Severi, Francesco | Encyclopedia.com

(*b* Arezzo, Italy, 13 April 1879; *d* Rome, Italy, 8 December 1961)

mathematics.

From 1898 until his death, Severi published more than 400 books and papers on mathematics, history of science, education, and philosophy. His most outstanding contributions, however, were in the field of algebraic geometry. Severi acquired the taste for elegant synthetic arguments while studying with Segre at the University of Turin, from which he graduated in 1900 under Segre's guidance. At Turin he became interested in algebraic and enumerative problems and developed a broad geometric eclecticism and a formidable dexterity in the projective geometry of higher spaces.

In the latter field Segre published (1894) an interesting reworking of geometry on an algebraic curve. In Italy, Bertini and Castelnuovo also contributed to this field, while in Germany, Brill and Max Noether, in the footsteps of Riemann, made more studies using different methods. A more invariant view, derived from the transformations introduced by Cremona thirty years earlier, led Castelnuovo and Enriques to lay the foundations of a similar theory for algebraic surfaces. This theory anticipated the work of Picard in France on the same subject.

Having served as *assistente* to Enriques at Bologna in 1902 and to Bertini at Pisa in 1903, Severi was drawn to these new developments. He attempted, with great success, to explain important and still unsolved problems along with work in new areas. He perfected the theory of birational invariants of algebraic surfaces and created an analogous (but more complex) theory for algebraic varieties of arbitrary dimension. The completion of this work was to take him another fifty years. Severi's work on algebraic geometry can best be described by dividing it into five sections, rather than maintaining a chronological order.

1. Enumerative and Projective Geometry, Intersections, and Questions on the Foundations of Algebraic Geometry. The proof of the principle of the "conservation of number," established heuristically by Schubert in the nineteenth century, was listed by Hilbert at the Paris Congress of 1900 as one of the fundamental unsolved problems of mathematics. Severi subsequently found and proved the conditions under which this principel is true. Thus he refined Schubert;'s wiork and also advanced it through the theorry of the base and the theory of characteristics.

Twenty years later this research by Severi inspired several mathematicians, including William V.D. Hodge, Wei L. Chow, and Bartel van der Waerden. Severi also introduced the important notion of the invariant order of an algebraic variety, which led to the theory of minimal models; and he studied improper double points of algebraic surfaces, characters of embedding of one variety in another, and generalizations of Bezout's theorem from intersections of plane curves to those of arbitrary varieties in higher projective spaces.

2. Series and Systems of Equivalence. This theory, created almost wholly by Severi, added to algebraic geometry many important entities, for example, the canonical varieties of arbitrary dimension—theory (completed later by Beniamino Segre and J. A. Todd) that has had considerable connections and implications in both algebra and topology. He generalized the theory of linear equivalence to arbitrary subvarieties of a given variety and also made lengthy fundamental studies of rational equivalence, algebraic equivalence, and algebraic correspondences between varieties.

3. Geometry on Algebraic Surfaces. At the beginning of the twentieth century, the geometry of algebraic surfaces had reached a dead end. Although Castelnuovo and Enriques had defined the genera, irregularity, and plurigenera of surgaces and had characterized those surfaces that are birationally equivalent to a plane or to a ruled surface, there were still several unsolved problems; and Picard's introduction of three types of integrals on a surface suggested many additional questions, a large number of which were successfully explained by Severi. Also, Severi reduced Picard's three types of integrals to normal form and found conditions of integrability for certain linear differential equations on a surface.

Severi introduced the notion of semiexact differentials of the first ype and, using Hodge's findings, surprisingly showed that they are always exact. An important property utilized in these investigations was the completeness of the characteristic series of a continuous complete system. Much effort was later required to establish this result in its correct generality.

4. Geometry on Algebraic Varieties. The extension from surfaces to varieties of three or more dimensions is no less difficult than that from curves to surfaces. In an early memoir (1909), Severi established the basis for the exteded theory with his study

of linear systems of hypersurfaces. He gave various definitions of the arithmetic genus of a variety and proved their equivalence, thus partially extending the Riemannn-Roch theorem and also Picard's theorem on the regularity of the adjoint.

Besides his work on the foundations of the general theory of algebraic varieties, Severi established the theory of irregularity and made important studies of continuous systems of curves in the plane and in higher projective spaces.

5. Abelian and Quasi-Abelian Varieties. The theory of Abelian varieties V_p, of dimension p, originated in 1889 with Picard, who investigated algebraic V_p possessing a continuous, transitive, Abelian group of ∞^p birational automorphisms. The infinitesimal transformations of the group led to p independent integrals. Picard maintained that these integrals were all of the first type, but Severi showed that this is not true for $p = 2$ if the group is not absolutely transitive.

The study of these V_p is connected with that of a particular type of functions of several complex variables— a generalization of elliptic functions. These functions are related to particular varieties, called Picard and Albanese varieties, to which Severi devoted several works. When the group of theV_p is transitive, but not necessarily absolutely transitive, V_p is called quasi-Abelian. Severi discussed these V_p in a lengthy paper written during the turbulent period October 1944—May 1945.

Algebraic geometry has undergone several revolutionary changes in the twentieth centuty that have led to many schools and to several widely differing methods of approach. Severi's work remains not merely a monument to him but also a valuable source from which all algebraic geometers continue to draw ideas. He himself characterized his approach to mathematical research with the following admonition:

Let us not pride ourselves too much on perfect rigour, which we today believe to be capable of reducing so large a part of mathematics to nothing, and let us not discard what does not appear quite as rigorous, for tomorrow we will certainly find imperfections in our perfection and from some brilliant, intuitive thought which had not yet the blessing of rigour will be drawn unthinkable results [Severi, "Intuizionismo e astrattismo nella matematica contemporanea," in *Atti del congresso. Unione matematica italiana* (Sept. 1948), p. 30].

BIBLIOGRAPHY

I . Original Works. Severi's works include *Vorlesungen über galgebraische Geometrie*, L. Löffler, ed. and trans. (Leipzig, 1921; repr., New Yor–London, 1968); "Geometria dell serie lineari," in *Trattato digeometria algebrica*, **1** , pt. 1 (1926), 145–169; *Serie, sistemi d'equivlenza e corrispondenze algebriche sulle varietá algebriche*, F. Conforto and E. Martinellie, eds. (Rome, 1942); *Funzioni quasi abeliane* (Vatican City, 1947); *Memorie scelte*, **B. Segre, ed** . (Bologna, 1950); *Geometria dei sistemi algebrichi sopra una superficie e sopra una varieta algebrica*, **II** (Rome, 1957), **III** (Rome, 1959); and *II teorema di Rimann-Roch per curve, superficie e varieta* (Berlin, 1958). Severi's mathematical papers, collected in seven volumes, will soon be published by the Lincei Academy (vols.**I** and **II** have already appeared in 1971 and 1974).

II . Secondary Literature. On Severi and his work, see B. Segre, *L'opera scientifica di Francesco Severi* (Rome, 1962), with complete bibliography.

Beniamino Segre