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(*b*. Rothenburg-ob-der-Tauber, Germany, 24 January 1798; *d*. Erlangen, Germany, 1 June 1867)

mathematics.

Staudt was the son of Johann Christian von Staudt, a municipal counsel, and Maria Albrecht. Rothenburg, famous for its many antiquities, was then a free imperial German city. The family had settled in Rothenburg as craftsmen as early as 1402. Various members became municipal councilmen in the sixteenth century and received a coat of arms. In 1700 Leopold I ennobled the family. Staudt's maternal ancestors, the Albrechts, also served as councilmen and burgomasters in the seventeenth and eighteenth centuries. Staudt's father was appointed a municipal legal officer by the Bavarian government in 1805, the year Rothenburg became part of the Kingdom of Bavaria.

After carefully supervising his early education, Staudt's parents sent him to the Gymnasium in Ansbach from 1814 to 1817. Then, drawn by the great reputation of Gauss, Staudt attended the University of Göttingen from 1818 to 1822. As a student he was surely well acquainted with Gauss's studies in number theory. His chief concern in these years, however, was theoretical and practical astronomy, to which he was also introduced by Gauss, who was then director of the observatory. As early as 1820 Staudt observed and computed the ephemerides of Mars and Pallas. His most comprehensive work in astronomy was the determination of the orbit of the comet discovered by Joseph Nicollet and Jean-Louis Pons in 1821. His precise calculations were highly praised by Gauss, and later observations led to only minor improvements. Staudt never returned to the field of astronomy, but it was on the basis of this early work that he received the doctorate from the University of Erlangen in 1822.

In the same year Staudt qualified at Munich as a mathematics teacher. His first assignment was at the secondary school in Würzburg. But with Gauss's intervention he was also able to lecture at the University of Würzburg. His lectures dealt with rather elementary topics. Because of insufficient support from the university, he transferred in 1827 to the secondary school in Nuremberg and taught there and at the Nuremberg polytechnical school until 1835. He finally achieved his primary goal when, on 1 October 1835, he was appointed full professor of mathematics at the University of Erlangen, where he remained until his death. He was unquestionably the leading mathematician at Erlangen, not least because of his outstanding human qualities. The latter, indeed, brought him many honorary posts in the university administration.

As at most German universities during this period, the level of mathematics instruction at Erlangen was not high, nor did the subject attract many students. It was not yet customary for mathematicians to discuss their own research in the classroom–a practices first introduced by Jacobi, at Königsberg. Accordingly, it was not until 1842 – 1843 that Staudt gave special lectures on his new geometry of position.

In 1832 Staudt married Jeanette Drechsler. They had a son, Eduard, and a daughter, Mathilde, who became the wife of a burgomaster of Erlangen. Staudt's wife died in 1848, and he never remarried. In his last years he suffered greatly from asthma.

Staudt was not a mathematician who astounded his colleagues by a flood of publications in a number of fields. He let his ideas mature for a long period before making them public, and his research was confined exclusively to projective geometry and to the only distantly related Bernoullian numbers. His fame as a great innovator in the history of mathematics stems primarily from his work in projective geometry, which he still called by the old name of "geometry of position," or *Geometrie der Lage*, the title of his principal publication (1847). This work was followed by three supplementary *Beiträge zur Geometrie der Lage* (1856–1860), which together contain more pages than the original book (396 as compared with 216).

After centuries of dominance, Euclidean geometry was challenged by Poncelet and Gergonne, who created projective geometry during the first third of the nineteenth century. These two mathematicians found that, through the use of perspective, circles and squares and other figures could be transformed into arbitrary conic sections and quadrilaterals and that a metric theorem for, say, the circle could be transformed into a metric theorem for conic sections. The most important contributions made by Poncelet (whose main writings appeared between 1813 and 1822) and Gergonne were the polarity theory of the conic sections and the principle of duality. Jakob Steiner, in his fundamental work *Systematische Entwicklung der Abhängigkeit geometrischer Gestalten voneinander* (1832), then introduced the projective production of conic sections and second-degree surfaces that is now named for him.

In their writings, however, all three of these pioneers failed to adhere strictly to the viewpoint of projective geometry, which admits only intersection, union, and incidence of points, straight lines, and planes. Staudt, in his 1847 book, was the first to adopt a fully rigorous approach. Without exception, his predecessors still spoke of distances, perpendiculars, angles, and other entities that play no role in projective geometry. Moreover, as the name of that important relationship indicates, in accounting for the cross ratio of four points on a straight line, they all made use of line segments. In contrast, Staudt stated in the preface to his masterpiece his intention of establishing the "geometry of position" free from all metrical considerations, and in the body of the book he constructed a real projective geometry of two and three dimensions.

Naturally, in Staudt's book these geometries are not founded on a complete axiom system in the modern sense. Rather, he adopted from Euclid's system everything that did not pertain to interval lengths, angles, and perpendicularity. Although it was not necessary, he also retained Euclid's parallel postulate and was therefore obliged to introduce points at infinity. This decision, while burdening his treatment with a constant need to consider the special positions of the geometric elements at infinite distance, altered nothing of the basic structure of geometry without a metric. Using only union and intersection of straight lines in the plane, Staudt constructed the fourth harmonic associated with three points on a straight line. Correspondingly, with three straight lines or planes of a pencil he was able to construct the fourth harmonic element. Although he did not give the theorem that name, he used Desargues's theorem to prove that his construction was precise.

Using the relationship of four points in general position on a plane to four corresponding points or straight lines on another plane or on the same plane, Staudt defined a collineation–or, as the case may be, a correlation–between these planes. Analogously he also pointed out spatial collineations and correlations. In this instance he made use of Möbius's network construction, which enabled him to obtain, from four given points of a plane, denumerably many points by drawing straight lines through point pairs and by making straight lines intersect. He then associated the points derived in this way with correspondingly constructed points and straight lines of the other plane. Felix Klein later noted that a continuity postulate is still required in order to assign to each of the infinitely many points of the first plane its image point (or image lines) on the other plane.

From the time of his first publications, Staudt displayed a grasp of the importance of the principle of duality. For every theorem he stated its converse. (As was customary, he generally gave the theorem on one half of the page and its converse on the other half.) In discussing the autocorrelations of P_2 and P_3, he succeeded in obtaining the polarities and also the null correlations that had previously been discovered by Gaetano Giorgini and Möbius. For example, he described a plane polarity as a partiuclar type of autocorrelation that yields a triangle in which each vertex is associated with the side opposite. On this basis, Staudt formulated the definition of the conic sections and quadrics that bears his name: they are the loci of those points that, through a polarity, are incident with their assigned straight lines or planes. This definition is superior to the one given by Steiner. For instance, in Staudt's definition the conic section appears as a point locus together with the totality of its tangents. Steiner, in contrast, required two different productions: one for the conic section K and another for the totality of its tangents, that is, for the dual figures associated with K. A Conic section defined in Staudt's manner can consist of the empty set; that is, it can contain no real points–accordingly. Klein applied the term *nullteilig* to.

In a coordinate geometry it is easy to extend the domain of the real points to the domain of the complex points with complex coordinates. Employing the concepts of real geometry, Staudt made an essential contribution to synthetic geometry through his elegantly formulated introduction of the complex projective spaces of one, two, and three dimensions. This advance was the principal achievement contained in his *Beiträge zur Geometrie der Lage*. He conceived of the complex points of a straight line P_1 by means of the so-called elliptic involution of the real range p_1 of P_1, which can also be described as those involuted autoprojectivities of p_1 among which pairs of corresponding points intersect each other. It can be shown by calculation that such an elliptic involution has two complex, conjugate fixed points; and Staudt had to furnish the elliptic involutions with two different orientations, so that ultimately he could interpret the oriented elliptic involutions on the real range of P_1 as points of P_1. The degenerate parabolic involutions are to be associated with the real points of P_1. In this way he also extended the real projective planes P_2 and P_3 to complex P_2 and P_3. He then showed–not an easy feat–that P_2 and P_3 satisfy the connection axioms of projective geometry. Among the lines P_1 of P_3 he found three types: those with infinitely many real points, those with only one real point, and those with no real point. He carefully classified the quadrics of P_3 according to the way in which straight lines of these three types lie on them.

Staudt favored the use of the second type of complex line of P_3 as a model of the complex numbers P_1. He applied the term *Wurf* ("throw") to a point quadruple and gave the procedure for finding sums and products in the set of these throws–or, more precisely, the set of the equivalence classes of projectively equivalent throws. Here he approached the projective foundation of the complex number field and the projective metric determination. Staudt termed certain throws neutral: those with real cross ratio, a property that can be determined computationally. Then, for three given points–*A, B, C,* in P_1–he designated as a chain the set of all those points of P_1 that form a neutral throw with *A, B, C*. These sets and their generalization to complex P_n are called "Staudt chains." In part three of the *Beiträge*, Staudt also dealt with third- and fourth-order spatial curves in the context of the theory of linear systems of equations.

At the time of their publication, Staudt's books were considered difficult. This assessment arose for several reasons. First, since he sought to present a strictly systematic construction of synthetic geometry, he did not present any formulas; more over, he refused to employ any diagrams. Second, he cited no other authors. Finally, although his theory of imaginaries was remarkable, it was extremely difficult to manipulate in comparison with algebraic equations. Accordingly, little significant progress could have been expected from its adoption in the study of figures more complicated than conic sections and quadrics.

Staudt is also known today for the Staudt-Clausen theorem in the theory of Bernoulli numbers. These numbers–B_n (n= 1,2,...)–appear in the summation formulas of the *n*th powers of the first *h* natural numbers: they also arise in analysis, for example, in the series expansion *x* cot *x*. The B_n are rational numbers of alternating sign, and the Staudt-Clausen theorem furnished the first significant indication of the law of their formation. In formulating the theorem, for the natural number *n* there is a designated uneven prime number, *p*, called Staudt's prime number, such that *p*–1 divides the number 2*n*. Then, according to the theorem, $(-1)^n B_n$ is a positive rational number, which, aside from its integral component, is a sum of unit fractions, among the denominators of which appear precisely the number 2 and all Staudt prime numbers for *n*. Staudt published his theorem in 1840; it was also demonstrated, independently, in the same year by Thomas Clausen, who was working in Altona. Staudt published two further, detailed works in Latin on the theory of Bernoulli numbers (Erlangen, 1845); but these writings never became widely known and later authors almost never cited them.

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Werner Burau