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(*b*. Huzioka[?]. Japan. 1642[?]: *d*. Edo [now Tokyo], Japan, 24 October 1708)

mathematics.

Knowledge of Seki's life is meager and indirect. His place of birth is variously given as Huzioka or Edo, and the year as 1642 or 1644. He was the second son of Nagaakira Utiyama, a samurai: his mother's name is unknown. He was adopted by the patriarch of the Seki family, an accountant, whom he accompanied to Edo as chief of the Bureau of Supply. In 1706, having grown too old to fulfill the duties of this office, he was transferred to a sinecure and died two years later.

Seki is reported to have begun the study of mathematics under Yositane Takahara, a brilliant disciple of Stgeyosi Mōri. Mōri is the author of *Warizansyo* ("A Book on Division," 1622), believed to be the first book on mathematics written by a Japanese, but little is known about Takahara. Of particular influence on Seki's mathematics was Suan-hsüeh ch'i-mêng ("Introduction to Mathematical Studies," 1299), compiled by Chu Shih-chieh, a collection of problems solved by a method known in Chinese as *¨tien-yuan shu* ("method of the celestial element"): this method makes it possible to solve a problem by transforming it into an algebraic equation with one variable. Kazuyuki Sawaguchi, allegedly the first Japanese mathematician to master this method, used it to solve 150 problems proposed by Masaoki Satö in his *Sanpö Kongenki* ("Fundamentals of Mathematics," 1667). He them organized these problems, together with their solutions, into the seven-volume collection *Kokon sanpöki* ("Ancient and Modern Mathematics," 1670). At the end of the last volume Sawaguchi presented fifteen problems that he believed to be unsolvable by means of *t¨ien-yuan shu*.

Seki's solutions to these problems were published in 1674 as *Hatubi sanpō*. Because mathematicians of those days were unable to grasp how the solutions had been accomplished. Katahiro Takebe (1664–1739), a distinguished disciple of Seki, published *Hatubi sanpō endan genkai* ("An Easy Guide to the *Hatubi sanpō*," 1685), the work that made Seki's method known.

In Chinese mathematics operations were performed by means of instruments called *suan-ch'ou* ("calculating rods") and therefore could not treat any algebraic expressions except those with one variable and numerical coefficients. Seki introduced Chinese ideographs and wrote them to the right of a vertical-such as $|a$-where a is used, for typographical reasons, instead of a Chinese ideograph, Seki called this notation *bōsyohō* ("method of writing by the side") and used it as the basis of his *endan zyutu* ("*endan* method"). *Endan zyutu* enabled him to represent known and unknown quantities by Chinese ideographs and led him to form equations with literal coefficients of any degree and with several variables. The technique was later renamed *tenzan zyutu* by Yosisuke Matunaga (1693–1744), the third licensee of the secret mathematical methods of Seki's school.

The *Hatubi sanpō* did not include Seki's principal theorems, which were kept secret; but in order to initiate students he arranged the theorems systematically. Parts of these works and some more extended theorems were collected and published posthumously by his disciples as *Katuyō sanpö*-which, with *Sekiryū sanpō sitibusyo* ("Seven Books on the Mathematics of Seki's School." 1907), is sufficient to grasp his mathematics.

Seki first treated general theories of algebraic equations. Since his method of side writing was inconvenient for writing general equations of degree n, he worked with equations of the second through fifth degrees. But his treatment was so general that his method could be applied to equations of any degree. Seki attempted to find a means of solving second-degree algebraic equations with numerical coefficients but, having no concept of algebraic solution, directed his efforts at finding an approximate solution. He discovered a procedure, used long before in China, that was substantially the same as Horner's. The notion of a discriminant of an algebraic equation also was introduced by Seki. Although he had no notion of the derivative, he derived from an algebraic expression $f(x)$ another expression that was the equivalent of $f'(x)$ in modern notation. Eliminating x from the pair of equations $f(x) = 0$ and $f'(x) = 0$, he obtained what is now called a discriminant. With the help of this expression Seki found double roots of the equation $f(x) = 0$. He also developed a method similar to Newton's by which an approximate value of the root of a numerical equation can be computed. Seki's *tenzan zyutu* was important in the treatment of problems that can be transformed into the solution of a system of simultaneous equations. In order to solve the problem of elimination, he introduced determinants and gave a rule for expressing them diagrammatically. For third-order equations his formula was basically similar to Sarrus'.

The method that Seki named *syōsahō* was intended to determine the coefficients of the expression $y = a_1 + a_2 x^2 + ... + a_n x^n$, when n values of $x_1, x_2, ..., x_n$ of x and the corresponding n values of $y_1, y_2, ..., y_n$ of y are given. Since his notation was not suitable for the general case corresponding to an arbitrary n, Seki treated the case that corresponds to the special value of n. His solution was similar to the method of finite difference.

Another method, *daskei zyutu*, also is important is Seki's mathematics. Its purpose is to find values of $s_p = 1^p + 2^p + ... + n^p$ for $p = 1, 2, 3, ...$ Using the *syōashō*., Seki calculated $s_1, s_2, ..., s_6$ for a particular value of n:

The numbers 1/6, 1/30, 1/42 are Bernoulli numbers, which were introduced in his *Ars conjectandi* (1713).

Enri ("principle of the circle"), one of Seki's important contributions, consists of rectification of the circumference of a circle, rectification of a circular arc, and cubature of a sphere. In the rectification of a circumference. Seki considered a circle of diameter 1 and an inscribed regular polygon of 2^n sides. He believed that the inscribed polygon gradually loses its angularities and finally becomes a circumference of the circle when the number of sides is increased without limit. He therefore calculated the perimeter c_i of the regular polygon of 2^i sides, where i represents any integer not greater than 17, and devised a method by which he was able to obtain a better result from c_{15}, c_{16}, c_{17}, The formula was

where

$c_{15} = 3.1415926487769856708$,
$c_{16} = 3.1415926523565913571$,
and
$c_{17} = 3.141592653288902775$.

Therefore, $s = 3.14159265359$, where s is the circumference of the circle; this value is accurate except for the last figure. Also in connection with this problem Seki created a method of approximation called *reiyaku zyutu*, by which he theoretically obtained 355/113 as an approximate value of π, a value found much earlier in China.

In the rectification of a circular arc, Seki considered an arc of which the chord is 8 and the corresponding sagitta is 2. He considered an inscribed open polygon of 2^{15} sides and, using the above formula, calculated the approximate value of the arc as 9.272953. In the cubature of a sphere, Seki calculated the volume of a sphere of diameter 10 and obtained as an approximate value. Using the approximate value of π as 355/113, Seki observed that if d is the diameter of this sphere.

Certain other geometrical problems, which do not belong among the *enri*, concern ellipses. Seki believed that any ellipse can be obtained through cutting a suitable circular cylinder by a plane. In order to obtain the area of an ellipse, he cut off from a circular cylinder two segments with generatrices of equal length such that one has elliptical bases and the other has circular bases. These two cylinders have the same volume. By equating the volumes of these two pieces of the cylinder. Seki obtained the result that the area of an ellipse is $\pi/4 \times AA' \times BB'$ where AA' BB' denote, respectively, the major and minor axis of an ellipse Among the problems of cubature, that of a solid generated by revolving a segment of circle about a straight line that lies on the same plane as the segment is noteworthy. His result in this area had generality, and the theorem that he established was substantially the same as that now called the theorem of Pappus and Guldin.

BIBLIOGRAPHY

I. Original Works. Seki's published writings are *Hatubi sanpō* (Edo, 1674) and *Katuyō sanpō* (Tokyo-Kyoto, 1709), copies of which are owned by the Mathematical Institute, Faculty of science, Kyoto University. *Sekiryü sanpō sitibusyo* ("Seven Books on the Mathematics of Seki's School"; Tokyo, 1907), is a collection of Seki's paper (1683–1685) that were transmitted from master to pupils, who were permitted to copy them. An important collection of MS papers, "Sanbusyō" ("Three Selected Papers"), is discussed by Fujiwara (see below). See also *Collected Papers of Takakazu Seki* (Osaka, 1974).

II. Secondary Literature. See the following, listed chronologically: Katahiro Takebe, *Hatubi sanpö endan genkai* ("An Easy Guide to the Hatubi sanpö": n.p., 1685), a copy of which is owned by the Mathematical Institute, Faculty of Science, Kyoto University, with a facs. of the text of *Hatubi sanpō* and an explanation of Seki's method of solution: Dairoku Kikuchi, "Seki's Method of Finding the Length of an Arc of a Circle," in *Proceedings of the Physico-Mathematical Society of Japan* **8** , no. 5 (1899), 179–198; and Matsusaburo Fujiwara, *Mathematics of Japan Before the Meiji Era*, **II** (Tokyo, 1956), 133–265, in Japanese. There are many other works that treat Seki's mathematics, such as Yoshio Mikami. *The Development of Mathematics in China and Japan* (Leipzig—New York, 1913: repr. New York. n.d.), but only those of scientific importance are cited here.

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