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(*b*. London, England, 3 September 1814; *d*. London, 15 March 1897)

mathematics.

Although Sylvester is perhaps most widely remembered for his indefatigable work in the theory of invariants, especially that done in conjunction with Arthur Cayley, he wrote extensively on many other topics in the theory of algebraic forms. He left important theorems in connection with Sturm's functions, canonical forms, and determinants; he especially advanced the theory of equations and the theory of partitions.

James Joseph (Joseph then being his surname) was born into a Jewish family originally from Liverpool. The son of Abraham Joseph, who died while the boy was young, James was the sixth and youngest son of nine children, at least four of whom later assumed the name Sylvester for a reason not now apparent.

Until Sylvester was fifteen, he was educated in London, at first in schools for Jewish boys at Highgate and at Islington, and then for five months at the University of London (later University College), where he met Augustus De Morgan. In 1828 he was expelled "for taking a table knife from the refectory with the intention of sticking it into a fellow student who had incurred his displeasure."[1] In 1829 Sylvester went to the school of the Royal Institution, in Liverpool, where he took the first prize in mathematics by an immense margin and won a prize of $500, offered by the Contractors of Lotteries in the United States, for solving a problem in arrangements. At this school he was persecuted for his faith to a point where he ran away to Dublin. There, in the street, he encountered R. Keatinge, a judge and his mother's cousin, who arranged for his return to school.

Sylvester now read mathematics for a short time with Richard Wilson, at one time a fellow of St. John's College, Cambridge, and in October 1831 he himself entered that college, where he stayed until the end of 1833, when he suffered a serious illness that kept him at home until January 1836. After further bouts of illness, Sylvester took the tripos examination in January 1837, placing second. Since he was not prepared to subscribe to the Thirty-Nine Articles of the Church of England, he was not allowed to take the degree or compete for Smith's mathematical prizes—still less secure a fellowship. He went, therefore, to Trinity College, Dublin, where he took the B.A. and M.A. in 1841. (He finally took the equivalent Cambridge degrees in 1872–1873, the enabling legislation having been passed in 1871.)

In 1838 Sylvester went to what is now University College, London, as De Morgan's colleague. He seems to have found the chair in Natural Philosophy uncongenial. In 1839, at the age of twentyfive, he was elected a fellow of the Royal Society on the strength of his earliest papers, written for *Philosophical Magazine* as soon as he had taken his tripos examination. The first four of these concern the analytical development of Fresnel's theory of the optical properties of crystals, and the motion of fluids and rigid bodies. His attention soon turned to more purely mathematical topics, especially the expression of Sturm's functions in terms of the roots of the equation.

From University College, Sylvester moved in 1841 to a post at the University of Virginia. There are many lurid and conflicting reports of the reasons for his having returned to England in the middle of 1843. He apparently differed from his colleagues as to the way an insubordinate student should be treated. He now left the academic world for a time, and in 1844 was appointed Actuary and Secretary to the Equity and Law Life Assurance Company. He apparently gave private tuition in mathematics, for he had Florence Nightingale as a pupil. In 1846, the same year that Cayley entered Lincoln's Inn, Sylvester entered Inner Temple and was finally called to the bar in November 1850. Cayley and Sylvester soon struck up a friendship. At his Oxford inaugural lecture many years later (1885), Sylvester spoke of Cayley, "who though younger than myself, is my spiritual progenitor—who first opened my eyes and purged them of dross so that they could see and accept the higher mysteries of our common mathematical faith." Both men referred on occasion to theorems they had derived separately through the stimulus of their conversations in the intervals between legal business.

In 1854 Sylvester was an unsuccessful candidate both for the chair of mathematics at the Royal Military Academy, Woolwich, and for the professorship in geometry at Gresham College, London. The successful candidate for the former position soon died, and with the help of Lord Brougham. Sylvester was appointed. He held this post from September 1855 to July 1870. At the same time he became editor, from its first issue in 1855, of the Quarterly Journal of Pure and Applied Mathematics, successor to the *Cambridge and Dublin Mathematical Journal*. Assisted as he was by Stokes, Cayley, and Hermite, there was no change in editorship until 1877.

In 1863 Sylvester replaced the geometer Steiner as mathematics correspondent to the French Academy of Sciences. Two years later he delivered a paper on Newton's rule (concerning the number of imaginary roots of an algebraic equation) at King's

College, London. A syllabus to the lecture was the first mathematical paper published by De Morgan's newly founded London Mathematical Society, of which Sylvester was president from 1866 to 1868. In 1869 he presided over the Mathematical and Physical Section of the British Association meeting at Exeter. His address was prompted by T. H. Huxley's charge that mathematics was an almost wholly deductive science, knowing nothing of experiment or causation. This led to a controversy carried on in the pages of *Nature*, relating to Kant's doctrine of space and time; Sylvester, however, was not at his best in this kind of discussion. He reprinted an expanded version of his presidential address, together with the correspondence from *Nature*, as an appendix to *The Laws of Verse* (1870). The thoughts of Matthew Arnold, to whom the book was dedicated, are not known. Sylvester had some slight renown throughout his life, especially among his close friends, for his dirigible flights of poetic fancy; and his book was meant to illustrate the quasi-mathematical "principles of phonetic syzygy." Five original verses introduce a long paper on syzygetic relations, and he used his own verse on several other mathematical occasions.

Sylvester translated verse from several languages. For example, under the nom de plume "Syzygeticus," he translated from the German "The Ballad of Sir John de Courcy";[2] and his *Laws of Verse* includes other examples of his work, which is no worse than that of many a non-mathematician. It could be argued, however, that it was worse in a different way. One of his poems had four hundred lines all rhyming with "Rosalind," while another had two hundred rhyming with "Winn." These were products of his later residence in Baltimore. Sylvester had perhaps a better appreciation of music, and took singing lessons from Gounod.

In 1870 Sylvester resigned his post at Woolwich, and after a bitter struggle that involved correspondence in the *Times*, and even a leading article there (17 August 1871), he secured a not unreasonable pension. It was not until 1876, when he was sixty-one, that he again filled any comparable post. When he did so, it was in response to a letter from the American physicist Joseph Henry. The Johns Hopkins University opened in that year, and Sylvester agreed to accept a chair in mathematics in return for his travelling expenses and an annual stipend of $5,000 "paid in gold." "His first pupil, his first class" was G. B. Halsted. A colleague was C. S. Peirce, with whom, indeed, Sylvester became embroiled in controversy on a small point of priority. Peirce nevertheless later said of him that he was "perhaps the mind most exuberant in ideas of pure mathematics of any since Gauss." While at Baltimore, Sylvester founded the *American Journal of Mathematics*, to which he contributed thirty papers. His first was a long and uncharacteristic account of the application of the atomic theory to the graphical representation to the concomitants of binary forms (quadratics). He resigned his position at Johns Hopkins in December 1883, when he was appointed to succeed H. J. S. Smith as Savilian professor of geometry at Oxford.

Sylvester was seventy when he delivered his inaugural "On the Method of Reciprocants" (1 December 1885). By virtue of his chair he became a fellow of New College, where he lived as long as he was in Oxford. He collaborated with James Hammond on the theory of reciprocants (functions of differential coefficients the forms of which are invariant under certain linear transformations of the variables) and also contributed several original papers to mathematical journals before his sight and general health began to fail. In 1892 he was allowed to appoint a deputy, William Esson; and in 1894 he retired, living mainly at London and Tunbridge Wells. For a short period in 1896 and 1897 he wrote more on mathematics (for example, on compound partitions and the Goldbach-Euler conjecture). A little more than a fortnight after a paralytic stroke, he died on 15 March 1897 and is buried in the Jewish cemetery at Ball's Pond, Dalston, London.

Sylvester received many honors in his lifetime, including the Royal Medal (1861) and the Copley Medal (1880). It is of interest that in the receipt of such awards he followed rather than preceded Cayley, who was his junior. Sylvester received honorary degrees from Dublin (1865), Edinburgh (1871), Oxford (1880), and Cambridge (1890).

Sylvester never married. He had been anxious to marry a Miss Marston, whom he met in New York in 1842, on his first visit to America. (She was the godmother of William Matthew Flinders Petrie, from whom the story comes.) It seems that although she had formed a strong attachment for him, she refused him on the ground of religious difference, and neither of them subsequently married.

Sylvester's greatest achievements were in algebra. With Cayley he helped to develop the theory of determinants and their application to nonalgebraic subjects. He was instrumental in helping to turn the attention of algebraists from such studies as the theory of equations—in which he nevertheless did important work—to the theory of forms, invariants, and linear associative algebras generally. His part in this movement is often obscured by his flamboyant style. In 1888 P. G. Tait, in a rather strained correspondence with Cayley over the relations between Tait's solution of a quaternionic equation and Sylvester's solution of a linear matrix equation, wrote with some justice: "I found Sylvester's papers hard to assimilate. A considerable part of each paper seems to be devoted to correction of hasty generalizations in the preceding one!"[3]

A number of Sylvester's early writings concern the reality of the roots of numerical equations, Newton's rule for the number of imaginary roots, and Sturm's theorem. His first published researches into these matters date from 1839, and were followed by a steady stream of special results. In due course he found simple expressions for the Sturmian functions (with the square factors removed) in terms of the roots:

$$f_2(x) = \Sigma(a-b)^2\,(x-c)\,(x-d)\,\ldots$$

$$f_3(x) = \Sigma(a-b)^2\,(a-c)^2\,(b-c)^2\,(x-d)\,\ldots$$

Applying Sturm's process of the greatest algebraic common measure to two independent functions *f(x)* and *ϕ(x)*, rather than to *f(x)* and *f'(x)*, he found for the resulting functions expressions involving products of differences between the roots of the equations $f(x) = 0$, $\phi(x) = 0$. Assuming that the real roots of the two equations are arranged in order of magnitude, the functions are of such a character that the roots of the one equation are intercalated among those of the other.

In connection with Newton's rule, the method of Sturm's proof was applied to a quite different problem. Sylvester supposed *x* to vary continuously, and investigated the increase and decrease in the changes of sign.[4]

Newton's first statement of his incomplete rule for enumerating imaginary roots dates from 1665–1666.[5] Although valid, the rule was not justified before Sylvester's proofs of the complete rule.

Another problem of great importance investigated in two long memoirs of 1853 and 1864 concerns the nature of the roots of a quintic equation. Sylvester took the functions of the coefficients that serve to decide the reality of the roots, and treated them as the coordinates of a point in *n*-dimensional space. A point is or is not "facultative" according to whether there corresponds, or fails to correspond, an equation with real coefficients. The character of the roots depends on the bounding surface or surfaces of the facultative regions, and on a single surface depending on the discriminant.[6]

Sylvester showed an early interest in the theory of numbers when he published a beautiful theorem on a product formed from numbers less than and prime to a given number.[7] This he described as "a pendant to the elegant discovery announced by the ever-to-be-lamented and commemorated Horner, with his dying voice"; but unfortunately it was later pointed out to him by Ivory that Gauss had given the theorem in his *Disquisitiones arithmeticae* (1801).[8] It is impossible to do justice in a short space to Sylvester's numerous later contributions to the theory of numbers, especially in the partition of numbers. Sylvester applied Cauchy's theory of residues and originated the concept of a denumerant. He also added several results to Euler's treatment of the "problem of the virgins" (the problem of enumerating positive and integral solutions of indeterminate simultaneous linear equations); but his most novel contributions to the subject are to be found in his use of a graphical method. He represented partitions of numbers by nodes placed in order at the points of a rectangular lattice ("graph"). Thus a partition of 9(5+3+1) may be represented by the points of the rows in the lattice. The conjugate partition (3+2+2+1+1) is then found by considering the lattice of columns, a fact possibly first appreciated by N. M. Ferrers.[9] This

representation greatly simplified and showed the way to proofs of many new results in the theory of partitions not only by Sylvester but also by early contributors to his *American Journal of Mathematics*, such as Fabean Franklin.

One of Sylvester's early contributions to the *Journal*, "On Certain Ternary Cubic-Form Equations,"[10] is notable for the geometrical theory of residuation on a cubic curve and the chain rule of rational derivation: From an arbitrary point 1 on the curve it is possible to derive the singly infinite series of points (1,2,4,5,...3p±1) such that the chord through any two points, *m* and *n*, meets the curve again in a point ($m+n$ or $|m-n|$, whichever number is not divisible by 3) of the series. The coordinates of any point *m* are rational and integral functions of degree m^2 of those of point 1.

Like his friend Cayley, Sylvester was above all an algebraist. As G. Salmon said, the two discussed the algebra of forms for so long that each would often find it hard to say what properly belonged to the other. Sylvester, however, produced the first general theory of contravariants of forms.[11] He was probably the first to recognize that for orthogonal transformations, covariants and contravariants coincide. Moreover, he proved a theorem first given without proof by Cayley, and the truth of which Cayley had begun to doubt. It concerns a certain expression for a number ("Cayley's number") that cannot exceed the number of linearly independent semi-invariants (or invariants) of a certain weight, degree, and extent. Sylvester showed that Cayley's expression for the number of linearly independent ("asyzygetic") semi-variants of a given type is in fact exact.[12] The result is proved as part of Sylvester's and Cayley's theory of annihilators, which was closely linked to that of generating functions for the tabulation of the partitions of numbers.

Under the influence of Lie's analysis, algebraic invariance was gradually subordinated to a more general theory of invariance under transformation groups. Although Boole had used linear differential operators to generate invariants and covariants, Cayley, Sylvester, and Aronhold were the first to do so systematically. In the calculation of invariants, it may be proved that any invariant *I* of the binary for (quantic)

$f = a_o x^p + p a_1 x^{p-1} y + ... + a_p y^p$

should satisfy the two differential equations

$\Omega I = 0,$

$O\ I = 0,$

where Ω and *O* are linear differential operators:

Sylvester called these functions annihilators, built up a rich theory around them, and generalized the method to other forms,[13] With Franklin he exhibited generating functions for all semi-invariants, of any degree, for the forms they studied.[14] Related to these studies is Sylvester's expression, in terms of a linear differential equation, of the condition that a function be an orthogonal covariant or invariant of a binary quantic. Thus the necessary and sufficient condition that F be a covariant for direct orthogonal transformations is that F have as its annihilator

Sylvester played an important part in the creation of the theory of canonical forms. What may be his most widely known theorem states that a general binary form of odd order ($2n - 1$) is a sum of n ($2n - 1$)-th powers of linear forms. (Thus, for example, a quintic may be reduced to a sum of three fifth powers of linear forms.) Sylvester wrote at length on the canonical reduction of the general $2n$-ic. He showed that even with the ternary quartic, which has fifteen coefficients, the problem was far less simple than it appeared, and that such cannot be simply reduced to a sum of five fourth powers (again with fifteen coefficients). It is here that he introduced the determinant known as the catalecticant, which he showed must vanish if the general $2n$-ic is to be expressed as the sum of n perfect $2n$th powers of linear forms, together with (in general) a term involving the square of the product of these forms.[15]

Early in his study of the effects of linear transformations on real quadratic forms, Sylvester discovered (and named) the law of inertia of quadratic forms.[16] The law was discovered independently by Jacobi.[17] The theorem is that a real quadratic form of rank r may be reduced by means of a real nonsingular linear transformation to the form

where the index p is uniquely determined. (It follows that two real quadratic forms are equivalent under real and nonsingular transformation if and only if they have the same rank and the same index.)

Another memorable result in the theory of linear transformations and matrices is Sylvester's law of nullity, according to which if r_1 and r_2 are the ranks of two matrices, and if R is the rank of their product,

$R \leq r_1$,

$R \leq r_2$,

$R \geq r_1 + r_2 - n$,

where n is the order of the matrices. For Sylvester the "nullity" of a matrix was the difference between its order and rank, wherefore he wrote his law thus: "The nullity of the product of *two* (and therefore of any number of) matrices cannot be less than the nullity of any factor, nor greater than the sum of the nullities of the *several* factors which make up the product."[18]

Sylvester devised a method (the "dialytic method") for the elimination of one unknown between two equations

$f(x) \equiv a_0x^n + a_1x^{n-1} + \ldots + a_n = 0 (a_0 \neq 0)$,

$\phi(x) \equiv b_0x^m + b_1x^{m-1} + \ldots + b_m = 0 (b_0 \neq 0)$,

The method is simpler than Euler's well-known method. Sylvester formed n equations from $f(x)$ by separate and successive multiplication by $x^{n-1}, x^{n-2}, \ldots 1$, and m equations from $\phi(x)$ by successive multiplication by $x^{m-1}, x^{m-2}, \ldots 1$, From the resulting $m + n$ equations he eliminated the $m + n$ power of x, treating each power as an independent variable. The vanishing of the resulting determinant (E) is a necessary condition for f and ϕ to have a common root,but the method is deficient to the extent that the condition $E = 0$ is not proved sufficient. This type of approach was superseded in Sylvester's lifetime when Kronecker developed a theory of elimination for systems of polynomials in any number of variables, but elementary texts still quote Sylvester's method alongside Euler's and Bezout's.

Sylvester was inordinately proud of his mathematical vocabulary. He once laid claim to the appellation "Mathematical Adam," asserting that he believed he had "given more names (passed into general circulation) to the creatures of the mathematical reason than all the other mathematicians of the age combined."[19] Much of his vocabulary has been forgotten, although some has survived; but it would be a mistake to suppose that Sylvester bestowed names lightly, or that they were a veneer for inferior mathematics. His "combinants," for example, were an important class of invariants of several q-ary p-ics (q and p constant).[20] His "plagiograph" was less obscure under the title "skewn pantograph"; but under either name it was an instrument based on an interesting and unexpected geometrical principle that he was the first to perceive.[21] And in like manner one might run through his works, with their "allotrious" factors, their "zetaic" multiplication, and a luxuriant terminology between.

Sylvester thought his verse to be as important as his mathematics; but he was a poor judge, and the two had little in common beyond an exuberant vocabulary. His mathematics spanned, of course, a far greater range than it is possible to review here. One characteristic of this range is that it was covered without much recourse to the writings of contemporaries. As H. F. Baker has pointed out, in projective geometry Sylvester seems to have been ignorant of Poncelet's circular points at infinity, and not to have been attracted by Staudt's methods of dispensing with the ordinary notion of length. Sylvester's papers simply ignore most problems in the foundations of geometry. Remarkable as some of his writings in the theory of numbers, elliptic integrals, and theta functions are, he would have benefited from a closer reading of Gauss, Kummer, Cauchy, Abel, Riemann, and Weier-strass. Neither Lie's work on the theory of continuous groups nor the algebraic solution of the fifth-degree equation elicited any attention from him, and it is perhaps surprising that Cayley did not persuade him of their value. An illustration of Sylvester's self-reliance is found at the end of one of the last lengthy papers he composed, "On Buffon's Problem of the Needle," a new approach to this well-known problem in probabilities.[22] The paper was the outcome of conversations with Morgan Crofton, when Sylvester was his senior at Woolwich in the 1860's; yet an extension of Barbier's theorem, now proved by Sylvester, had been published in 1868 by Crofton himself. Sylvester's strength lay in the fact that he could acknowledge this sort of inadvertent duplication without significantly diminishing the enormous mathematical capital he had amassed.

NOTES

1. H. H. Bellot, *University College, London*, 1826–1926 (London, 1929), 38.

2.*Gentleman's Magazine*, n.s., **6** (Feb. 1871), 38–48.

3. C.G. Knott, *The Life and Scientific Work of Peter Guthrie Tait* (Cambridge, 1911), 159.

4. For this and the preceding doctrines, see especially "On a Theory of the Syzygetic Functions..." (1853), repr. in his *Collected Mathematical Papers* (henceforth abbreviated as **CMP**), **I** , no. 57, 429–586; and "Algebraical Researches, Containing a Disquisition on Newton's Rule ...," ibid., **II** , no. **74** (1864), 376–479. On Newton's rule also see ibid., **II** , no. **81** (1865), 493–494; no. **84** (1865–1866), 498–513; no. 108 (1871), 704–708; and **III** , no. 42 (1880), 414–425.

5. See D. T. Whiteside, ed., *The Mathematical Papers of Isaac Newton*, **I** (Cambridge, 1967), 524.

6. See in particular **CMP** , **I** , no. 57 (1853), 436.

7. Ibid., no. **5** (1838), 39.

8.*Disquisitiones arithmeticae* (1801), 76.

9.***CMP*, I** , no. **59** (1853), 597.

10. Ibid., **III** , no. **39** (1879–1880), 312–391.

11. Ibid., **I** , no. **33** (1851), 198–202.

12. Ibid., **IV** , no. **44** (1886), 515–519; see also no. **42** (1886). **458** .

13. See, for example, *ibid*., **III** , no. **18** (1878), 117–126; no. 27 (1878), 318–340; **IV** , no. **41** (1886), 278–302, esp. 288; no. 42 (1886), 305–513, esp. 451.

14. See especially ibid., **III** , no. 67 (1882), 568–622.

15. See the important memoir in *ibid*., **I** , no. **42** (1852), 284–327, with its amusing note to 293; "Meicatalecticizant would more completely express the meaning of that which, for the sake of brevity, I denote catalecticant."

16. Ibid., no. **47** (1852), 378–381; no. 57 (1853), 511; **IV** , no. 49 (1887), 532.

17.*Journal Für Mathematik*, **53** (1857), 275–281.

18.**CMP, IV** , no. 15 (1884), 134.

19. Ibid., no. **53** (1888), 588.

20. For some further details, see P. Gordan, *Vortesungen über Invariantentheorie*, **II** (Leipzig, 1887), 70–78.

21. CMP, **III** , no. 3 (1875), 26–34.

22. Ibid., **IV** , no. **69** (1890–1891), 663–679.

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I. Original Works. Sylvester published no lengthy volume of mathematics, although his books on versification and its mathematical principles are numerous, and include *The Law of Continuity as Applied to Versification...Illustrated by an English Rendering of "Tyr-rhena regnum," Hor. 3, 29 ...* (London, 1869); *The Laws of Verse, or Principles of Versification Exemplified in Metrical Translations* (London, 1870); *Fliegende Blätter* (Rosalind and Other Poems), a *Supplement to the Laws of Verse* (London, 1876); *Spring's Debut. A Town Idyll in Two Centuries of Continuous Rhyme* (Baltimore, 1880); *Retrospect. A Verse Composition by the Savilian Professor of Geometry ... Tr. Into Latin by Undergraduates of New College* (Oxford, 1884); and *Corolla versuum Cantatrici eximiac ... a professore Saviliano geometriae apud oxonienses* (Oxford, 1895).

Sylvester's mathematical papers are in *The Collected Mathematical Papers of James Joseph Sylvester*, H. F. Baker, ed., **4** vols. (Cambridge, 1904–1912). Thirty of his 87 known letters from American sources are in R. C. Archibald, "Unpublished Letters of James Joseph Sylvester and Other New Information Concerning His Life and Work," in *Osiris*, **1** (1936), 85–154. Archibald gives full bibliographical information on most of the verse writings. Sylvester wrote a sonnet to the Savilian professor of astronomy, Charles Pritchard, on the occasion of his receiving the gold medal of the Royal Astronomical Society, in *Nature*, **33** (1886), 516. One might have imagined that in his literary flamboyance he was imitating Disraeli, had he not also addressed a sonnet to Gladstone (1890).

Thirty-three of Sylvester's lectures were reported by James Hammond in *Lectures Containing an Exposition of the Fundamental Principles of the New Theory of Reciprocants Delivered During ....1886 Before the University of Oxford*, repr. from *American Journal of Mathematics* (Oxford–Baltimore, 1888), Lecture 34 is by Hammond. Sylvester contributed well over 300 different mathematical problems to *Educational Times*. These are calendared in *Collected Papers*, IV, 743–747; and several letters concerning the problems are printed in Archibald, *op. cit*., 124–128.

II. Secondary Litrature. H. F. Baker included a personal biography of Sylvester in his ed. of the collected papers. R. C. Archibald,*op. cit*., 91–95, lists 57 publications dealing with Sylvester's life and works. To these may be added R. C. Archibald, "Material Concerning James Joseph Sylvester," in *Studies and Essays Offered to George Sarton* (New York, 1947), 209–217; and R. C. Yates. "Sylevster at the University of Virginia," in *American Mathematical Monthly*, **44** (1937), 194–201.

The most useful discussions of Sylvester's life and work are A. Cayley, "Scientific Worthies **XXV** : James Joseph Sylvester," in *Nature*, **39** (1889), 217-219, repr. in *The Collected Mathematical Papers of Arthur Cayley*, XIII (Cambridge, 1897), 43–48; F. Franklin, *People and Problems, a Collection of Addresses and Editorials* (New York, 1908), 11–27, first printed in *Bulletin of the American Mathematical Society*, **3** (1897), 299–309; P. A. MacMahon, obituary in *Nature*, **55** (1897), 492–494; and obituary in *Proceedings of the Royal Society*, **63** (1898), ix–xxv; and M. Noether, obituary in *Mathematische Annalen*, **50** (1898), 133–156. Of these, Noether's article is mathematically the most useful. Cayley wrote much invaluable commentary on Sylvester's work, for which see the index to Cayley's collected papers. The *Johns Hopkins University Circulars* are a convenient source of biography, since the editors often reprinted articles on Sylvester that had first been published elsewhere (such as those cited above by Cayley and Franklin, and the first of MacMahon's).

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