Thabit Ibn Qurra, Al-S?abi? Al-H?arrani | Encyclopedia.com

(*b*. Harrān, Mesopotamia [now Turkey], 836; *d*. Baghdad, 18 February 901)

mathematics, astronomy, mechanics, medicine, philosophy.

Life. Thābit ibn Qurra belonged to the Sabian (Mandaean) sect, descended from the Babylonian star worshippers. Because the Sabians' religion was related to the stars they produced many astronomers and mathematicians. During the Hellenistic era they spoke Greek and took Greek names; and after the Arab conquest they spoke Arabic and began to assume Arabic names, although for a long time they remained true to their religion. Thābit, whose native language was Syriac, also knew Greek and Arabic. Most of his scientific works were written in Arabic, but some were in Syriac; he translated many Greek works into Arabic.

In his youth Thābit was a money changer in Harrān. The mathematician Muhammad ibn Mūsā ibn Shākir, one of three sons of Mūsā ibn Shākir, who was traveling through Harrān, was impressed by his knowledge of languages and invited him to Baghdad; there, under the guidance of the brothers, Thābit became a great scholar in mathematics and astronomy. His mathematical writings, the most studied of his works, played an important role in preparing the way for such important mathematical discoveries as the extension of the concept of number to (positive) real numbers, integral calculus, theorems in spherical trigonometry, analytic geometry, and non–Euclidean geometry. In astronomy Thābit was one of the first reformers of the Ptolemaic system, and in mechanics he was a founder of statics. He was also a distinguished physician and the leader of a Sabian community in Iraq, where he substantially strengthened the sect's influence. During his last years Thābit was in the retinue of the Abbasid Caliph al-Mu'tadid (892–902). His son Sinān and his grandsons Ib|rāhīm and Thābit were well–known scholars.

Mathematics. Thābit worked in almost all areas of mathematics. He translated many ancient mathematical works from the Greek, particularly all the works of Archimedes that have not been preserved in the original language, including *Lemmata, On Touching Circles,* and *On Triangles,* and Apolonius' *Conics*. He also wrote commentaries on Euclid's *Elements* and Ptolemy's *Almagest*.

Thābit's *Kitāb al-Mafrūdāt* ("Book of Data") was very popular during the Middle Ages and was included by Nasīr al-Dīn al-Tūsī in his edition of the "Intermediate Books" between Euclid's *Elements* and the *Almagest*. It contains thirty–six propositions in elementary geometry and geometrical algebra, including twelve problems in construction and a geometric problem equivalent to solution of a quadratic equation $(a + x)x = b$. *Maqāla fīistikhrāj al-a'dād al-mutahābba bi–suhūlat al-maslak ilā dhālika* ("Book on the Determination of Amicable Numbers") contains ten propositions in number theory, including ones on the constructions of perfect numbers (equal to the sum of their divisors), coinciding with Euclid's *Elements***IX,** 36, on the construction of surplus and "defective" numbers (respectively, those greater and less than the sum of their divisors) and the problem, first solved by Thābit, of the construction of "amicable" numbers (pairs of numbers the sum of the divisors of each of which is equal to the other). Thābit's rule is the following: If $p = 3 \cdot 2^{n-1} - 1$, $q = 3 \cdot 2^{n-1} - 1$, and $r = 9 \cdot 2^{2n-1} - 1$, r are prime numbers, then $M = 2^n \cdot pq$ and $N = 2^n \,.\, r$ are amicable numbers.

Kitāb fī Ta'līf al-nisab ("Book on the Composition of Ratios") is devoted to "composite ratios" (ratios of geometrical quantities), which are presented in the form of products of ratios. The ancient Greeks, who considered only the natural numbers as numbers, avoided applying arithmetical terminology to geometrical quantities, and thus they named the multiplication of ratios by "composition." Composition of ratios is used in the *Elements* (**VI,** 23), but is not defined in the original text; instead, only particular cases of composite ratios are defined (*Definitions***V,** 9–10). An addition by a later commentator (evidently Theon of Alexandria, in **VI,** 5) on composite ratios is done in a completely non–Euclidean manner.

Thābit criticizes *Elements***VI,** 5, and proposes a definition in the spirit of Euclid: for three quantities A, B, and C, the ratio A/B is composed of the ratios A/C and C/B, and for six quantities A,B,C,D,E,F the ratio A/B is composed of the ratios C/D and E/F, if there are also three quantities L,M,N, such that $A/B = L/M$, $C/D = L/N$, $E/F = N/M$. He later defines the "Multiplication of quantities by a quantity" and systematically applies arithmetical terminology to geometrical quantities. He also proves a number of theorems on the composition of ratios and solves certain problems concerning them . This treatise was important in preparing the extension of the concept of number to positive real numbers, produced in a clear form in the eleventh century by al-Bīrūnī (*al-Qānūn al-Mas'ūdī*) and al-Khayyāmī (*Sharh māashkhāla min musādarāt Kitāb Uqlīdis*).

In *Risāla fi Shakl al-qitā*‘ (“Treatise on the Secant Figure”) Thābit gives a new and very elegant proof of Menelaus’ theorem of the complete spherical quadrilateral, which Ptolemy had used to solve problems in spherical astronomy; to obtain various forms of this theorem Thābit used his own theory of composite ratios. In *Kitāb fi Misāhat qat‘ almakhrūt alladhī yusammaā al-mukāfi*’ (“Book on the Measurement of the Conic Section Called Parabolic”) Thābit computed the area of the segment of a parabola. First he proved several theorems on the summation of a numerical sequence from

He then transferred the last result to segments $a_k = (2k - 1)a, b_k = 2k . b$ and proved the theorem that for any ratio α/β, however small, there can always be found a natural n for which

which is equivalent to the relation him .

Thābit also applied this result to the segments and divides the diameter of the parabola into segments proportional to odd numbers; through the points of division he then takes chords conjugate with the diameter and inscribes in the segment of the parabola a polygon the apexes of which are the ends of these chords. The area of this polygon is valued by upper and lower limits, on the basis of which it is shown that the area of the segment is equal to 2/3 the product of the base by the height. A. P. Youschkevitch has shown that Thābit’s computation is equivalent to that of the integral and not as is done in the computation of the area in Archimedes’ *Quadrature of the Parabola*. The computation is based essentially on the application of upper and lower integral sums, and the proof is done by the method of exhaustion; there, for the first time, the segment of integration is divided into unequal parts.

In *Maqāla fī Misāhat al-3;mujassamāt al-mukāfiya* (“Book on the Measurement of Parabolic Bodies”) Thābit introduces a class of bodies obtained by rotating a segment of a parabola around a diameter: “parabolic cupolas” with smooth, projecting, or squeezed vertex and, around the bases, “parabolic spheres,” named cupolas and spheres. As in *Kitāb . . . al-mukāfī* he also proved theorems on the summing of a number sequence; a theorem equivalent to for any α, $0 < \alpha < 1$; and a theorem that the volume of the “parabolic cupola” is equal to half the volume of a cylinder, the base of which is the base of the cupola, and the height is the axis of the cupola: the result is equivalent to the computation of the integral .

Kitāb fī Misāhat al-ashkāl al-musattaha wa’lmujassama (“Book on the Measurement of Plane and solid Figures”) contains rules for computing the areas of plane figures and the surfaces and volumes of solids. Besides the rules known earlier there is the rule proved by Thābit in “another book,” which has not survived, for computing the volumes of solids with “various bases” (truncated pyramids and cones): if S_1 and S_2 are the areas of the bases and h is the height, then the volume is equal to .

Kitāb fi l–ta attī li–istikhrāj ‘amal al-masā’il alhandasiyya (“Book on the Method of solving Geometrical Problems”) examines the succcession of operations in three forms of geometrical problems: construction, measurement, and proof (in contrast with Euclid, who examined only problems in construction [“problems”] and in proof [“theorems”]. In *Risāla fi’l–hujja al-mansūba ilā Suqrāt fi’l–murabba wa qutrihi* (“Treatise on the Proof Attributed to Socrates on the Square and Its Diagonals”). Thābit examines the proof, described by Plato in *Meno,* of Pythagoras’ theorem for an isosceles right triangle and gives three new proofs for the general case of this theorem. In the first, from a square constructed on the hypotenuse, two triangles congruent to the given triangle and constructed on two adjacent sides of the square are taken out and are added to the two other sides of the square, and the figure obtained thus consists of squares constructed on the legs of the right triangle. The second proof also is based on the division of squares that are constructed on the legs of a right triangle into parts that form the square constructed on the hypotenuse. The third proof is the generalization of Euclid’s *Elements***VI,** 31. There is also a generalization of the Pythagorean theorem: If in triangle *ABC* two straight lines are drawn from the vertex *B* so as to cut off the similar triangles *ABE* and *BCD,* then $AB^2 + BC^2 = AC (AE + CD)$.

In *Kitāb fī ‘amal shakl mujassam dhī arba’ ‘ashrat qā ida tuhītu bihi kura ma‘lūma* (“Book on the Construction of a Solid Figure . . .”) Thābit constructs a fourteen–sided polyhedron inscribed in a given sphere. He next makes two attempts to prove Euclid’s fifth postulate: *Maqāla fī burhān al-musādara ’l–mashhūra min Uqlīdis* (“Book of the Proof of the Well–Known Postulate of Euclid”) and *Maqāla fī anna ’l–khattayn idhā ukhrijā ’alā zawiyatayn aqal min qā’imatayn iltaqayā* (“Book on the Fact That Two Lines Drawn [From a Transversal] at Angles Less Than Two Right Angles Will Meet”). The first attempt is based on the unclear assumption that if two straight lines intersected by a third move closer together or farther apart on one side of it, then they must, correspondingly, move farther apart or closer together on the other side. The “proof” consists of five propositions, the most important of which is the third, in which Thābit proves the existence of a parallelogram, by means of which Euclid’s fifth postulate is proved in the fifth proposition. The second attempt is based on kinematic considerations. In the introduction to the treatise Thābit criticizes the approach of Euclid, who tries to use motion as little as possible in geometry, asserting the necessity of its use. Further on, he postulates that in “one simple motion” (parallel translation) of a body, all its points describe straight lines. The “proof” consists of seven propositions, in the first of which, from the necessity of using motion, he concludes that equidistant straight lines exist; in the fourth proposition he proves the existence of a rectangle that is used in the seventh proposition to prove Euclid’s fifth postulate. These two treatises were an important influence on subsequent attempts to prove the fifth postulate (the latter in particular influenced Ibn al-Haytham’s commentaries on Euclid). Similar attempts later led to the creation of non-Euclidean geometry.

Kitāb fi Qutū‘ al-ustu wāna wa–basītihā (“Book on the Sections of the Cylinder and Its Surface”) examines plane sections of an inclined circular cylinder and computes the area of the lateral surfaces of such a cylinder between the two plane sections. The treatise contains thirty–seven propositions. Having shown in the thirteenth that an ellipse is obtained through right–angled compression of the circle, in the next Thābit proves that the area of an ellipse with semiaxes *a* and *b* is equal to the area of the circle of radius $\sqrt{ab}$; and in the propositions 15–17 he examines the equiaffine transformation, making the ellipse into a circle equal to it.

Thābit proves that in this case the areas of the segments of the ellipse are equal to the areas of the segments of the circle corresponding to it. In the thirty–seventh proposition he demonstrates that the area of the lateral surface of the cylinder between two plane segments is equal to the product of the length of the periphery of the ellipse that is the least section of the cylinder by the length of the segment of the axis of the cylinder between the sections. This proposition is equivalent to the formula that expresses the elliptical integral of the more general type by means of the simplest type, which gives the length of the periphery of the ellipse.

The algebraic treatise *Qawl fī Taṣḥīḥ masā'il al-jabr bi 'l–barāhī al-handasiyya* ("Discourse on the Establishment of the Correctness of Algebra Problems . . .") establishes the rules for solving the quadratic equations $x^2 + ax = b$, $x^2 + b = ax$, $x^2 = ax + b$, using *Elements* **II** , 5–6. (In giving the geometrical proofs of these rules earlier, AlKhwārizmī did not refer to Euclid.) In *Mas'ala fī s'amal al-mutawassitayn waqisma zāwiya malū ma bi–thalāth aqsām mutasāwiya* ("Problem of Constructing Two Means and the Division of a Given Angle Into Three Equal Parts"), Thābit solves classical problems of the trisection of an angle and the construction of two mean proportionals that amount to cubic equations. Here these problems are solved by a method equivalent to Archimedes' method of "insertion" which basically involves finding points of intersection of a hyperbola and a circumference. (In his algebraic treatise al-Khayyāmī later used an analogous method to solve all forms of cubic equations that are not equivalent to linear and quadratic ones and that assume positive roots.)

Thābit studied the uneven apparent motion of the sun according to Ptolemy's eccentricity hypothesis in *Kitāb fi Ibtā' al-haraka fi falak al-burūj wa sur'atihā bi–hasab al-mawādi' allati yakūnu fihi min al-falak al-khārij al-markaz* ("Book on the Deceleration and Acceleration of the Motion on the Ecliptic . . ."), which contains points of maximum and minimum velocity of apparent motion and points at which the true velocity of apparent motion is equal to the mean velocity of motion. Actually these points contain the instantaneous velocity of the unequal apparent motion of the sun.

A treatise on the sundial, *Kitāb fi ālāt al-sā' āt allatī tusammā rukhāmāt*, is very interesting for the history of mathematics. In it the definition of height h of the sun and its azimuth A according to its declination δ, the latitude ø of the city and the hour angle t leads to the rules sin h=dos(ø–δ)– versed sin t · cos δ · cos ø and , which are equivalent to the spherical theorems of cosines and sines for spherical triangles of general forms, the vertexes of which are the sun, the zenith, and the pole of the universe. The rules were formulated by Thābit only for solving concrete problems in spherical astronomy; as a general theorem of spherical trigonometry, the theorem of sines appeared only at the end of the tenth century (Mansūr ibn 'Irāq), while the theorem of cosines did not appear until the fifteenth century (Regiomontanus). In the same treatise Thābit examines the transition from the length of the shadow of the gnomon l on the plane of the sundial and the azimuth A of this shadow, which in essence represent the polar coordinates of the point, to "parts of longitude" x and "parts of latitude" y, which represent rectangular coordinates of the same point according to the rule $x=l\sin A$, $y=l\cos A$.

In another treatise on the sundial, *Maqāla fi sifat al-ashkāl allatī tahduthu bi–mamarr taraf zill al-miqyās fi sath al-ufug fi kull yawm wa fi kull balad*, Thābit examines conic sections described by the end of a shadow of the gnomon on the horizontal plane and determines the diameters and centers of these sections for various positions of the sun. In the philosophical treatise *Masā'il su'ila 'anhā Thābit ibn Qurra al-Harrānī* ("Questions Posed to Thābit. . ."), he emphasizes the abstract character of number (*adad*), as distinct from the concrete "counted thing" (*ma'dūd*), and postulates "the existence of things that are actually infinite in contrast with Aristotle, who recognized only potential infinity. Actual infinity is also used by Thābit in *Kitāb fi'l qarastūn* ("Book on Beam Balance").

Astronomy . Thābit wrote many astronomical works. We have already noted his treatise on the investigation of the apparent motion of the sun; his *Kitāb fī Sanat al-shams* ("Book on the Solar Year") is on the same subject. *Qawl fi īdāh al-wajh alladhī dhakara Batlamyūs*. . .concerns the apparent motion of the moon, and *Fī hisāb ru'yat al-ahilla*, the visibility of the new moon. In what has been transmitted as *De motu octave spere* and *Risāla ilā Ishāq ibn Hunayn* ("Letter to. . .") Thābit states his kinematic hypothesis, which explains the phenomenon of precession with the aid of the "eighth celestial sphere" (that of the fixed stars); the first seven are those of the sun, moon, and five planets. Thābit explains the "trepidation" of the equinoxes with the help of a ninth sphere. The theory of trepidation first appeared in Islam in connection with Thābit's name.

Mechanics and Physics . Two of Thābit's treatises on weights, *Kitāb fi Sifat al-wazn wa–ikhtilāfihi* ("Book on the Properties of Weight and Nonequilibrium") and *Kitāb fi'l–Qarastūn* ("Book on Beam Balance"), are devoted to mechanics. In the first he formulates Aristotle's dynamic principle, as well as the conditions of equilibrium of a beam, hung or supported in the middle and weighted on the ends. In the second treatise, starting from the same principle. Thābit proves the principle of equilibrium of levers and demonstrates that two equal loads, balancing a third, can be replaced by their sum at a midpoint without destroying the equilibrium. After further generalizing the latter proposition for the case in which "as many [equal] loads as desired and even infinitely many" are hung at equal distances, Thābit considers the case of equally distributed continuous loads. Here, through the method of exhaustion and examination of upper and lower integral sums, a calculation equivalent to computation of the integral The result obtained is used to determine the conditions of equilibrium for a heavy beam.

Thābit's work in natural sciences includes *Qawl fi'l–Sabab alladhī ju'ilat lahu miyāh al-bahr māliha* ("Discourse on the Reason Why Seawater Is Salted"), extant in manuscript, and writings on the reason for the formation of mountains and on the striking of fire from stones. He also wrote two treatises on music.

Medicine . Thābit was one of the best–known physicians of the medieval East. Ibn al-Qiftī, in *Ta'rikh al-hukamā*, tells of Thābit's curing a butcher who was given up for dead. Thābit wrote many works on Galen and medicinal treatises, which are almost completely unstudied. Among these treatises are general guides to medicine–*al-Dhakhira fī ilm al-tibb* ("A Treasurey of Medicine"), *Kitāb al-Rawda fi l–tibb* ("Book of the Garden of Medicine"), *al-Kunnash* ("Collection")–and works on the circulation of the blood, embryology, the cure of various illnesses–*Kitāb fi 'ilm al-'ayn* . . . ("Book on the Science of the Eye…"), *Kitāb fi'l–jadarī wa'l–hasbā* ("Book on Smallpox and Measles"), *Risāla fī tawallud al-hasāt* ("Treatise on the Origin of Gallstones"), *Risāla fi'l–bayād alladhī yazharu fi'l–badan* ("Treatise on Whiteness . . .in the Body")–and on medicines. Thābit also wrote on the anatomy of birds and on veterinary medicine (*Kitāb al-baytara*), and commented on *De plantis*, ascribed to Aristotle.

Philosophy and Humanistic Sciences . Thābit's philosophical treatise *Masā'il su'ila 'anhā Thābit ibn Qurra al-Harrānī* comprises his answers to questions posed by his student Abū Mūsā ibn Usayd, a Christian from Iraq. In another extant philosophical treatise, *Maqāla fī talkhīs mā atā bihi Aristūtālis fī kitābihi fi Mā ba'd al-tābī'a*, Thābit criticizes the views of Plato and Aristotle on the motionlessness of essence, which is undoubtedly related to his opposition to the ancient tradition of not using motion in mathematics. Ibn al-Qiftī (*op. cit*., 120) says that Thābit commented on Aristotle's *Categories, De interpretatione*, and *Analytics*. He also wrote on logic, psychology, ethics, the classification of sciences, the grammar of the Syriac language, politics, and the symbolism in Plato's *Republic*. Ibn al-Qiftī also states that Thābit produced many works in Syriac on religion and the customs of the Sabians.

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