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b. (Garden, Stirlingshire, Scotland, 1692; *d*. Edinburgh, Scotland, 5 December 1770)

mathematics

Stirling was the third son of Archibald Stirling and his second wife, Anna Hamilton, and grandson of Lord Garden of Keir. The whole family supported the Jacobite cause, and Archibald Stirling was in prison on a charge of high treason (of which he was later acquitted) while his son attended Glasgow University. James Stirling matriculated at Balliol College, Oxford, in 1711, without taking the oath. He himself was acquitted of the charge of "cursing King George" at the assizes. He seems to have left Oxford in 1716, after refusing to take the oaths needed to continue his scholarship. He did not graduate.

The previous year John Keill had mentioned Stirling's achievements in a letter to Newton, and at about the same time Stirling became acquainted with John Arbuthnot, the well-known mathematician, physician, and satirist. Such connections enabled him to publish (in Oxford) his first book, *Lineae tertii ordinis Neutonianae, sive illustratiotractatus D. Neutoni de enumeratione linearum tertii ordinis* (1717). The eight-page subscription list included Newton himself, besides many Oxford men. The book was dedicated to Nicholas Tron, the Venetian ambassador, who had become a fellow of the Royal Society in 1715, the same year in which Newton's correspondent, the Abbé Conti, was also elected. Stirling may then have held a teaching appointment in Edinburgh,[1] but the fame brought him by his book and the influence of his Venetian friends soon secured him a post in Venice. In 1718 Stirling submitted, through Newton, his first Royal Society paper, "Methodus differentialis Newtoniana illustrata," and in August 1719 he wrote from Venice thanking Newton for his kindness and offering to act as intermediary with Nikolaus I Bernoulli.

Little else is known about Stirling's stay in Venice, although his return to Britain is supposed to have been hastened because he had learned some secrets of the glass industry and may have feared for his life. By mid-1724 he had returned to Scotland, and a few months later he settled in London. In 1726 Newton helped secure Stirling's fellowship in the Royal Society and at about this time Stirling succeeded Benjamin Worster as one of the partners of the Little Tower Street Academy,[2] conducted by William Watts. This was one of the most successful schools in London; and, although he had to borrow money to pay for the mathematical instruments he needed, Stirling's finances improved. He helped to prepare *A Course of Mechanical and Experimental Philosophy* (to give it the title of a syllabus published in 1727) that included mechanics, hydrostatics, optics, and astronomy, a course very much in the tradition of Keill and Desaguliers, the leading scientific lecturer at that time. Stirling gave up some of his leisure to write his main work, *Methodus differentialis*, which appeared in 1730. A little later, through his friend Arbuthnot, Stirling was brought in as an adviser to Henry St. John, Lord Bolingbroke, since he was considered to be one of the few persons capable of understanding the financial calculations of Sir Robert Walpole. The latter's electoral victory of 1734 led to Bolingbroke's retirement to France.[3]

Given his reputation it was not surprising that Stirling was asked to reorganize the work of the Scottish Mining Company in the lead mines at Leadhills, Lanarkshire, near the border with Dumfries. Stirling was a successful administrator and spent most of his time after 1735 in the remote village. He married Barbara Watson of Thirty-acres, near Stirling; their only child, a daughter, married her cousin Archibald Stirling, who succeeded Stirling as manager at Leadhills.

Although Stirling continued his mathematical correspondence–with John Machin, Alexis-Claude Clairaut, Leonhard Euler, and Martin Folkes, among others–it is clear that most of his energy was spent in mining affairs. His most influential mathematical correspondent, Colin Maclaurin, died in 1746, largely as a result of his efforts in defending Edinburgh against the Jacobite rebellion of the preceding year; Stirling's own political principles prevented him from succeeding to the Edinburgh chair left vacant at Maclaurin's death.[4] In 1748 Stirling was elected to the Berlin Academy of Sciences, even though his directly mathematical activities had ceased; he resigned his fellowship in the Royal Society in 1754. In 1752 he was presented with a silver teakettle for conducting the first survey of the Clyde by the town council of Glasgow, where he also apparently acted as a teacher of bookkeeping, navigation, geography, practical mathematics, and French.[5] In his later years he became too frail to move about easily; he died on a visit to Edinburgh for medical treatment.

Stirling's *tractatus* of 1717 won him a considerable reputation. In it, after a considerable amount of introductory material, Stirling proved Newton's enumeration of seventy-two species of cubic curves and added four more. François Nicole and Nikolaus I Bernoulli then added two more curves, in 1731 and 1733, respectively, the latter in a letter to Stirling.

Stirling next turned from cubics to differences, the other main topic of Newton's *Analysis* (1711). But these studies were interrupted by his moving from Oxford to Venice, from which he wrote to give permission for publication (without an intended supplement) of his 1719 paper "Methodus differentialis." This paper should not be confused with a later book of similar title, but it may be considered a precursor to it, since the book represents the further development and fuller treatment of the same ideas. Some of the same results are given in both; the so-called Newton-Stirling central difference formula,[6] which was also discussed by Cotes, is especially noteworthy.

The *Methodus differentialis: sive tractatus de summatione et interpolatione serierum infinitarum* of 1730 consists of a relatively brief introduction followed by two parts, on summation and interpolation. The work was sufficiently important to be reprinted twice during Stirling's lifetime, in 1753 and 1764, and to be published in an English translation in 1749.[7] The translation was made by Francis Holliday, who was then master of a grammar school near Retford, Nottinghamshire, as well as editor of *Miscellanea curiosa mathematica*, one of a number of relatively short-lived popular mathematical serials[8] published during the mid-eighteenth century. (The translator's preface shows that Holliday had originally intended to publish the translation in his serial and indicates that he planned to follow *The Differential Method* with other translations of Stirling's work as well; perhaps the reception of the book was insufficiently favorable for these other plans to materialize.)

In his preface, Stirling indicated that Newton, too, had considered the problem of speeding the convergence of series by transformations involving differences. De Moivre had made progress with a recurring series, but his methods could be generalized to other series in which "the relation of the terms is varied according to some regular law." The most useful representation of terms was in a series of factorials, positive or negative. Manipulation often required conversion of factorials into powers, and Stirling gave tables of the coefficients for this conversion. He then showed that the columns of the tables gave the coefficients for the inverse expressions, of powers in factorials; those for positive (negative) powers are now called "Stirling Numbers of the first [or second] kind" in his honor.[9] The so-called Stirling series

is equivalent to the expansion of $(z^2+nz)^{-1}$ in negative factorials, which is the last example given in his introduction.

Stirling explained that part one of the *Methodus differentialis*, "on the summation of series," was designed to show how to transform series in order to make them converge more rapidly and so to expedite calculation. As an example[10] he gave the series

studied by Brouncker in connection with the quadrature of the hyperbola; Stirling concluded that "if anyone would find an accurate value of this series to nine places . . . they would require one thousand million of terms; and this series converges much swifter than many others. . . ." Another example[11] was the calculation—"which Mr. Leibnitz long ago greatly desired"—of

Stirling's sixth proposition was effectively an early example of a test for the convergence of an infinite product; he gave many examples of problems, now solved by the use of gamma functions, that illustrated his aim. The last section of the first part of the book contains an incomplete development of De Moivre's principles used in recurring series; for linear relations with polynomial coefficients connecting a finite number of terms, Stirling reduced the solution to that of a corresponding differential equation.

Stirling continued to show his analytical skill in part two, on the interpolation of series. As an example[12] of interpolation at the beginning of a series, he took the gamma series $T_{n+1} = nT_n$, with $T_1 = 1$, to find the term $T_{3/2}$ intermediate between the two terms T_1 and T_2 and calculated the result to ten decimal places: his result is now written $\Gamma(1/2) = \sqrt{\pi}$. Stirling's other results are now expressed using gamma functions or hypergeometric series. He also discussed[13] the sum of any number of logarithms of arguments in arithmetical progression and obtained the logarithmic equivalent of the result, sometimes called Stirling's theorem, that

Just before leaving London, Stirling contributed a short article to the *Philosophical Transactions of the Royal Society* entitled "Of the Figure of the Earth, and the Variation of Gravity on the Surface." In it he stated, without proof, that the earth was an oblate spheroid, supporting Newton against the rival Cassinian view. This paper was unknown to Clairaut, who submitted a paper partly duplicating it from Lapland, where he was part of the expedition under Maupertuis that proved Newton's hypothesis.[14] Although Stirling contributed another technical paper ten years later, it is clear that his new post in Scotland did not give him an opportunity to pursue his mathematical activities in any depth and that his significant work was confined to the 1720's and 1730's.

NOTES

1. W. Steven, *History of George Heriot's Hospital*, F. W. Bedford, ed. (Edinburgh, 1859), 307, mentions James Stirling as assistant master, elected 12 August 1717.

2. N. Hans, *New Trends in Education in the Eighteenth Century* (London 1951), 82–87, gives the best account of the Academy, but his dates for Stirling and Patoun are unreliable.

3. The connection with Bolingbroke is given by Ramsay (see bibliography), 308–309, but is ignored in most accounts.

4. A. Grant, *The Story of the University of Edinburgh*. II (London, 1884), 301.

5. Glasgow City Archives and *Glasgow Courant*, Nov. 1753, Nov. 1754, and Nov. 1755, reported by M. J. M. McDonald and J. A. Cable respectively.

6. D. T. Whiteside, ed., *The Mathematical Papers of Isaac Newton*, IV (Cambridge, 1971), 58, n. 19.

7. The Latin and English versions had 153 and 141 pages, respectively; references in Tweedie to the Latin ed. can be converted to those to the latter, given here, by subtracting about ten.

8. R. C. Archibald, "Notes on some Minor English Mathematical Serials," in *Mathematical Gazette*, **14** , no. 200 (April 1929), 379–400.

9. Stirling, *Differential Method*, 17, 20. A useful, modern textbook, C. Jordan's *Calculus of Finite Differences*, 2nd ed. (New York, 1947), devotes ch. 4 to Stirling's numbers. Jordan and Tweedie give details of the articles by N. Nielsen that stress the significance of Stirling.

10.*Differential Method*, 23 – 25.

11.*Ibid*., 27 – 28.

12.*Ibid*., 99 – 103.

13.*Ibid*., 123 – 125.

14. I. Todhunter, *A History of the Mathematical Theories of Attraction*. I (London, 1873), ch. 4.

BIBLIOGRAPHY

I. Original Works. Stirling's works are listed in the text. His major work is *Methodus differentialis: sivetractatus de summatione et interpolatione serierum infinitarum* (London, 1730). The family papers are at the General Register House, Edinburgh; they contain disappointingly few mathematical papers, but more about Stirling's mining activities.

II. Secondary Literature. The main authority is the unindexed volume C. Tweedie, *James Stirling: A Sketch of His Life and Works Along With His Scientific Correspondence* (Oxford, 1922); also J. O. Mitchell's *Old Glasgow Essays* (Glasgow, 1905), repr. from "James Stirling Mathematician," *Glasgow Herald* (1886); and J. Ramsay's *Scotland and Scotsmen in the Eighteenth Century*, A. Allardyce, ed., II (Edinburgh, 1888), 306–326. Other works are detailed in the notes and in Tweedie.

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