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(*b*. Edmonton, Middlesex, England, 18 August 1685; *d*. London, England, 29 December 1731)

mathematics.

Brook Taylor was the son of John Taylor of Bifrons House, Kent, and Olivia, daughter of Sir Nicholas Tempest, Bart. The family was fairly well-to-do, and was connected with the minor nobility. Brook's grandfather, Nathaniel, had supported Oliver Cromwell. John Taylor was a stern parent from whom Brook became estranged in 1721 when he married a woman said to have been of good family but of no fortune. In 1723 Brook returned home after his wife's death in childbirth. He married again in 1725 with his father's approval, but his second wife died in childbirth in 1730. The daughter born at that time survived.

Taylor's home life seems to have influenced his work in several ways. Two of his major scientific contributions deal with the vibrating string and with perspective drawing. His father was interested in music and art, and entertained many musicians in his home. The family archives were said to contain paintings by Brook, and there is an unpublished manuscript entitled *On Musick* among the Taylor materials at St. John's College, Cambridge. This is not the paper said to have been presented to the Royal Society prior to 1713, but a portion of a projected joint work by Taylor, Sir Isaac Newton, and Dr. Pepusch, who apparently was to write on the nonscientific aspects of music.

Taylor was tutored at home before entering St. John's College in 1701, where the chief mathematicians were John Machin and John Keill. Taylor received the LL.B. degree in 1709, was elected to the Royal Society in 1712, and was awarded the LL.D. degree in 1714. He was elected secretary to the Royal Society in January 1714, but he resigned in October 1718 because of ill health and perhaps because of a loss of interest in this rather confining task. He visited France several times both for the sake of his health and for social reasons. Out of these trips grew a scientific correspondence with Pierre Rémond de Montmort dealing with infinite series and Montmort's work in probability. In this Taylor served on some occasions as an intermediary between Montmort and Abraham De Moivre. W. W. Rouse Ball reports that the problem of the knight's tour was first solved by Montmort and De Moivre after it had been suggested by Taylor.[1]

Taylor published his first important paper in the *Philosophical Transactions of the Royal Society* in 1714, but he had actually written it by 1708, according to his correspondence with Keill. The paper dealt with the determination of the center of oscillation of a body, and was typical both of Taylor's work and of the times, in that it dealt with a problem in mechanics, used Newtonian dot notation, and led to a dispute with Johann **I** Bernoulli.

The period of 1714–1719 was Taylor's most productive, mathematically. The first editions of both his mathematical books, *Methodus incrementorum directa et inversa* and *Linear Perspective*, appeared in 1715. Their second editions appeared in 1717 and 1719 respectively. He also published thirteen articles, some of them letters and reviews, in the *Philosophical Transactions* during the years 1712–1724. These include accounts of experiments with capillarity, magnetism, and the thermometer. In his later years Taylor turned to religious and philosophical writings. His third book, *Comtemplatio philosophica*, was printed posthumously by his grandson in 1793.

Taylor is best known for the theorem or process for expanding functions into infinite series that commonly bears his name. Since it is an important theorem, and since there is disagreement as to the amount of credit that should be given to him for its development, an outline of his derivation of the theorem will be given here. The discussion of Proposition **VII** , Theorem **III** of the *Methodus incrementorum* includes the statement:

If z grows to be $z + nz$ then x equals

Taylor used dots below the variables to represent increments or finite differences, and dots above to represent Newton's fluxions.

The above statement is a notationally improved version of Newton's interpolation formula as given in Lemma 5 of Book **III** of his *Principia*. This formula had first appeared in a letter from James Gregory to John Collins in 1670.[2] Taylor had derived this formula inductively from a difference table written in terms of x and its successive differences.

Next, Taylor made the substitutions

to derive the statement: “as z growing becomes $z+v$, x likewise growing becomes

The final step in the derivation and Taylor’s original statement of the theorem, which in modern notation is

is finally derived in Corollary II to Theorem III as follows: “for evanescent increments [write] the fluxions which are proportional to them and make all of equal, then as with time flowing uniformly z becomes $z+v$, so will x become

This becomes the modern form of Taylor’s series when we realize that with “time flowing uniformly” is a constant, , and v is the increment in the independent variable.

Taylor’s first statement of this theorem had been given in a letter of 26 July 1712 to John Machin, which has been reprinted by H. Bateman. In it Taylor remarked that this discovery grew out of a hint from Machin given in a conversation in Child’s Coffeehouse about the use of “Sir Isaac Newton’s series” to solve Kepler’s problem, and “Dr. Halley’s method of extracting roots” of polynomial equations, which had been published in the *Transactions* for 1694.

This shows Taylor’s fairness, care, and familiarity with the literature. He used his formula to expand functions in series and to solve differential equations, but he seemed to have no foreshadowing of the fundamental role later assigned to it by Lagrange nor to have any qualms about the lack of rigor in its derivation. Colin Maclaurin noted that the special case of Taylor’s series now known as Maclaurin’s theorem or series was discussed by Taylor on page 27 of the 1717 edition of the *Methodus*. The term “Taylor’s series” was probably first used by L’Huillier in 1786, although Condorcet used both the names of Taylor and d’Alembert in 1784[3].

Although infinite series were in the air at the time, and Taylor himself noted several sources and motivations for his development, it seems that he developed his formula independently and was the first to state it explicitly and in a general form. Peano based his claim for Johann I Bernoulli’s priority on an integration in which Bernoulli used an infinite series in 1694[4]. Pringsheim showed that it is possible to derive Taylor’s theorem from Bernoulli’s formula by some changes of variable. However, there seems to be no indication that Taylor did this, nor that Bernoulli appreciated the final form or generality of the Taylor theorem. Taylor’s Proposition XI, Theorem IV, on the other hand, is directly equivalent to Bernoulli’s integration formula. However, Taylor’s derivation differs from Bernoulli’s in such a way as to entitle him to priority for the process of integration by parts.

Taylor was one of the few English mathematicians who could hold their own in disputes with Continental rivals, although even so he did not always prevail. Bernoulli pointed out that an integration problem issued by Taylor as a challenge to “non-English mathematicians” had already been completed by Leibniz in *Acta eruditorum*. Their debates in the journals occasionally included rather heated phrases and, at one time, a wager of fifty guineas. When Bernoulli suggested in a private letter that they couch their debate in more gentlemanly terms, Taylor replied that he meant to sound sharp and “to show an indignation”.

The *Methodus* contained several additional firsts, the importance of which could not have been realized at the time. These include the recognition and determination of a singular solution for a differential equation[5], a formula involving a change in variables and relating the derivatives of a function to those of its inverse function, the determination of centers of oscillation and percussion, curvature, and the vibrating string problem. The last three problems had been published earlier in the *Philosophical Transactions*, as had been a continued fraction for computing logarithms.

Newton approached curvature by way of the determination of the center of curvature as the limit point of the intersection of two normals. Although this was not published until 1736, Taylor was familiar with Newton’s work, since, after applying his own formula, Taylor remarked that the results agreed with those given by Newton for conic sections. Taylor, however, conceived of the radius of curvature as the radius of the limiting circle through three points of a curve, and associated curvature with the problem of the angle of contact dating back to Euclid. He then used curvature and the radius of curvature in giving the first solution for the normal vibrations of the simplest case of the plucked string. In propositions XXII and XXIII he showed that under his conditions each point will vibrate in the manner of a cycloidal pendulum, and he determined the period in terms of the length and weight of the string and a weight supported by the string. There is little doubt that Taylor’s work influenced later writers since, for example, Bernoulli cited Taylor in letters to his son Daniel on this topic.

The *Methodus* qualifies Taylor as one of the founders of the calculus of finite differences, and as one of the first to use it in interpolation and in summation of series.

Taylor contributed to the history of the barometer by explaining a derivation of the variation of atmospheric pressure as a logarithmic function of the altitude, and he also contributed to the study of the refraction of light.

Like all of Taylor’s writing, his book on linear perspective was so concise that Bernoulli characterized it as “abstruse to all and unintelligible to artists for whom it was more especially written”[6]. Even the second edition, which nearly doubled the forty-two pages of the first, showed little improvement in this matter. Its effect, nevertheless, was very substantial, since it passed through four editions, three translations, and twelve authors who prepared twenty-two editions of extended expositions based on Taylor’s concepts. He developed his theory of perspective in a formal and rigorous fashion in a sequence of theorems and proofs. The most outstanding and original of his ideas in this field were his definition and use of vanishing points and vanishing ideas for all lines and planes, and his development of a theory and practice for the inverse problem of perspective that later served as a basis for work by Lambert and for the development of photogrammetry. Taylor also made free use of the idea of associating infinitely distant points of intersection with parallel lines, and he sought to devise methods for doing geometric constructions directly in perspective.

A study of Brook Taylor's life and work reveals that his contribution to the development of mathematics was substantially greater than the attachment of his name to one theorem would suggest. His work was concise and hard to follow. The surprising number of major concepts that he touched upon, initially developed, but failed to elaborate further leads one to regret that health, family concerns and sadness, or other unassessable factors, including wealth and parental dominance, restricted the mathematically productive portion of his relatively short life.

NOTES

1. W. W. Rouse Ball, *Mathematical Recreations and Essays* (London, 1912), p. 175.

2. H. W. Turnbull, *James Gregory Tercentenary Memorial Volume* (London, 1939), pp.119–120.

3. Gino Loria, *Storia delle matematiche*, 2nd ed, (Milan, 1950), p. 649.

4. G. Peano, *Formulario mathematico*, 5th ed. (Turin, 1906–1908), pp. 87.

5. E. L. Ince, *Ordinary Differential Equations* (New York, 1944), p. 87.

6.*Contemplatio philosphica*, p. 29, quoted from *Acta eruditorum*.

BIBLIOGRAPHY

I. Original Works The major source of biographical data as well as the only publication of his philosophical book is *Contemplatio philosophica*: *A Posthumous Work of the late Brook Taylor, L.L.D. F.R.S. Some Time Secretary of the Royal Society to Which Is Prefixed a Life of the Author by his Grandson, Sir William Young, Bart., F.R.S. A.S.S. with an appendix containing Sundry Original Papers, Letters from the Count Raymond de Montmort, Lord Bolingbroke, Mercilly de Villette, Bernoulli, & c.* (London, 1793).

This book and the mathematical letters appended to it are reproduced in Heinrich Auchter, *Brook Taylor der Mathematiker und Philosoph* (Würzburg, 1937). Both of these books have a picture of Taylor as secretary of the Royal Society (1714) as a frontispiece. This picture may be derived from a plaque since it is signed "R. Earlem, Sculp". It is labeled "From an Original Picture in the Possession of Lady Young". A nearly identical picture labeled "J. Dudley, Sculp", is reproduced in *The Mathematics Teacher*, **27** (January 1927), 4. It is also labeled "London, Published March 26, 1811 by J. Taylor, High Holborn".

Charles Richard Wild, in *A History of the Royal Society* (London, 1848), lists a portrait of Taylor painted by Amiconi among the portraits in possession of the Royal Society, but *The Record of the Royal Society*, 3rd ed. (London, 1912), records in its "List of Portraits in Oil in Possession of the Society" "Brook Taylor L.L.D. F.R.S. (1685–1731). Presented by Sir W. Young, Bart., F.R.S. Painter Unknown".

The two editions of Taylor's *Methodus* cited above were both published in London, as were the editions of his *Linear Perspective*. Complete data on the editions and extensions of this book are contained in P. S. Jones, "Brook Taylor and the Mathematical Theory of Linear Perspective", in *The American Mathematical Monthly*, **58** (Nov. 1951), 597–606.

Additional data on Taylor's correspondence is to be found in H. Bateman. "The Correspondence of Brook Taylor", in *Bibliotheca Mathematica*, 3rd ser., **7** (1906–1907), 367–371; Edward M. Langley, "An Interesting Find", in *The Mathematical Gazette*, **IV** (July 1907), 97–98: Ivo Schneider, "Der Mathematiker Abraham de Moivre", in *Archive for History of Exact Sciences*, **5** (1968/1969), 177–317.

II. Secondary Literature. For details of one of Taylor's disputes see Luigi Conte, "Giovanni Bernoulli e le sfida di Brook Taylor", in *Archives de l'histoire des sciences*, **27** (or 1 of new series), 611–622.

The most extensive history of Taylor's theorem is Alfred Pringsheim, "Zur Geschichte des Taylorschen Lehrsatzes", in *Bibliotheca mathematica*, 3rd ser., **I** (Leipzig, 1900), 433–479.

Phillip S. Jones