *d*. Paris, France, 20 June 1963)

mathematics.

Salem was born to Jewish parents of Spanish origin, and the family moved to Paris when he was fifteen. His father, Emmanuel Salem, a well-known lawyer, died in 1940. His mother, Fortunée, and other members of his family died in a concentration camp.

Salem studied law at the University of Paris and mathematics at the Sorbonne, and received the *licence* in each field in 1919. He completed the engineering course at the École Centrale des Arts et Manufactures, where he studied mathematics under Jacques Hadamard, in 1921. Though he preferred mathematics, Salem went into banking. From 1921 to 1938 he worked for the Banque de Paris et des Pays-Bas, becoming a manager in 1938. He and his wife, Adriana, were married in 1923; they had two sons and a daughter.

Among Salem's avocations were music (he played the violin and liked to join quartets), fine arts, literature (mainly Italian and French), and sports (he skied and rode horseback). His main interest, however, was mathematics. He chose Fourier series, a field rather neglected in France at the time (except by Denjoy and S. Mandelbrojt), worked on special and difficult topics, and wrote a series of short papers published in *Comptes-rendus de l'Académie des sciences*. Denjoy insisted that he write a dissertation, for which Salem received the doctorate in mathematics from the Sorbonne in 1940.

Salem was mobilized in 1939, sent to England as assistant to Jean Monnet, and demobilized in June 1940. He took his family to Cambridge, Massachusetts, after a short stay in Canada. His doctorate enabled Salem to obtain a position at the Massachusetts Institute of Technology, where he taught from 1941 to 1955, achieving a full professorship by the latter year. There he met a number of mathematicians interested in Fourier series: Norbert Wiener, J. D. Tamarkin, D. C. Spencer, and Antoni Zygmund. He and Zygmund became close friends and wrote a number of joint papers.

Salem visited Paris every year after the war and gave a course at the Sorbonne from 1948 to 1955. He became professor there in 1955 and moved to Paris, where he played a major role in the renewal of interest in Fourier series.

An extraordinary teacher, Salem lectured in both English and French in his elementary and research courses. His writings contained nothing superfluous, nothing hidden; they were easy to comprehend even when they dealt with the most intricate mathematics—such as the interplay between harmonic analysis and the theory of algebraic numbers. His work is likely to remain a masterpiece of difficult mathematics and beautiful exposition.

After Salem's death his wife established an international prize, awarded every year to a young mathematician who has made an exceptional contribution to the theory of Fourier series or a related field. The list of those who have received the prize, first awarded in 1968, is impressive and testifies to the explosive activity in this area of mathematics.

Salem's works range over topics in the theory of Fourier series, number theory, geometrical set theory, and probability theory. The following two results of Salem's convey the flavor of his work. The first was discovered by him but was developed with others until the final form was achieved. The second has been the source of considerable research.

The first is related to a problem going back to Heinrich Eduard Heine and Georg Cantor (1870). Given a set E of $[0, 2\pi]$, does the convergence of a trigonometric series out of E imply that all coefficients are 0? If the answer is yes, E is called a U set (set of uniqueness); if not, an M set (set of multiplicity). The fact that the empty set is a U set is not obvious; it was proved by Cantor, together with a number of results from which set theory originated. Now consider a set E of a special type, associated with a given number ξ between 0 and ½: E is closed, and can be decomposed into two disjoint subsets homothetic to E with a ratio ξ. The case $\xi = 1/3$ is the classic triadic Cantor set, which is obtained from a closed interval I by removing the open middle third, then repeating this dissection on the remaining intervals, and so on; what remains is E. From the works of Rajchman and Nina Bari (about 1920) it was known that E is a U set when $1/\xi$ is an integer, and an M set when $1/\xi$ is a non-integral rational number. What Salem discovered (stated in 1943, proved with Antoni Zygmund in 1955, using a new approach developed by I. I. Piatecki-Shapiro) is a complete and surprising answer: E is a U set if and only if $1/\xi$ is an algebraic integer (that is, a root of a polynomial with integral coefficients, the leading term having the coefficient 1), such that all other roots of the polynomial lie inside the unit circle of the complex plane. This class of numbers is a closed set on the line; the numbers in question were called Pisot-Vijayaraghayan numbers by Salem, and their class was called S (for Salem) by C. Pisot.

The second problem was posed by A. Beurling and solved by Salem in 1950. Given a number a between 0 and 1, does there exist a closed set on the line whose Hausdorff dimension is α and that carries a measure μ whose Fourier transform $\mu(u)$ is dominated by $|u|^{-\alpha/2}$? From previous results on Hausdorff dimension found by O. Frostman, it was known that $\alpha/2$ is the critical index. The answer is affirmative and the result interesting, but the proof is of greater interest because it contains the first introduction of a random measure into harmonic analysis; no measure is exhibited as an example, but almost all measures (with a convenient probability space) fit the requirement. Today the study of random measures forms an active field in its own right and has provided new proofs of Salem's theorem. Brownian images, local times, and occupation densities can be studied through the Salem approach and have yielded new results in probability theory.

BIBLIOGRAPHY

Salem's books are *Essais sur les séries trigonométriques* (Paris, 1940); *Algebraic Numbers and Fourier Series* (Belmont, Calif., 1963); *Ensembles parfaits et séries trigonométriques* (Paris, 1963), written with Jean-Pierre Kahane; and *Oeuvres mathématiques* (Paris, 1967), see especially 295-304, 311-315, 481-493, and 590-592 for problems discussed in text. The latter volume also includes a useful preface by Antoni Zygmund and an introduction by Jean-Pierre Kahane and Antoni Zygmund.

Jean-pierre Kahane