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(*b*. Athens, *ca*. 417 b.c.; *d*. Athens, 369 b.c.)

mathematics.

The son of Euphronius of Sunium, Theaetetus studied under Theodorus of Cyrene and at the Academy with Plato. Although no writing of his has survived, Theaetetus had a major influence in the development of Greek mathematics. His contributions to the theory of irrational quantities and the construction of the regular solids are particularly recorded; and he probably devised a general theory of proportion–applicable to incommensurable and to commensurable magnitudes–before the theory developed by Eudoxus and set out in book V of Euclid's Elements.

The Suda lexicon has two entries[1] under the name Theaetetus:

"Theaetetus, of Athens, astronomer, philosopher, disciple of Socrates, taught at Heraclea. He was the first to write on (or construct) the so–called five solids. He lived after the Peloponnesian war",

"Theaetetus, of Heraclea in Pontus, philosopher, a pupil of Plato".

Some have supposed that these notices refer to the same person, but it is more probable, as G. J. Allman[2] conjectures, that the second Theaetetus was a son or other relative of the first sent by him while teaching at Heraclea to study at the Academy in his native city.

Plato clearly regarded Theaetetus with a respect and admiration second only to that which he felt for Socrates. He made him a principal character in two dialogues, the eponymous *Theaetetus* and the *Sophist*; and it is from the former dialogue that what we know about the life of Theaetetus is chiefly derived[3]. In the dialogue Euclid of Megara gets a servant boy to read to his friend Terpsion a discussion between Socrates, Theodorus, and Theaetetus that Plato recorded soon after it took place on the day that Socrates faced his accusers, that is, in 399 b.c. Since Theaetetus is there referred to as a μειϱάκιον ("a youth"), it is implied that he was an adolescent, say eighteen years old, that is, he was born about 417 b.c.[4]. His father, we are told, left a large fortune, which was squandered by trustees; but this did not prevent Theaetetus from being a liberal giver. Although Theaetetus was given the rare Greek compliment of being καλος τε καὶἀγαθός ("a thorough gentleman"), it was the beauty of his mind rather than of his body that impressed his compatriots; for, like Socrates, he had a snub nose and protruding eyes. Among the many young men with whom Theodorus had been acquainted, he had never found one so marvelously gifted; the lad's researches were like a stream of oil flowing without sound. Socrates predicted that Theaetetus would become notable if he came to full years. In the preface to the dialogue Euclid relates how he had just seen Theaetetus being carried in a dying condition from the camp at Corinth to Athens; not only had he been wounded in action, after acquitting himself gallantly, but he had contracted dysentery. This would be in the year 369 b.c., for the only other year in that century in which Athens and Corinth were at war, 384 b.c., would hardly allow time for Theaetetus' manifold accomplishments[5].

The Theaetetus is devoted to the problem of knowledge; and the *Sophist*, apart from a method of definition, to the meaning of nonbeing. Although Theaetetus plays a major part in both discussions, there is no reason to think that he was a philosopher in the usual sense of the word. Plato merely used him as a vehicle for thoughts that he wanted expressed. That the two *Suda* passages use the term "philosopher" proves nothing, since the lexicon regularly calls mathematicians philosophers[6].

In the summary of the early history of Greek geometry given by Proclus, and probably taken from Eudemus, Theaetetus is mentioned along with Leodamas of Thasos and Archytas of Tarentum as having increased theorems and made an advance toward a more scientific grouping[7], the zeal for which is well shown in the mathematical passage that Plato introduces into the *Theaetetus*[8]. In this passage Theaetetus first relates how Theodorus demonstrated to him and the younger Socrates (a namesake of the philosopher) in each separate case that is a surd. He adds: "Since the number of roots[9] seemed to be infinite, it occurred to us to try to gather them together under one name by which we could call all the roots", Accordingly Theaetetus and the younger Socrates divided all numbers into two classes. A number that could be formed by multiplying equal factors they likened to a square and called "square and equilateral". The other numbers–which could not be formed by multiplying equal factors, but only a greater by a less, or a less by a greater–they likened to an oblong and called "oblong numbers". The lines forming the sides of equilateral numbers they called "lengths", and the lines forming oblong numbers they called "roots". "And similarly", concluded the Theaetetus of the dialogue, "for solids", which can only mean that they attempted a similar classification of cube roots.

The classification may now seem trivial, but the discovery of the irrational was a fairly recent matter[10] and involved a complete recasting of Greek mathematics: and Theaetetus was still only a young man. His more mature work on the subject is recorded in a commentary on the tenth book of Euclid's Elements, which has survived only in Arabic and is generally identified with the commentary that Pappus is known to have written. In the introduction to this commentary it is stated:[11]

The aim of Book X of Euclid's treatise on the Elements is to investigate the commensurable and the incommensurable, the rational and irrational continuous quantities. This science had its origin in the school of Pythagoras, but underwent an important development at the hands of the Athenian, Theaetetus, who is justly admired for his natural aptitude in this as in other branches of mathematics. One of the most gifted of men, he patiently pursued the investigation of the truth contained in these branches of science, as Plato bears witness in the book which he called after him, and was in my opinion the chief means of establishing exact distinctions and irrefutable proofs with respect to the above–mentioned quantities. For although later the great Apollonius, whose genius for mathematics was of the highest possible order, added some remarkable species of these after much laborious application, it was nevertheless Theaetetus who distinguished the roots which are commensurable in length from those which are incommensurable, and who divided the more generally known irrational lines according to the different means, assigning the medial line to geometry, the binomial to arithmetic, and the apotome to harmony, as is stated by Eudemus the Peripatetic.

The last sentence gives the key to the achievement of Theaetetus in this field. He laid the foundation of the elaborate classification of irrationals, which is found in Euclid's tenth book: and in particular Theaetetus discovered, and presumably named, the medial, binomial, and apotome. The medial is formed by the product of two magnitudes, the binomial ("of two names") by the sum of two magnitudes, and the apotome (implying that something has been cut off) by the difference of two magnitudes. It is easy to see the correlation between the medial and the geometric mean, for the geometric mean between two irrational magnitudes,[12]*a, b,* is and is medial. It is also easy to see the correlation between the binomial and the arithmetic mean, for the arithmetic mean between *a,b,* is (½*a* + ½*b*); and this is a binomial. It is not so easy to see the connection between the apotome and the harmonic mean; but a clue is given in the second part of the work, where the commentator returns to the achievement of Theaetetus and observes that if the rectangle contained by two lines is a medial, and one of the sides is a binomial, the other side is an apotome. This in turn recalls Euclid, *Elements* X,112, and amounts to saying that the harmonic mean between *a, b,* that is, $2ab/ (a+b)$, can be expressed as

This leads to the question how much of Euclid's tenth book is due to Theaetetus. After a close examination, B.L. van der Waerden concluded that "The entire book is the work of Theaetetus",[13] There are several reasons, however, for preferring to believe that Theaetetus merely identified the medial, binomial, and apotome lines, correlating them with the three means, as the Arabic commentary says, and that the addition of ten other species of irrationals, making thirteen in all, or twenty–five when the binomials and apotomes are further subdivided, is the work of Eulid himself. A scholium to the fundamental proposition **X.9** ("the squares on strainght lines commensurable in length have to one another the ratio which a square number has to a square number ...") runs as follows: "This theorem is the discovery of Theaetetus, and Plato recalls it in the *Theaetetus*, but there it is related to particular cases, here treated generally",[14] This would be a pointless remark if Theaetetus were the author of the whole book.

The careful distinction made in the "Eudemian summary" between Euclid's treatment of Eudoxus and Theaetetus is also relevant, Euclid, says the author, "put together the elements, arranging in order many of Eudoxus' theorems, perfecting many of Theaetetus', and bringing to irrefutable demonstration the things which had been only loosely proved by his predecessors".[15] The implication would seem to be that book **V** is almost entirely the discovery of Eudoxus save in its arrangement, but book **X** is partly due to Theaetetus and partly to Euclid himself. The strongest argument for believing that Theaetetus had an almost complete knowledge of the Euclidean theory of irrationals is that the correlation of the apotome with the harmonic mean implies a knowledge of book **X** .112; but it is relevant that the genuine text of Euclid probably stops at book **X.III** with the list of the thirteen irrational straight lines.[16]

A related question is the extent to which the influence of Theaetetus can be seen in the arithmetical books of Euclid's *Elements*, **VII-IX** . Euclid **X.9** depends on **VIII.11** ("Between two square numbers there is one mean proportional number . . ."), and **VIII.11** depends on **VII.17** and **VII.18** (in modern notation, $ab : ac = b : c$, and $a : b = ac : bc$). H. G. Zeuthen has argued[17] that, these propositions are an inseparable part of a whole theory established in book **VII** and in the early part of book **VIII** , and that this theory must be due to Theaetetus with the object of laying a sound basis for his treatment of irrationals. It is clear, however, as T. L. Heath has pointed out,[18] that before Theaetetus both Hippocrates and Archytas must have known propositions and definitions corresponding to these in books **VII** and **VIII** ; and there is no reason to abandon the traditional view that the Pythagoreans had a numerical theory of proportion that was taken over by Euclid in his arithmetical books. Theaetetus merely made use of an existing body of knowledge.

Theaetetus' work on irrationals is closely related to the two other main contributions to mathematics attributed to him. The only use made of book **X** in the subsequent books of Euclid's *Elements* is to express the sides of the regular solids inscribed in a sphere in terms of the diameter. In the case of the pyramid, the octahedron, and the cube, the length of the side is actually determined; in the case of the icosahedron, it is shown to be a minor; and in the case of the dodecahedron, to be an apotome. It is therefore significant that in the passage from the *Suda* lexicon (cited above) Theaetetus is credited as the first to "write upon" or "Theaetetus is credited as the first to "write upon" or "construct" the so–called five solids (πρωητος δὲ πέντε καλούμενα στερεὰ ἔγραψε). It is also significant that at the end of the mathematical passage in the *Theaetetus* he says that he and his companion proceeded to deal with solids in the same way as with squares and oblongs in the plane. Probably on the authority of Theophrastus, Aëtius[19] attributed the discovery of the five regular solids to the Pythagoreans; and Proclus[20] actually attributes to Pythagoras himself the "putting together" (σύστασις) of the "cosmic figures" use of them in the *Timaeus* to build the universe;[21]and no doubt the σύστασις is to be understood as a "putting together" of triangles, squares, and pentagons in order to make solid angles as in that dialogue rather than in the sense of a formal construction. Theaetetus was probably the first to give a theoretical construction for all the five regular solids and to show how to inscribe them in a sphere. A scholium to Euclid, *Elements***XIII** , actually attributes to Theaetetus rather than to the Pythagoreans the discovery of the octahedron and icosahedron.[22] On the surface this is puzzling, since the octahedron is a more elementary figure than the dodecahedron, which requires a knowledge of the pentagon; but many objects of dodecahedral form have been found from days much earlier than Pythagoras,[23] and the Pythagorean Hippasus is known to have written on "the construction of the sphere from the twelve pentagons",[24] (It would be in this work, if not earlier, that he would have encountered the irrational, and for his impiety in revealing it, he was drowned at sea.) If the Pythagoreans knew the dodecahedron, almost certainly they knew also the octahedron and probably the icosahedron; and the scholium quoted above may be discounted. The achievement of Theaetetus was to give a complete theoretical construction of all five regular solids such as we find in Euclid, *Elements***XIII** ; and Theaetetus must be regarded as the main source of the book, although Euclid no doubt arranged the maerials in his own impeccable way and put the finishing touches.[25]

The theory of irrationals is also linked with that of proportionals. When the irrational was discovered, it involved a recasting of the Pythagorean theory of proportion, which depended on taking aliquot parts, and which consequently was applicable only to rational numbers, in a more genmagnitudes. Such a general theory was found by Eudoxus and is embodied in Euclid, *Elements* V. But in 1933[26] Oskar Becker gave a new interpretaion of an obscure passage in Aristotle's *Topics*.[27] He suggested that the theory of proportion had already been recast in a highly ingenious form; and if so, the indication is that it was so recast by Eudoxus' older contemporary Theaetetus.

In the passage under discussion Aristotle observes that in mathematics some things are not easily proved for lack of a definition–for example, that a straight line parallel to two of the sides of a parallelogram divides the other two sides and the area in the same ratio: but if the definition is given, it becomes immediately clear, "for the areas have the same ανταναίρεσις as the sides, and this is the definition of the same ratio". What does the Greek word mean?. The basic meaning is "a taking away", and the older commentators up to Heath

and the Oxford translation supposed that it meant "a taking away of the same fraction". In the figure *EF* is the straight line parallel to the sides *AB, DC* of the parallelogram *ABCD*, and *AE, BF* are the same parts of *AD, BC* respectively as the parallelogram *ABFE* is of the parallelogram *ABCD*. This would be in accordance with the Pythagorean theory of proportion, and the passage would contain nothing significant. But Becker drew attention to the comment by Alexander of Aphrodisias on this passage; he uses the word ἀνθυφαίρεσις and observes that this is what Aristotle means by ἀνταναίρεσις. This might not in itself prove very much–the meaning could still be much the same–if it were not, as Becker also noted, that Euclid, although he does not employ the noun ἀνθυφαίρεσις, does in four places[29] use the verb, ἀνθυφαίρεῖν, and–this is the really significant fact–uses it to describe the process of finding the greatest common measure between two magnitudes. In this process the lesser magnitude is subtracted from the greater as many times as possible until a magnitude smaller than itself is left, and then the difference is subtracted as many times as possible from the lesser until a difference smaller than itself is left, and so on continually (ἀνθυφαιρουουμένου δὲ ἀεὶ τοῦἐλάσσονος ἀπὸ τουη μείζονος). In the case of commensurable magnitudes the process comes to an end after a finite number of steps, but in the case of incommensurable magnitudes the process never comes to an end. A mathematician as acute as Theaetetus would realize that this could be made a test of commensurability (as it is in Euclid, *Elements***X.2**) and that by adopting a definition of proportion based on this test he could have a theory of proportion applicable to commensurable no less than incommensurable magnitudes.[30]

It is possible that such a general theory was evolved before Eudoxus by some person other than Theaetetus, but in view of Theaetetus' known competence and his interest in irrationals, he is the most likely author. The attribution becomes even more credible if Zeuthen's explanation of how Theodorus proved the square roots of to be irrational is accepted (see the article on Theodorus of Cyrene); for according to his conjecture Theodorus used this method in each particular case, and Theodorus was the teacher of Theaetetus. Although there is no direct evidence that Theaetetus worked out such a pre–Eudoxan theory of proportion, the presumption in favor is strong; and it has convinced all recent commentators.

It is not known whether Theaetetus made any discoveries outside these three fields. In the "Eudemian summary" Proclus says:[31] "Hermotimus of Colophon advanced farther the investigations begun by Eudoxus and Theaetetus: he discovered many propositions in the elements and compiled some portion of the theory of loci", While it is clear that Theaetetus studied mathematics under Theodorus, it is uncertain whether he did so a Cyrene or at Athens. It may be accepted that at some time he taught in Heraclea, and he may have been the teacher of Heraclides Ponticus.[32]

NOTES

1.*Suda Lexicon*, Ada Adler, ed., I, pt. 2 (Leipzig, 1931), Θ 93 and 94, p. 689.6–9.

2. G. J. Allman, "Theaetetus", in *Hermathena*, **6** (1887), 269–278, repr. in *Greek Geometry From Thales to Euclid* (London–Dublin, 1889), 206–215.

3. Plato, *Theaetetus, Platonis opera*, J. Burnet, ed., I (Oxford, 1899), 142a-148b; Plato, Loeb Classical Library, H. N. Fowler, ed., **VII** (London–Cambridge, Mass., 1921; repr. 1967), 6.1–27.24.

4. The birth of Theaetetus has usually been placed in 415 B.C.,or even as late as 413, which would make him not more than sixteen years Old in 399; but the instances given in H. G. Liddell, R. Scott, and H. Stuart Jones,*A Greek English Lexicon* (Oxford. 1940), s.vv. μειράκιον and ἔφηβος, show clearly that a μειράκιον would not be younger than eighteen and might be nearly as old as twenty–one. A sentence in the Chronicle of Eusebius–preserved in the Armenian and in Jerome's Latin version–*Sancti Hieronymi interpretatio chronicae Eusebii Pamphili* in *Patrologia Latina*. J. P. migne, ed., vol. **XXVII** = S. Hieronymi, vol. **VIII** (Paris, 1846), cols. 453–454–which would place the central point of Theaetetus' activity in the third year of the 85th Olympiad (438 B.C)–must be dismissed as an error. Eusebius' statement is repeated by George Syncellus *Corpus scriptorum historiae Byzantinae*, B. G. Niebuhr,ed., pt. 7.1;*Georgius Syncellus et Nicephorus*, G. Dindorff, ed., 1 (Bonn, 1829), p. 471.9.

5. It is one of Eva Sachs's principal achievements in her pioneering inaugural dissertation, *De Theaeteto Atheniensi mathematico* (Berlin, 1914), 16–40, to have established this point irrefutably against E. Zeller and others.

6. But Malcolm S. Brown, in "Theaetetus: Knowledge as Continued Learning," in *Journal of the History of Philosophy*, **7** (1969), 359, maintains that Theaetetus "influenced both the course of mathematics and that of philosophy". Brown quotes Sachs, *op. cit*., p. 69, in support: "Ille re vera philosophus fuit perfectus": but it is doubtful if Sachs physician Brown seizes on the statement of Theaetetus at the beginning of his conversation with Socrates: "When I make a mistake you will correct me" (*Theaetetus*, 146c.). Brown sees in the mathematical work of Theaetetus a process of successive approximations, which can be construed as "containing errors which are being corrected". He holds also that there is an epistemological analogue, "a well–di–arriving at a final answer, would nevertheless pernult of an improvement (even an indefinite improvement) of opinion"; and he believes that in this dialogue at least Plato yielded somewhat to the suggestion of Theaetetus that "knowledge is continued learning" (p. 379).

7. Proclus,*In primum Euclidis*, G. Friedlein, ed. (Leipzig, 1873; repr., Hildesheim. 1967). p. 66.14–18; English trans., Glenn R. Morrow, Proclus;*A Commentary on the First Book of Euclid's Elements* (Princeton, 1970), p. 54.11–14.

8. Plato, *Theaetetus, Platonis opera*, J. Burnet, ed., I (Oxford, 1899), 147c-148b; Plato, Loeb Classical Library, H. N. Fowler, ed., **VII** (London–Cambridge, Mass., 1921; repr., 1967), pp. 24.9–27.24.

9. The Greek word is δυνάμεις, which at a latter date could only mean "squares"; but here its meaning would appear to be "roots", and we can only suppose that at this early stage in Greek mathematics the terminology had not become fixed. It is not necessary with Paul Tannery ("Sur la langue mathématique de Platon", in *Annales de la Faculté des lettres de Bordeaux*, **1** "1884". 96, repr. in *Mémoires scientifiques*, **2** [1912], 92) to alter δύναμις without any MS authority to δυναμένη, the later technical expression for a square root. For a very full discussion of a different interpretation. See árpád Szabó, *Anfánge der griechischen Mathematik* (Munich–Vienna, 1969), 14–22, 43–57. Szabó holds that δύναμις means *Quadratwert eines Rechtecks* ("square value of a rectangle"), that is, the square equivalent in area to a rectangle. This interpretation has attractions, but the fact that Plato categorically describes δυνάμεις as γραμμαί, "lines", and sets δύναμις in oposition to μῆκος, a rational length, seems fatal to it. But Szabó establishes that δύναμις cannot be power in general.

10. But not so recently as the time of Plato himself. Even if the Athenian stranger in the *Laws* is identified with Plato, it is reading too much into his words αὐτὸς ἀκούσας ὀψέ ποτε τὸπερὶ ταῦτα ἡμῶν πάθος (1819d 5–6) to suppose that the irrational was not discovered until the fourth century B.C. Likewise the statement in the "Eudemian summary", in Proclus, *op.cit.,* p. 65. 19–21, that Pythagoras "discovered the matter of the irrationals", (τὴν τῶν ἀλόγων πραγματεὶαν . . . ἀνεῦρεν) must be wrong, and there is almost certainly a textual error– αλόγων for ἀναλόγων ("proportionals"). The existence of irrational magnitudes was almost certainly discovered, as Greek tradition asserted,by Hippasus of Metapontum in the middle of the fifth century B.C. The best discussion of the date is Kurt von Fritz, "The Discovery of Incommensurability by Hippasus of Metapontum", in *Studies in Presocratic Philosophy*,

David J. Furley and R. E. Allen, eds., I (London–New York, 1970), 382–412. For an attempt to show that the discovery was made in the closing years of the fifth century, see Eric Frank, *Platon und die sogenannten Pythagoreer* (Halle, 1923), Árpád Szabó, *op. cit*., pp. 60–69, 111–118, 238, seeks to show that the irrational was discovered in the study of mean proportionals as opposed to the prevailing theory that it arose from the study of diagonals of squares after the discovery of "Phythagoras' theorem".

11. The translation is based in the main on that of William Thomson, in William Thomson and Gustav Junge, *The Commentary of Pappus on Book X of Euclid's Elements* (Cambridge, Mass., 1903; repr., New York, 1968), 63; but his "powers" (that is, the squares), although a faithful rendering of the Arabic, has been modified, since "roots" appears to be the meaning. The ambiguity of the Greek δὑναμις, before the terminology became fixed, is reflected in the Arabic.

12. It would be going beyond the evidence to attribute to Theaetetus the Euclidean notion (*X, Definition 3*) that a straight line may be rational but commensurable only in square with a rational straight line; that is, that if *r* is a rational straight line and *m, n* integers with *m/ n* in its lowest terms not a square, then is rational. T. L. Heath observes, "It would appear that Euclid's terminology here differed as much from that of his predecessors as it does from ours", and he aptly cites the expression of Plato (following the Pythagoreans), in the *Republic* 546c 4–5; ἄρρητος διάμετρος τοῦ πεμπάδος ("the irrational diameter of five") for the diagonal of a square of side five units; that is, for Plato, and presumably for Theaetetus, as for us, is irrational, whereas Euclid would have called it "rational but commensurable in square only". Eva Sachs takes a contrary view, *Die fünf platonischen körper*, p. 105, but without satisfactory reasons.

13. B. L. van der Waerdin, *Science Awakening*, 2nd ed. (Groningen,1956[?]), p. 172. In full, he writes: "Has the same Theaetetus who studied the medial, the binomial and the apotome, also defined and investigated the ten other irrationalities, or were those introduced later on? It seems to me that all of this is the work of one mathematician. For, the study of the 13 irrationalities is a unit. The same fundamental idea prevails throughout the book, the same methods of proof are applied in all cases. Propositions X.17 and 18 concerning the measurability of the roots of a quadratic equation precede the introduction of binomial and apotome, but these are not used until the higher irrationalities appear on the scene. The theory of the binomial and the apotome is almost inextricably interwoven with that of the 10 higher irrationals. Hence–the entire book is the work of Theaetetus". The conclusion does not follow. The unity may be due to Euclid himself, using some propositions already proved, adding refinements of his own, and welding the whole into one, as Proclus testifies. The division of irrationals into medial, binomial, and apotome can perfectly well be separated from the subdivisions into more complex irrationals. If it were true that X.17 and 18 are not used until after the introduction of the binomial and the apotome, this would prove nothing since they are in their correct logical position; and for that matter, the whole of book X is not used again until book XIII; but, in fact, X.18 is used in X.33, whereas the binomial is not introduced until X.36 and the apotome until X.73.

14.*Euclidis opera omnia*, J. L. Heiberg and H. Menge, eds., V (Leipzig, 1888), Scholium 62 in Elementorum Librum X, p. 450. 16–18. There is good reason to believe that the scholiast is Proclus. See H. Knoche, *Untersuchungen über die neu aufgefundenen Scholien des Proklus Diadochus zu Euclids Elementen* (Herford, 1865), p. 24; and J. L. Heiberg, "Paralipomena zu Euclid", in *Hermes*, **38** (1903), p. 341.

15. Proclus, *op, cit*., p. 68.7–10; Eng. trans. *op. cit*., p. 56.19–23.

16. J. L. Heiberg gives conclusive reasons for bracketing propositions 112–115, in *Euclidis opera omnia*, J.L. Heiberg and H. Menge, eds., V. P. IXXXV, and concludes; "non dubito, quin hae quoque propositiones 112–115 e doctrina Apollonii promptae sint; nam antiquae sunt et bonae, hoc saltim constare putaverim, eas ab Euclide scriptas nonesse".

17. H. G. Zeuthen, "Sur la constitution des livres arithétiques des Eléments d'Euclide et leur rapport à la question de l'irrationalité", in *Oversigt over det Kongelige Danske Videnskabernes Selskabs Forhandlinger* (1910), 395–435.

18. Thomas Heath, *A History of Greek Mathematics*, I (Oxford, 1921), 211.

19. Aëtius, Placita, II, 6, 5, in H. Diels, *Doxographi Graeei* (Berlin, 1879), p. 334; and *Die Fragmente der Vorsokratiker*, H. Diels and W. Kranz, eds., 6th ed., I (Dublin–Zurich, 1951; repr., 1969), p 403.8–12.

20. Proclus, op, cit., p. 65.20–21; Eng. trans., op. cit., p. 53.5 Morrow translates the Greek work as "structure".

21. Plato, *Timaeus* 53c–55c; *Platonis opera*, J. Burnet, ed., IV (Oxford, 1915); Loeb Classical Library, *Plato, Timaeus etc*., R. G. Bury, ed. (London–Cambridge, Mass., 1929; repr., 1966), pp. 126.16–134.4

22.*Euclidis opera omnia*, J. L. Heiberg and H. Menge, eds., V (Leipzig, 1888), Scholium I in Elementorum Librum XIII, p. 654.1–10

23. One, discovered in 1885 at Monte Loffa in the Colli Euganei near Padua, of Etruscan origin, is dated between 1000 and 500 B. C. (F. Lindemann, "Zur Geschichte der Polyeder und der Zahlzeichen," in *Sitzungsberichte der Bayerischen Akademie der Wissenschaften zu München*, 26 (1897), 725.

24. Iamblichus, *De communi mathematica scientia* 25, N. Festa, ed. (Leipzig, 1891), 77.18–21; *De vita Pythagorica* 18.88, A. Nauck, ed. (Leipzig, 1884;repr., 1965).

25. In the course of a full discussion Eva Sachs, in *Die fünf platonischen Körper* (Berlin, 1917), asserts (p. 105) that the construction of the five solids in Euclid, *Elements*, XIII, 13–17, springs from Theaetetus. She approves H. Vogt, in *Bibliotheca mathematica*, **9** , 3rd ser. (1908–1909), p. 47 for controverting Paul Tannery, *La géométrie Grecque* (Paris, 1887), p. 101, who would ascribe the construction of the five solids to the Pythagoreans while leaving to Theaetetus the calculation of the relation of the sides to the radius of the circumscribing sphere; for how, she and Vogt ask, can the exact construction be accomplished without a prior knowledge of this relation?. The question how much of Euclid's book XIII is due to Theaetetus is bound up with the difficult question how much, if any, is due to the Aristaeus who is mentioned by Hypsicles in the so-called *Elements*, Book XIV. J. L. Heiberg and H. Menge, eds., vol. V, p. 6.22–23, as the author of a book entitled *Comparison of the Five Figures*, and whether this Aristaeus is to be identified with Aristaeus the Elder, author of a formative book on solid loci, that is, conics. T. L. Heath, in *The Thirteen Books of Euclid's Elements*, III (Cambridge, 1908; 2nd ed., 1925; repr., New York, 1956), p. 439, following C. A. Bretschneider, *Die Geometrie und die Geometer vor Eukleides* (Leipzig, 1870), p. 171, took the view that "as Aristaeus's work was the newest and latest in which, before Euclid's time, this subject was treated, we have in Euclid XIII at least a partial recapitulation of the contents of the treatise of Aristaeus"; but Eva Sachs, *op. cit*., p. 107, denies this conclusion.

26. Oskar Becker, "Eudoxos Studien I: Eine voreudoxische Proportionenlehre und ihre Spuren bei Aristoteles und Euklid", in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, **2B** (1933), 311–333. To some extent the theory had already been adumbrated independently by H. G. Zeuthen, "Hvorledes Mathematiken i Tiden fra Platon til Euklid", in *Kongelige Danske Videnskabernes Selskabs Skriften*, **5** (1915), 108, and E. J. Dijksterhuis, *De Elementen van Euclides*, I (Groningen, 1929), 71, as Becker himself recognizes in *Das mathematischen Denken der Antike* (Göttingen, 1957), p. 103, n. 25. Becker failed to convince T. L. Heath, Mathematics in Aristotle (Oxford, 1949; repr., 1970), 80–83, who in the absence of confirmatory evidence could "only regard Becker's article as a highly interesting speculation" (p. 83). It has also been criticized by K. Reidemeister, Das exakte Denken der Griechen (Hamburg, 1949), p. 22, and by árpád Szabö. "Ein Beleg fur die voreudoxische Proportionlehre?" in *Archiv für Begriffsgeschichte*, **9** (1964), 151'171, and in his *Anfänge der griechischen Mathematik* (Munich–Vienna). 134'135, 180–181. The theory received support, however, from a Leiden dissertation by E. B. Plooij, *Euclid's Conception of Ratio as Criticized by Arabian Commentators* (Rotterdam, 1950). Becker rejected the criticisms in *Archiv für Begriffsgeschichte*, **4** (1959), p. 223, and adhered to his theory in his book *Grundlagen der Mathematik in geschichtlicher Entwicklung* (Bonn, 1954; 2nd ed., 1964). His theory has been wholeheartedly endorsed by B. L. van der Waerden, *Science Awakening*, 2nd ed. (Groningen, 1956[?]), 175–179; by Kurt von Fritz in "The Discovery of Incommensurability of Hippasus of Metapontum", in *Studies in Presocratic Philosophy*, David J. Furley and R. E. Allen, eds., I (London–New York, 1970), 408–410, esp. note 87; but his statement that Heath "still called the definition 'metaphysical'" is unfair, since Heath said it was "'metaphysical' (as Barrow would say)," and in any case this was in *The Thirteen Books of Euclid's Elements*, II (Cambridge, 1908; 2nd ed., 1925; repr., New York, 1956), p. 121, written before Becker's theory was enunciated; by Malcolm S. Brown, *op. cit*., pp. 363–364; and by Wilbur Knorr, *The Evolution of the Euclidean Elements* (Dordrecht, 1975).

27. Aristotle, *Topics* VIII, 3, 158B 29'159A 1.

28. Alexander of Aphrodisias, *commentarium in Topica*, Strache and Wallies, eds., in *Commentaria in Aristotelem Graeca*, II (Berlin, 1891), 545. 12–17.

29.*Euclid, Elements*, VII.1, VII.2. X.2, and X.3, J. L. Heiberg, ed., II (Leipzig, 1884), 188.13–15, 192.6–7; III(Leipzig, 1886), 12–14, 10.4–5; E. S. Stamatis, ed. (post J. L. Heiberg), II (Leipzig, 1970), 105.8–9, 107.3–4; III (Leipzig, 1972), 3.19–20, 5.8–9.

30. The Arabian commentator al–Māhānī(fl.ca.897), dissatisfied with Euclid's definition, worked out for himself an "anthyphairetic" definition, as was recognized by E. B. Plooij. *op. cit*. For al–Nayrīzī, see *Anaritii in decem libros priores Elementorum Euclidis ex interpretatione Gherardi Cremonensis*, M. Curtze, ed., in *Euclidis opera omnia*, J. L. Heiberg and H. Menge, eds., *Supplementum*, pp. 157'160.

31. Proclus, *op. cit*., p. 67.20–23; English trans., *op. cit*., p. 56.9–12

32. Eva Sachs, *De Theaeteto Atheniensi Mathematico*, p. 64, following Ulrich von Wilamowits–Moellendorf.

BIBLIOGRAPHY

No original writing by Theaetetus has survived, even in quotation, although his work is undoubtedly embedded in Euclid, *Elements*, X and XIII.

Secondary literature includes G. J. Allman, “Theaetetus”, in *Hermathena*, **6** (1887), 269–278, repr. in *Greek Geometry From Thales to Euclid* (London–Dublin, 1889), 206–215; Oskar Becker, “Eudoxos Studien I: Eine voreudoxische Proportionenlehre und ihre Spuren bei Aristoteles und Euklid”, in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, **2B** (1933), 311–333; *ibid*., **3B** (1934), 533–553, repr. in O. Becker, ed., *Zur Geschichte der griechischen Mathematik* (Darmstadt, 1965); in *Archiv für Begriffsgeschichte*. **4** (1959), 223; and in *Grundlagen der Mathematik in geschichtlicher Entwicklung* (Bonn, 1954; 2nd ed., 1964), 78–87; Malcolm S. Brown, “Theaetetus: Knowledge as Continued Learning”, in *Journal of the History of Philosophy*, **7** (1969), 359–379; Kurt von Fritz, “The Discovery of Incommensurability by Hippasus of Metapontum”, in *Annals of Mathematics*, **46** (1945), 242–264; “Platon, Theatet und die antike Mathematik”, in *Phiologus*, **87** (1932), 40–62, 136–178; and David J. Furley and R. E. Allen, eds., “The Discovery of Incommensurability by Hippasus of Metapontum”, in *Studies in Presocratic Philosophy***I** (London–New York, 1970), 382’412.

See also Thomas Heath, *A History of Greek Mathematics*, I (Oxford, 1921), 203–204, 209–212; Pauly–Wissowa, Real–Encyclopädie der classischen Altertums-wissenschaft, 2nd ser., V, cols. 1351–1372; Eva Sachs, *De Theaeteto Atheniensi mathematico* (Inaugural diss., Berlin, 1914); Die fünf platonischen Körper (Berlin, 1917), 88–119; árpád Szabó, “Ein Beleg für die voreudoxische Proportionenlehre?” in Archiv für Begriffsgeschichte, 9 (1964), 151–171; “Die Fruhgeschichte der Theorie der Irrationalitaten”, in *Anfänge der griechischen Mathematik*, pt. 1 (Munich–Vienna, 1969), 38’130; “Die voreuklidische Proportionlehre”, *ibid*., pt. 2. pp. 131–242; Heinrich Vogt, “Die Entdeckungs–geschichte des Irrationalen nach Plato und anderen Quellen des 4. Jahrhunderts”, in *Bibliotheca mathematica*, 3ser., 10 (1909–1910), 97–155; “Zur Entdeckungs-geshichte des Irrationalen”, *ibid*., **14** (1913–1914), 9–29; B. L. van der Waerden, *Ontwakende Wetenschap* (Groningen, 1950), also in English, Arnold Dresden, trans., *Science Awakening* (Groningen, 1954; 2nd ed., [?], 1956), 165’179; A. Wasserstein, “Theaetetus and the History of the Theory of Numbers”, in *Classical Quarterly*, n.s. **8** (1958), 165–179; H. G. Zeuthen, “Notes sur l’histoire des mathématiques VIII; Sur la constitution des livres arithmétiques des Eléments d’Euclide et leur rapport à la question de l’irrationalité”, in *Oversigt over det Kongelige Danske Vindenskabernes Selskabs Forhandlinger* (1910), 395–435; “Sur les connaissances géométriques des Grecs avant la reforme platonicienne de la géométrie”, *ibid*, (1913), 431–473; and “Sur l’origine historique de la connaissance des quantités irrationelles”, *ibid*. (1915), 333–362; and Wilbur Knorr, *The Evolution of the Euclidean Elements* (Dordrecht, 1975), chs. 7,8.

See also the Bibliography of the article on Theodorus of Cyrene.

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