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(*b*. Kazuya Village, near Gifu, Japan, 21 April 1875; *d*. Tokyo, Japan [?], 29 February 1960)

*mathematics*.

Teiji Takagi came from a family of landowners and government officials of a rural part of Gifu prefecture in central Japan. His mother, Tsune Takagi, was married to Mitsuzo Kinomura, of the town of Kitagata, but she returned to her family's home to bear her child and never returned to her husband, allegedly because she was repelled by the leeches on his farm. The boy's uncle, Kansuke Takagi, who was unable to have children of his own, adopted Teiji as his son, a common Japanese practice at the time for propertied families without heirs. The boy was frail and, under the severe discipline of his adoptive father, spent his childhood in study rather than in play with other children. He was an outstanding pupil, advancing through the six-year curriculum of the village school in three years, then graduating five years later, first in his class, from the middle school in Gifu. From 1891 to 1894 he attended the Third National Senior High School in Kyoto.

In 1894 Takagi entered the Imperial University in Tokyo, having already chosen mathematics as his field of study while in Kyoto. After his graduation in 1897, he began graduate study at the same university, but a prestigious government scholarship soon enabled him to go to Germany for three years; he elected to spend the first half of this period in Berlin, the second half in Göttingen.

Takagi read David Hilbert's famous report on the theory of algebraic numbers while he was in Berlin, and he developed an interest in pursuing the subject in the same direction as Hilbert himself—toward Kronecker's *Jugendtraum* ("dream of his youth") theorem on the abelian extensions of imaginary quadratic number fields. Hilbert, however, had shifted to other interests, so his direct influence on Takagi's work during Takagi's year-and-a-half stay in Göttingen was not great. Nonetheless, Takagi believed that his development was profoundly affected by Hilbert, especially by the spirit of mathematical study that Hilbert and Klein generated in turn-of-the-century Göttingen.

Soon after his return home in December 1901, Takagi married Toshi Tani, the sister-in-law of his landlord, in Tokyo on 6 April 1902. The couple had five daughters and three sons. He completed his doctorate at the Imperial University of Tokyo (26 December 1903) on the basis of a paper he had written in Göttingen on a special case of Kronccker's *Jugendtraum* theorem, namely, the case in which the imaginary quadratic field is the one obtained by adjoining *i*, the square root of -1. This paper and a few brief notes were published in 1902 and 1903, but there was no other published work in the following years prior to World War I. In 1904 he was made full professor at the university.

Class field theory grew out of attempts by Hilbert and Heinrich Weber, among others, to understand and prove the Jugendtraum theorem and other works of Leopold Kronecker. Weber gave the original definition of a "class field" (*Klassenkörper*) in terms of a rather complicated construction, the essence of which was to describe the way in which prime ideals of an algebraic number field (called the "ground field") factor in an abelian extension field (that is, a normal extension field whose Galois group over the ground field is an abelian group). Hilbert recast the theory, simplifying it by using a less general definition : Hilbert defined a class field to be what is now called an "absolute" class field, that is, a maximal abelian extension that is "unramified." (An extension is called unramified if the factorization of a prime ideal of the ground field never contains a repeated factor in the extension field.) The existence of class fields in Hilbert's sense was proved by Furtwängler in 1907.

Takagi returned to Weber's original viewpoint, but, instead of considering the way in which primes factor in an abelian extension field, considered the question of determining which ideals of the ground field are relative norms of ideals in the extension field. His main theorems were: (1) For any abelian extension, the ideals of the ground field that are relative norms of ideals of the extension field can be determined by simple multiplicative congruence conditions; (2) conversely, given a set of congruence conditions of the type Takagi describes, there is an abelian extension field in which these congruences determine the ideals that are relative norms of ideals in the extension field; (3) the prime ideals that figure in the muliplicative congruences coincide with the prime divisors of the discriminant of the extension; (4) the Galois group of the extension is isomorphic to the multiplicative group of classes of ideals of the ground field described by the multiplicative congruences; and (5) the way in which a prime ideal factors in the extension field depends only on its image in the Galois group—in particular, the prime ideals that factor into prime ideals of the first degree are precisely the ones that are relative norms (corresponding to the identity of the Galois group).

Takagi, having discovered these sweeping and unexpected theorems in isolation from his German colleagues, doubted they were correct. In fact, by his own account, he was sure they were wrong and spent a great deal of effort looking for an error in his reasoning. By the time the war was over and communication reestablished, however, he had convinced himself and had completed his theory. He set it forth in two long papers (1920, 1922) that constitute, in number of pages, half the volume of his published papers in languages other than Japanese, and contain most of his creative work.

At the International Congress of Mathematicians in Strasbourg in 1920, Takagi gave a brief presentation of his results. Whether because the German mathematicians had been excluded from participation in the congress, or because his presentation was so diffident, the significance of Takagi's work seems not to have been grasped by anyone present. Only in 1922, when C. L. Siegel persuaded Emil Artin to read the first of Takagi's two great papers, did the importance of his work begin to be recognized. A few years later. Helmut Hasse's expository treatises on class field theory made Takagi's theory known to the mathematical world, establishing it in its proper place as a revolutionary advance in algebraic number theory.

Beginning in 1930 Takagi wrote a number of textbooks in Japanese (he had also written one in 1901) dealing with algebra, analysis, the history of mathematics in the nineteenth century, and number theory. These books grew out of his long teaching career at the Imperial University and reportedly had a great effect on the education of later generations of Japanese mathematicians. In addition, he wrote a number of popular works that reached a wide audience and did much to stimulate interest in mathematics in Japan, particularly among young people.

# BIBLIOGRAPHY

I. Original Works. "Über eine Theorie des relativ Abelschen Zahlkorpers," in *Journal of the College of Science, Imperial University of Tokyo*, **41** (1920), 1–133; and "Über das Reciprocitatsgesetz in einem beliebigen algebraischen Zahlkorper," *ibid*., **44** (1922), 1–50. See also *The Collected Papers of Teiji Takagi*, S. Kuroda, ed. (Tokyo, 1973).

II. Secondary Literature. Kin-ya Honda, "Teiji Takagi: A Biography," in *Commentarii mathematici Universitatis Sancti Pauli*, **24** , no. 2 (1975), 141–167.

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