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(*b*. Louvain, Belgium, 14 August 1866; *d*. Louvain, 2 March 1962)

mathematics.

Vallée-Poussin’s father was for nearly forty years professor of mineralogy and geology at Louvain. Young Vallée-Poussin entered the Jesuit College at Mons, but he found the teaching in some subjects, notably philosophy, unacceptable. He turned to engineering, although, after obtaining his diploma, he devoted himself to pure mathematics. Since boyhood he had been encouraged in mathematics by Louis-Philippe Gilbert and in 1891 he became Gilbert’s assistant at the University of Louvain. Gilbert died in 1892 and, at the age of twenty-six, Vallée-Poussin was elected to his chair. He remained all his life at Louvain.

Vallée-Poussin made a very happy marriage with the gifted daughter of a Belgian family whom he met on holiday in Norway in 1900.

As the outstanding Belgian mathematician of his generation, Vallée-Poussin received many tributes. In accordance with custom, he was honored by celebrations at Louvain in 1928 after thirty-five years in his chair, and again in 1943 after fifty years. On the former occasion, the king of the Belgians conferred on Vallée-Poussin the rank of baron. The Belgian Royal Academy elected him a member in 1909, and he became an associate member of the Paris Académie des Sciences in 1945. He was also a commander of the Legion of Honor and honorary president of the International Mathematical Union.

Vallée-Poussin’s earliest investigations were concerned with topics of analysis suggested by his own teaching. He proved in an elegant and general form theorems in the differential and integral calculus. In 1892 his memoir on differential equations was awarded a *couronne* by the Belgian Royal Academy. He quickly showed his analytical power in a spectacular way by his researches into the distribution of primes. After nearly a century of conjectures and proofs of partial results, the prime number theorem—that π the number of primes $p \le x$, is asymptotically $x/\log x$—was proved independently by Hadamard and by Vallée-Poussin in 1896. The two proofs look very different, but each is achieved by difficult arguments of complex function theory applied to the zeta function of Riemann. Vallée-Poussin extended his researches to cover the distribution of primes in arithmetical progressions and primes represented by binary quadratic forms. He also made an advance of the first importance in the original prime number theorem by assigning an upper estimate to the difference between $\pi(x)$ and the logarithmic integral li x, which remained for twenty years the closest known. Apart from his two later papers on the zeta function in 1916, Vallée-Poussin left to others the development of the ideas that he had introduced into the theory of numbers.

Although the proof of the prime number theorem was Vallée-Poussin’s highest achievement, his main impact on mathematical thought was his *Cours d’analyse*, a model of style, economy, and lucidity. The sweeping changes that Vallée-Poussin made in successive editions of his work reflected his current interests. The first edition expounded the traditional calculus, differential equations, and differential geometry; it was just too early for the Lebesgue integral. Two sizes of type were used, the larger for a basic course and the smaller for supplementary matter suited to mathematical specialists. In the second edition the part in small type was greatly expanded to take in set theory, measure and the Lebesgue intergal, bounded variation, the Jordan curve theorem, and trigonometric series up to the theorems of Parseval des Chênes and Fejér. The third edition of volume I (1914) introduced the Stolz-Fréchet definition of the differentiability of $f(x, y)$. The third edition of volume II was burned when the German army overan Louvain. It would have pursued the discussion of the Lebesgue integra.

Vallée-Poussin, invited to Harvard and to Paris in 1915 and 1916, expanded this work into the Borel tract, *Intégrales de Lebesgue* . . ., which bears the marks of successive refinements of treatment. The second edition of the tract included analytc sets (Lusin, Souslin) and the Stieltjes integral. Vallée-Poussin’s *Cours d’analyse* itself reverted after 1919 to a basic course without the small print.

In the decade after 1908 Vallée-Poussin made fundamental advances in the theory of approximation to functions by algebraic and trigonometric polynomials. The fact that any continous function $f(x)$ can be thus approximated uniformly in a closed interval had been proved in 1885 by Weierstrass by intergrating the product of $f(u)$ and a peak function $K(u,x)$, which rises steeply to its maximum at $u = x$. Vallée-Poussin (and Landau independently) applied this singular integral method with $K(u,x)$ of the form

$\{1 - (u - x)^2\}^n$ or $\{\cos 1/2(u - x)\}^{2n}$

to obtain results about the closeness of approximation to $f(x)$ by polynomials of assigned degree under Hypotheses about f and its dervatives.

The Lebesgue integral gave new life to the theory of trigonometric series, and Vallée-Poussin proved a number of results that have become classic, notably his uniqueness theorem, his test for convergence, and a method of summation that is stronger than all the Cesàro methods.

During the first quarter of the twentieth century, Vallée-Poussin's interests were dominated by the Borel-Lebesgue revolution and were centered on the real variable. (His is the one *Cours d'analyse* that contains no complex function theory.) After 1925 he turned again to the complex variable, in particular to potential theory and conformal representation. He collected his contributions in a book *Le potentiel logarithmique* (1949), the publication of which was held up by the war. By the time the book appeared some of his ideas had been superseded by those of a younger school of French analysts.

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J. C. Burkill