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(*b* Saluzzo, Italy, 20 August 1863; *d.* Turin, Italy, 18 May 1924)

mathematics.

Segre studied under Enrico D'Ovidio at the University of Turin, where he formed a long friendship with his fellow student Gino Loria. Segre submitted his doctoral dissertation in 1883, when he was only twenty; and in the same year he was named assistant to the professor of algebra and to the professor of [analytic geometry](#). Two years later he became an assistant in descriptive geometry, and from 1885 to 1888 he replaced Giuseppe Bruno in the courses on projective geometry. In 1888 he succeeded D'Ovidio in the chair of higher geometry, a post he held without interruption until his death.

Segre was much influenced by D'Ovidio's course on the geometry of ruled spaces (1881–1882). D'Ovidio started from the ideas of Plücker, which had been taken up and developed by [Felix Klein](#). According to these ideas, the geometry of ruled space is equivalent to the study of a quadratic variety of four dimensions imbedded in a linear space of five dimensions. In his lectures D'Ovidio examined the works of Veronese on the projective geometry of hyperspaces and those of Weierstrass on bilinear and quadratic forms. These topics inspired much of Segre's research, beginning with his thesis. The latter consists of two parts: a study of the quadrics in a linear space of arbitrary dimension and an examination of the geometry of the right line and of its quadratic series. Before completing his thesis, Segre collaborated with Loria on a twenty-two-page article in French that they sent to Klein, who published it in *Mathematische Annalen* (1883). A long and active correspondence between Segre and Klein then ensued.

Segre's mathematical work can be divided into four distinct areas, all of which are linked by a common concern with the problem of space. The first of these areas comprises Segre's articles on the geometric properties that are invariant under linear transformations of space. In this connection Segre showed the value of investigating hyperspaces in the study of three-dimensional space S_3 . For example, a ruled surface of S_3 , which is composed of right lines, can be represented by a curve in S_5 ; it thus becomes possible to reduce the classification of surfaces to that of curves. The insufficiencies of the earlier theories proposed by A. Möbius, Grassmann, Cayley, and Cremona were thus soon revealed.

According to Segre, a ruled surface in a space S_2 can also be considered a variety of ∞^2 points distributed on ∞^1 right lines. Further, Segre generalized the theory of the loci formed by ∞^1 right lines S_n to the theory of the loci formed by ∞^1 planes. He took as his point of departure certain problems on bundles of quadrics that Weierstrass and L. Kronecker had treated in a purely algebraic manner.

At this time it was known that the intersection of two quadrics of S_3 is a quartic the projection of which from a point exterior to it onto a plane is a quartic with two double points. John Casey and Gaston Darboux had shown that its study is useful for that of fourth-order surfaces, called cyclides. Segre reexamined and generalized the problem by placing the two quadrics in a space S_4 . He also investigated the locus resulting from the intersection of two quadrics of S_3 and discovered that it is no longer a surface but rather a three-dimensional variety that can be interpreted as a complex quadratic of S_3 . From this result he confirmed in an elegant manner the famous fourth-order surface with sixteen double points, which had been found by

Kummer in 1864 and bear his name. Before Segre's findings, the study of this surface required the use of extremely complicated algebraic procedures.

Segre next began a series of works on the properties of algebraic curves and ruled surfaces subjected to birational transformations. Alfred Clebsch, Paul Gordan, Alexander Brill, and Max Noether had already studied these transformations with a view toward giving a geometric interpretation to the theory of Abelian functions. Segre showed the advantage gained by operating in a hyperspace. His article of 1896 on the birational transformations of a surface contains the invariant that Zeuthen had encountered under another form in 1871, now called Zeuthen-Segre invariant.

Segre's interest in 1890 in the properties of the Riemann sphere led him to a third area of research: the role of imaginary elements in geometry. He laid the basis of a new theory of hyperalgebraic entities by representing complex points of S_n by means of the ∞^{2n} real points of one of the varieties V_{2n} . (This variety has since been named for Segre.) Certain of Segre's hyperalgebraic, and he was led to enlarge the concept of a point. To this end, he introduced points that he called bicomplex, which correspond to the ordinary complex points of the real image. Their coordinates are bicomplex numbers constructed with the aid of the two unities i and j , such that:

$$i \cdot j = j \cdot i$$

and

$$i^2 = j^2 = -1.$$

Later, in 1912, Segre returned to this subject, when, utilizing the works of Von Staudt, he studied another type of complex geometry.

Darboux's *Leçons sur la théorie générale des surfaces*, which Segre often used in his courses, inspired him to investigate (from 1907) infinitesimal geometry. Extending the work of Darboux, Segre studied a certain class of surfaces in S_n defined by second-order linear partial differential equations. These surfaces are described by a moving point of which the homogeneous coordinates-functions of two independent parameters u and v -are the solutions of a second-order partial differential equation. Among the surfaces of a hyperspace, Segre was particularly interested in those that lead to a Laplace equation. In an article of 1908 on the conjugate tangents of a surface, he established a relationship between the points of the tangent plane and those of the planes passing through the origin. To establish this relationship he employed infinitesimals of higher order in a problem concerning the neighborhood of a point. This procedure led him to introduce a new system of lines, analogous to those studied by Darboux, traced on the surface: they were named Segre lines, and their differential equation was established by Fubini. It may be noted that Segre's last publication dealt with differential geometry. Segre wrote a long article on hyperspaces for the *Encyklopädie der mathematischen Wissenschaften*, containing all that was then known about such spaces. A model article, it is notable for its clarity and elegance.

Segre became a member of the Academy of Turin in 1889. He long served on the editorial board of the *Annali di matematica pura ed applicata*, on which he was succeeded by his former student Severi of the University of Rome.

Through his teaching and his publications, Segre played an important role in reviving an interest in geometry in Italy. His reputation and the new ideas he presented in his courses attracted many Italian and foreign students to Turin. Segre's contribution to the knowledge of space assures him a place after Cremona in the ranks of the most illustrious members of the new Italian school of geometry.

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