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(*b*. Hermsdorf, Silesia [now Sobiecin, Poland], 25 January 1843; *d* Berlin, Germany, 30 November 1921)

Mathematics.

Schwarz, the son of an architect, was the leading mathematician in Berlin in the period following Kronecker, Kummer, and Weierstrass. He may be said to represent the link between these great mathematicians and the generation active in Germany in the first third of the twentieth century, a group that he greatly influenced. After attending the Gymnasium in Dortmund, he studied chemistry in Berlin at the Gewerbeinstitut (now the Technische Universität) but, under the influence of Kummer and Weierstrass, soon changed to mathematics. Schwarz received the doctorate in 1864 and then completed his training as a *Mittelschule* teacher. Immediately thereafter, in 1867, he was appointed assistant professor at Halle. In 1869 he became a full professor at the Eidgenössische Technische Hochschule in Zurich and in 1875 assumed the same rank at the University of Göttingen. Schwarz succeeded Weierstrass at the University of Berlin in 1892 and lectured there until 1917. During this long period, teaching duties and concern for his many students took so much of his time that he published very little more. A contributing element may have been his propensity for handling both the important and the trivial with the same thoroughness, a trait also evident in his mathematical papers. Schwarz was a member of the Prussian and Bavarian academies of sciences. He was married to a daughter of Kummer.

Schwarz's greatest strength lay in his geometric intuition, which was brought to bear in his first publication, an elementary proof of the chief theorem of axonometry, which had been posed by Karl Pohlke, his teacher at the Gewerbeinstitut. The influence of Weierstrass, however, soon led Schwarz to place his geometric ability in the service of analysis: and this synthesis was the basis of his contribution to mathematics. Schwarz tended to work on narrowly defined, concrete, individual problems, but in solving them he developed methods the significance of which far transcended the problem under discussion.

Schwarz's most important contribution to the history of mathematics was the "rescue" of some of Riemann's achievements. The demonstrations had been justly challenged by Weierstrass. The question centered on the "main theorem" of conformal (similar in the least parts) mapping, which stated that every simply connected region of the plane can be conformally mapped onto a circular area. In order to prove it, Riemann had employed the relation of the problem to the first boundary-value problem of potential theory (Dirichlet's problem), which requires a solution of the partial differential equation $\Delta u = 0$ with prescribed values at the boundary of the region. Dirichlet believed he had disposed of this problem with the observation (Dirichlet's principle) that such a function yields an extreme value for a certain double integral; Weierstrass had objected that the existence of a function which can do that is not at all self-evident but must be demonstrated.

Schwarz first solved the mapping problem explicitly for various simple geometric figures—the square and the triangle—and then in general for polygons. He also treated the conformal mapping of polyhedral surfaces onto the spherical surface. These results enabled him to solve the two problems mentioned, that is, to present the first completely valid proofs for extended classes of regions by approximating the given region by means of polygons. These works contained the first statement of principles that are now familiar to all: the principle of reflection: the "alternating method," which provides a further method for the approximation of solution functions, and "Schwarz's lemma."

Schwarz also worked in the field of minimal surfaces(surfaces of least area), a characteristic problem of the calculus of variation. Such a surface must everywhere have zero mean curvature, and in general all surfaces with this property are termed minimal surfaces. The boundary-value problem requires in this case that a minimal surface be passed through a given closed space curve, a procedure that can be carried out experimentally by dipping a wire loop into a soap solution. Following his preference for concrete geometrical problems, Schwarz first solved the problem explicitly for special space curves, mostly consisting of straight sections, of which the curve composed of four out of six edges of a tetrahedron has become the best known.

In his most important work, a *Festschrift* for Weierstrass' seventieth birthday, Schwarz set himself the task of completely answering the questior. of whether a given minimal surface really yields a minimal area. Aside from the achievement itself, which contains the first complete treatment of the second variation in a multiple integral, this work introduced methods that immediately became extremely fruitful. For example, a function was constructed through successive approximations that Picard was able to employ in obtaining his existence proof for differential equations. Furthermore, Schwarz demonstrated the existence of a certain number, which could be viewed as the (least) eigenvalue for the eigenvalue problem of a certain differential equation (these concepts did not exist then). This was done through a method that Schwarz's student Erhard Schmidt later applied to the proof of the existence of an eigenvalue of an integral equation—a procedure that is one of the most important tools of modern analysis. In this connection Schwarz also employed the inequality for integrals that is today known as “Schwarz’s inequality.”

Algebra played the least role in Schwarz’s work: his dissertation, however, was devoted to those surfaces developable into the plane that are given by algebraic equations of the first seven degrees. Much later he answered the question: In which cases does the Gaussian hypergeometric series represent an algebraic function? In approaching this matter, moreover, he developed trains of thought that led directly to the theory of auto-morphic functions, which was developed shortly afterward by Klein and Poincaré.

Of a series of minor works, executed with the same devotion and care as the major ones, two that involve criticism of predecessors and contemporaries remain to be mentioned. Schwarz presented the first rigorous proof that the sphere possesses a smaller surface area than any other body of the same volume. Earlier mathematicians, particularly Steiner, had implicitly supposed in their demonstrations the existence of a body with least surface area. Schwarz also pointed out that in the definition of the area of a curved surface appearing in many textbooks of his time, the method employed for determining the length of a curve was applied carelessly and that it therefore, for example, led to an infinitely great area resulting for so simple a surface as a cylindrical section.

BIBLIOGRAPHY

I. Original Works. Schwarz’s writings were collected in his *Gesammelte mathematische Abhandlungen*, 2 vols. (Berlin, 1890). He also compiled and edited *Nach Vorlesungen und Aufzeichnungen des Hrn. K. Weierstrass*, 12 pts. (Göttingen, 1881–1885), brought together in the 2nd ed. (Berlin, 1893), and also in French (Paris, 1894).

II. Secondary Literature. See *Mathematische Abhandlungen Hermann Amandus Schwarz zu seinem fünfzigjährigen Doktorjubiläum gewidmet von Freunden und Schülern*, C. Carathéodory, G. Hessenberg, E. Landau, and L. Lichtenstein, eds. (Berlin, 1914), with portrait: L. Bieberbach, “H. A. Schwarz,” in *Sitzungsberichte der Berliner mathematischen Gesellschaft*, **21** (1922), 47–51, with portrait and list of works not included in *Gesammelte Abhandlungen*; C. Caratheodory, “Hermann Amandus Schwarz,” in *Deutsches biographisches Jahrbuch*. III (1921), 236–238; G.Hamel, “Zum Gedächtnis an Hermann Amandus Schwarz,” in *Jahresberichte der Deutschen Mathematikervereinigung*, **32** (1923), 6–13, with portrait and complete bibliography; F.Lindemann, obituary in *Jahrbuch der bayerischen Akademie der Wissenschaften* (1922–1923), 75–77; R. von Mises, “H. A. Schwarz,” in *Zeitschrift für angewandte Mathematik und Mechanik*, **1** (1921); and E. Schmidt, “Gedächtnisrede auf Hermann Amandus Schwarz,” in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin* (1922), 85–87.

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