

# Weierstrass, Karl Theodor Wilhelm I

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(*b.* Ostenfelde, Westphalia, Germany, 31 October 1815; *d.* Berlin, Germany, 19 February 1897), *Mathematics*.

Weierstrass was the first child of Wilhelm Weierstrass, secretary to the mayor of Ostenfelde, and Theodora Vonderforst, who were married five months before his birth. The family name first appeared in Mettmann, a small town between Düsseldorf and Elberfeld; since the sixteenth century they had been artisans and small merchants. Weierstrass' father, an intelligent, educated man with knowledge of the arts and sciences, could have held higher posts than he actually did; little is known about his mother's family. Weierstrass had a brother and two sisters, none of whom ever married. When Weierstrass was eight his father entered the Prussian taxation service; and as a result of his frequent transfers, young Karl attended several primary schools. In 1829, at the age of fourteen, he was accepted at the Catholic Gymnasium in Paderborn, where his father was assistant and subsequently treasurer at the main customs office.

A distinguished student at the Gymnasium, Weierstrass received several prizes before graduating. Unlike many mathematicians, he had no musical talent; nor did he ever acquire an interest in the theater, painting, or sculpture. He did, however, value lyric poetry and occasionally wrote verses himself. In 1828, a year after his mother's death, Weierstrass' father remarried. At the age of fifteen Weierstrass reportedly worked as a bookkeeper for a merchant's wife—both to utilize his abilities and to ease the strain of his family's financial situation. A reader of Crelle's *Journal für die reine und angewandte Mathematik* while in his teens, he also gave his brother Peter mathematical coaching that does not seem to have proved helpful: Weierstrass' proofs were generally "knocking," his brother later admitted.

After leaving the Gymnasium in 1834, Weierstrass complied with his father's wish that he study public finance and administration, and entered the University of Bonn. The course of Studies that he pursued was planned to permit him to obtain a background in law, administration, and economics—the requisites for those seeking higher administrative posts in Prussia. The study of mathematics or related areas was his first choice, however; and the conflict between duty and inclination led to physical and mental strain. He tried, in vain, to overcome his problems by participating in carefree student life, but he soon came to shun lectures and to restrict himself to studying mathematics on his own, beginning with the *Mécanique céleste* of Laplace. Weierstrass was fortunate in having an understanding adviser in astronomy, mathematics, and physics, Dietrich von Münchow. However, Münchow was of the old school and, because he gave only elementary lectures, was remote from the advances of modern mathematics.

Around this time Weierstrass read Jacobi's *Fundamenta nova theoriae functionum ellipticarum* (1829); the work proved difficult for him, based, as it was, on prior knowledge of Legendre's *Traité des fonctions elliptiques*, published shortly beforehand. A transcript of Christof Gudermann's lecture on modular functions rendered the theory of elliptic variables understandable to him and inspired him to initiate his own research. In a letter to Sophus Lie of 10 April 1882, Weierstrass explained his definitive decision to study mathematics:

For me this letter [from Abel to Legendre], when I became aware of it in Crelle's *Journal* [6 (1830), 73–80] during my student years, was of the utmost importance. The immediate derivation of the form of representation of the function given by Abel and designated by him by  $\lambda(x)$ , from the differential equation defining this function, was the first mathematical task I set myself; and its fortunate solution made me determined to devote myself wholly to mathematics; I made this decision in my seventh semester [winter semester 1837-1838], although originally I undertook the study of public finance and administration [N. H. Abel, *Mémorial* (1902), 108].

After eight semesters Weierstrass left the university without taking the examination. Although his father was greatly disappointed, a family friend, president of the court of justice of Paderborn, persuaded him to send Weierstrass to the Theological and Philosophical Academy at Münster, where he would be able to take the teacher's examination after a short time. Weierstrass enrolled on 22 May 1839. Helped and encouraged by Gudermann— Weierstrass was the only university student at his lectures on elliptic functions, he left Münster that autumn to prepare for the state examination.

In January 1840 Weierstrass' father assumed the more remunerative post of director of the salt-works and the family moved to Westernkotten, near Lippstadt. Two months later Weierstrass registered for the examination, and at the beginning of May he received the philological, pedagogical, and mathematical problems for the written examination. The first mathematical problem was one that Weierstrass himself had requested, the representation of elliptic functions. Following Abel, for whose work he had always had the highest regard, Weierstrass presented in his examination an important advance in the new theory of elliptic functions, and this work contains important starting points for his subsequent investigations. Gudermann recognized

the significance of his accomplishment and wrote in his evaluation that Weierstrass was “of equal rank with the discoverers who were crowned with glory.” The school superintendent was somewhat more restrained. When Weierstrass later read Gudermann’s complete critique, he admitted that had he learned of it earlier, he would have published the work immediately and most certainly would have obtained a university chair sooner. He considered it especially fine of Gudermann to have praised him so highly, even though his work contained sharp criticism of Gudermann’s method. It is of interest that one of the other mathematical problems that Weierstrass was assigned, on elementary geometry, gave him much difficulty, at least according to his brother’s account.

After having passed the oral examinations in April 1841, Weierstrass taught for a one-year probationary period at the Gymnasium in Münster before transferring to the Catholic [secondary school](#) in Deutsch-Krone, West Prussia (1842-1848), and then to the Catholic Gymnasium in Braunsberg, East Prussia (1848-1855). In addition to mathematics and physics, he taught German, botany, geography, history, gymnastics, and even calligraphy. (Reminiscent of this is his peculiar  $q$  of the Weierstrassian  $p$ -function.) In recalling the misery of these years, Weierstrass remarked that he had neither a colleague for mathematical discussions nor access to a mathematical library, and that the exchange of scientific letters was a luxury that he could not afford. The “unending dreariness and boredom” would have been unbearable for him without hard work, and his every free minute was devoted to mathematics.

Fortunately he found an understanding senior colleague in Ferdinand Schultz, director of the Braunsberg Institute. Weierstrass was in Deutsch-Krone during the Revolution of 1848. Dirichlet had stated that a mathematician could only be a democrat, and Weierstrass’ beliefs were not contrary to this belief. Commissioned to oversee the belletristic section of the local newspaper, he approved reprinting the freedom songs of Georg Herwegh—under the eyes of the censor.

Although not involved in nationalistic struggles, Weierstrass was no stranger to national feelings. He aspired to neither title nor decorations and was reluctant to exchange the simple title of professor for the more pretentious one of privy councillor. His religious views were moderate and tolerant, and he eschewed political as well as religious bigots. Reared a Catholic, he paid homage in a public speech as rector to the cultural significance of the Reformation. In his philosophical outlook he was a frank adherent of Kant and an opponent of Fichte and Schelling.

Weierstrass’ first publications on Abelian functions, which appeared in the Braunsberg school prospectus (1848-1849), went unnoticed; but the following work, “Zur Theorie der Abelschen Functionen” (Crelle’s *Journal*, **47** [1854], 289–306), elicited enormous interest and marked a decisive turning point in his life. In this memoir he demonstrated the solution to the problem of inversion of the hyperelliptic integrals, which he accomplished by representing Abelian functions as the quotients of constantly converging power series. Many of his results were only hinted at in this work, for since 1850 he had suffered painful attacks of vertigo, which lasted up to an hour and subsided only after a tormenting attack of vomiting. These attacks, which contemporaries called brain spasms, recurred for about twelve years and made it impossible for him to work. Although the 1854 paper was merely a preliminary statement, Liouville called it “one of those works that marks an epoch in science.” On 31 March 1854 the University of Königsberg awarded Weierstrass an honorary doctorate. He was promoted to senior lecturer at Braunsberg and in the fall of 1885 was granted a year’s leave to continue his studies. Firmly determined not to return to the school, he applied in August 1855 for the post of Kummer’s successor at the University of Brelay—an unusual mode of procedure. (Kummer had been called to Berlin to succeed Dirichlet, who had assumed Gauss’s chair at Göttingen.) Weierstrass did not receive the appointment at Breslau.

In the famous “Theorie der Abelschen Functionen” (Crelle’s *Journal*, **52** [1856], 285–380), which contains an excerpt from the previously mentioned examination work, Weierstrass proved what previously he had only hinted. According to Hilbert, he had realized one of the greatest achievements of analysis, the solution of the Jacobian inversion problem for hyperelliptic integrals. There was talk of appointment to a post in Austria, but before formal discussions could take place Weierstrass accepted on 14 June 1856 an appointment as professor at the Industry Institute in Berlin, a forerunner of the Technische Hochschule. While he did not have to return to the Gymnasium in Braunsberg, his hopes for appointment to the University of Berlin had not been realized. In September 1856, while attending a conference of natural scientists in Vienna, Weierstrass was offered a special professorship at any Austrian university of his choice. He was still undecided a month later, when he was invited to the University of Berlin as associate professor. He accepted. On 19 November 1856 he became a member of the Berlin Academy. It was not until July 1864 that he was able to leave the Industry Institute and assume a chair at the university.

Having spent the most productive years of his life teaching elementary classes, far from the centers of scientific activity, Weierstrass had found time for his own research only at the expense of his health. Heavy demands were again made on him at Berlin, and on 16 December 1861 he suffered a complete collapse; he did not return to scientific work until the winter semester of 1862-1863. Henceforth he always lectured while seated, consigning the related work at the blackboard to an advanced student. The “brain spasms” were replaced by recurrent attacks of bronchitis and phlebitis, which afflicted him until his death at the age of eighty-one. Nevertheless, he became a recognized master, primarily through his lectures. He delayed publication of his results not—as has often been charged—because of a “basic aversion to printer’s ink” but, rather, because his critical sense invariably compelled him to base any analysis on a firm foundation, starting from a fresh approach and continually revising and expanding.

It was only gradually that Weierstrass acquired the masterly skill in lecturing extolled by his later students. Initially his lectures were seldom clear, orderly, or understandable. His ideas simply streamed forth. Yet his reputation for lecturing on new theories attracted students from around the world, and eventually some 250 students attended his classes. Since no one else offered the

same subject matter, graduate students as well as university lecturers were attracted to Berlin. Moreover, he was generous in suggesting topics for dissertations and continuing investigations.

One of Weierstrass' first lectures at Berlin was on the application of Fourier series and integrals to problems of mathematical physics. But the lack of rigor that he detected in all available works on the subject, as well as the fruitlessness of his own efforts to surmount this deficiency, frustrated him to the degree that he decided not to present this course again. It was not until 1885 that he took up the representation of single-valued functions of a real variable by means of trigonometric series, stressing that "he had considered the needs of mathematical physics." Here again are manifest his proverbial striving for the characteristic "Weierstrassian rigor" that virtually compelled him to carry his investigations to an ever higher degree of maturity and completion. His position concerning the applications of his research was clarified in his inaugural speech at the Berlin Academy on 9 July 1857, in which he stated that mathematics occupies an especially high place because only through its aid can a truly satisfying understanding of natural phenomena be obtained. To some degree his outlook approached that of Gauss, who believed that mathematics should be the friend of practice, but never its slave.

Over the years Weierstrass developed a great lecture cycle: "Introduction to the Theory of Analytic Functions"; "Theory of Elliptic Functions," sometimes beginning with the differential calculus, at other times starting with the theory of functions, the point of departure being the algebraic addition theorem; "Application of Elliptic Functions to problems in Geometry and Mechanics"; "Theory of Abelian Functions"; "Application of Abelian Functions to the Solution of Selected Geometric Problems"; and "Calculus of Variations." Within this cycle Weierstrass erected the entire structure of his mathematics, using as building blocks, only that which he himself had proven.

During seven semesters (1864-1873) Weierstrass also lectured on synthetic geometry, thereby honoring his promise to [Jakob Steiner](#) before the latter's death in 1863. Steiner's discussions, which Weierstrass had read in Crelle's *Journal* as a student, had especially stimulated his interest; and he was one of the few people in Berlin with whom the old crank had remained on good terms. These lectures were given only out of a sense of obligation, however—not from any interest in the subject; for Weierstrass considered geometric demonstrations to be in very poor taste. If, as has been alleged, he sometimes permitted himself to clarify a point by using a diagram, it was carefully erased.

In addition to lecturing, Weierstrass introduced the first seminar devoted exclusively to mathematics in Germany, a joint undertaking with Kummer at the University of Berlin in 1861. Here again he developed many fruitful concepts that were frequently used by his students as subjects for papers. In his inaugural lecture as rector of the University of Berlin, Weierstrass called for lecturers to "designate the boundaries that had not yet been crossed by science ...from which positions further advances would then be made possible." The lecturer should neither deny his students "a deeper insight into the progress of his own investigations, nor should he remain silent about his own errors and disappointments."

Weierstrass' students included Heinrich Bruns, Georg Frobenius, Georg Hettner, Ludwig Kiepert, Wilhelm Killing, Johannes Knoblauch, Ernst Kötter, Reinhold von Lilienthal, Hans von Mangoldt, Felix Müller, Eugen Netto, Friedrich Schottky, Ludwig Stickelberger, and Wilhelm Ludwig Thomé. Auditors or participants in the seminar included Paul Bachmann, Oskar Bolza, Friedrich Engel, Leopold Gegenbauer, August Gutzmer, Lothar Heffter, Kurt Hensel, Otto Hölder, Adolf Hurwitz, [Felix Klein](#), Adolf Kneser, Leo Koenigsberger, Fritz Kötter, Mathias Lerch, Sophus Lie, Jacob Lüroth, Franz Mertens, [Hermann Minkowski](#), Gösta Mittag-leffler, Hermann Amandus Schwarz, and Otto Stolz. The philosopher [Edmund Husserl](#)—insofar as he was a mathematician—was also a student of Weierstrass.

Weierstrass was not without his detractors: [Felix Klein](#), for instance, remarked that he and Lie had merely fought for their own points of view in the seminars. Most of Weierstrass' students, however, accepted his theories as an unassailable standard. Doubts were not permitted to arise, and checking was hardly possible since Weierstrass cited very few other sources and arranged his methodical structure so that he was obliged to refer only to himself. Independent opinions, such as Klein's, were the exception.

Weierstrass' criticism of Riemann's basic concept of the theory of functions, namely the application and use of the principle of Dirichlet, resulted in the fact that until the twentieth century his approach to the theory of functions, starting with the power series, was preferred to Riemann's, which originated with complex differentiation, Weierstrass formulated his credo in a letter to his student H. A. Schwarz (3 October 18756):

The more I ponder the principles of function theory—and I do so incessantly—the more I am convinced that it must be founded on simple algebraic truths and that one is therefore on the wrong path if, instead of building on simple and fundamental algebraic propositions, one has recourse to the "transcendental" (to put it briefly), no matter how impressive at first glance, for example, seem the reflections by means of which Riemann discovered so many of the most important properties of algebraic functions. It is self-evident that any and all paths must be open to a researcher during the actual course of his investigations; what is at issue here is merely the question of a systematic theoretical foundation.

Although Weierstrass enjoyed considerable authority at Berlin, he occasionally encountered substantial resistance from his colleagues; and such criticism hurt him deeply. In the late 1870's his relations with his close friend [Leopold Kronecker](#) cooled considerably when Kronecker imparted to Weierstrass his antipathy for the work of [Georg Cantor](#). Weierstrass had been one of the first to recognize the value of Cantor's accomplishments and had in fact stimulated his work on the concept of countability. Kronecker, by contrast, proclaimed that he had set himself the task "of investigating the error of every conclusion used in the

so-called present method of analysis.” Weierstrass’ reaction to Kronecker’s attack may well have been excessive; but in 1885 he decided to leave Germany and go to Switzerland, believing that everything for which he had worked was near collapse. Determined to prevent such a catastrophe, he resolved to remain in Berlin after all. The choice of his successor and publication of his works were problems still to be resolved—and his successor would have to be endorsed by Kronecker. Kronecker’s death in 1891 cleared the path for the appointment of Hermann Amandus Schwarz.

But the publication of Weierstrass’ writings was another matter. He was satisfied with neither the circulating transcripts of his lectures nor with the textbooks that followed his concepts and that he had, to some degree, authorized; and his major ideas and methodology remained unpublished. In 1887, having already edited the works of Steiner and Jacobi, Weierstrass decided to publish his own mathematical lifework, assured of the help of the younger mathematicians of his school. He lived to see only the first two volumes appear in print (1894, 1895). According to his wishes, volume IV was given preferential treatment, and it appeared in 1902. The altered title, “Lectures on the Theory of Abelian Transcendentals” —they had always been called the theory of Abelian functions—more accurately reflects the scope of his lectures. Volume III was published the following year. Twelve years elapsed before the appearance of volume V (“Lectures on Elliptic Functions”) and volume VI (“Selected Problems of Geometry and Mechanics to be Solved With the Aid of the Theory of Elliptic Functions”), and another dozen years before volume VII (“Lectures on the Calculus of Variations”). All of Weierstrass’ efforts to ensure the publication soon after his death of a complete edition of his works were fruitless: volume VIII—X, intended to contain works on hyperelliptic functions, a second edition of his lectures on elliptic functions, and the theory of functions, remain unpublished.

In 1870, at the age of fifty-five, Weierstrass met the twenty-year-old Russian [Sonya Kovalevsky](#), who had come to Berlin from Heidelberg, where she had taken her first semester under Leo Koenigsberger. Unable to secure her admission to the university, Weierstrass taught her privately; and his role in both her scientific and personal affairs far transcended the usual teacher-student relationship. In her he found a “refreshingly enthusiastic participant” in all his thoughts, and much that he had suspected or fumbled for became clear in his conversations with her. In a letter to her of 20 August 1873, Weierstrass wrote of their having “dreamed and been enraptured of so many riddles that remain for us to solve, on finite and infinite spaces, on the stability of the world system, and on all the other major problems of the mathematics and the physics of the future.” It seemed to him as though she had “been close . . . throughout [his] entire life . . . and never have I found anyone who could bring me such understanding of the highest aims of science and such joyful accord with my intentions and basic principles as you!” Through his intercession she received the doctorate *in absentia* at Göttingen in 1874.

Yet their friendship did not remain untroubled. Her links with socialist circles, her literary career as author of novels, and her advocacy of the emancipation of women strongly biased the judgment of her contemporaries, and resulted in defamation of the friendship. On the other hand, many of his letters to her were unanswered. At one juncture she remained silent for three years. He was instrumental in her obtaining an appointment as lecturer in mathematics at Stockholm in 1883 and a life professorship in mathematics in 1889. The misinterpretation of their relationship and her early death in 1891 brought him additional physical suffering. During his last three years he was confined to a wheelchair, immobile and dependent. He died of pneumonia.

In his inaugural speech to the Berlin Academy, Weierstrass characterized his scientific activity as having centered on the search for “those values of a wholly new type of which analysis had not yet had an example, their actual representation, and the elucidation of their properties.” One of his earliest attempts at solving this problem was a treatise (1841) on the representation of an analytic function exhibiting an [absolute value](#) that lies between two given boundaries. It contained the Cauchy integral proposition and the Laurent proposition. It was published only fifty-three years later, however, when it became clear that Weierstrass at the age of twenty-six had already had at his disposal the principles of his theory of functions, to the development of which he subsequently devoted his lifework. Yet his contribution to reestablishing the theory of analytic functions ultimately served only to achieve his final aim: the erection of a general theory of Abelian integrals (all integrals over algebraic functions) and the consideration of their converse functions, the Abelian functions.

What Weierstrass considered to be his main scientific task is now held to be less important than his accomplishments in the foundation of his theory. The special functions which he investigated, and the theory of which he lucidly elaborated or transformed, now elicit less interest than his criticism, rigor, generally valid concepts, and the procedures and propositions, of the theory of functions. Weierstrass’ name remains linked to his preliminary proposition, approximation propositions, double series proposition, proposition of products, and fundamental proposition—as well as the Casorati-Weierstrass proposition. Hundreds of mathematicians were influenced by his uncompromising development of a systematic foundation and his pursuit of a fixed plan after appropriate preparation of detail; and they in turn instilled in their students Weierstrass’ concepts of the necessity of clarity and truth, and his belief that the highest aim of science is to achieve general results. Admired by Poincaré for his “unity of thought,” Weierstrass was the most important nineteenth-century German mathematician after Gauss and Riemann.

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Kurt-R. Biermann