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(*b*. Columbia, Missouri, 26 November 1894; *d*. Stockholm, Sweden, 18 March 1964

mathematics.

Wiener was the son of Leo Wiener, who was born in Byelostok, Russia, and Bertha Kahn. Although a child prodigy, he matured into a renowned mathematician rather slowly. At first he was taught by his father. He entered high school at the age of nine and graduated two years later. After four years in college, he enrolled at the Harvard Graduate School at the age of fifteen in order to study zoology. That soon turned out to be a wrong choice. He next tried philosophy at Cornell. "A philosopher in spite of himself," Wiener took a Ph.D. at Harvard in 1913 with a dissertation on the boundary between philosophy and mathematics. A Harvard traveling fellowship paid his way to Europe. Bertrand Russell was his chief mentor at Cambridge and advised him to learn more mathematics. Neither the examples of Hardy and Littlewood at Cambridge,however, nor those of Hilbert and Landau at Göttingen, converted him to mathematics. Back in the United States in 1915, Wiener tried various jobs teaching philosophy, mathematics, and engineering. In the spring of 1919 he got a position in the mathematics department of the Massachusetts Institute of Technology, not then particularly distinguished in that discipline. An assistant professor in 1924, associate in 1929, and full professor in 1932, he remained at MIT until his retirement. Although his genius contributed to establishing the institute's present reputation, he could never comfort himself over the failure of other American universities, and particularly of Harvard to show much interest in him. He traveled a great deal, to Europe and to Asia, and his visits to Germany in the interwar years left their traces in many anecdotes told in Continental circles. His *Cybernetics* made him a public figure. President Lyndon Johnson awarded him the National Medal of Science two months before his death. He died during a trip to Sweden and left two daughters. His wife was the former Margaret Engemann.

In appearance and behavior, Norbert Wiener was a baroque figure, short, rotund, and myopic, combining these and many qualities in extreme degree. His conversation was a curious mixture of pomposity and wantonness. He was a poor listener. His self-praise was playful, convincing, and never offensive. He spoke many languages but was not easy to understand in any of them. He was a famously bad lecturer.

Wiener was a great mathematician who opened new perspectives onto fields in which the activity became intense, as it still is. Although most of his ideas have become standard knowledge, his original papers, and especially his books, remain difficult to read. His style was often chaotic. After proving at length a fact that would be too easy if set as an exercise for an intelligent sophomore, he would assume without proof a profound theorem that was seemingly unrelated to the preceding text, then continue with a proof containing puzzling but irrelevant terms, next interrupt it with a totally unrelated historical exposition, meanwhile quote something from the "last chapter" of the book that had actually been in the first, and so on. He would often treat unrelated questions consecutively, and although the discussion of any one of them might be lucid, rigorous, and beautiful, the reader is left puzzled by the lack of continuity. All too often Wiener could not resist the temptation to tell everything that cropped up in his comprehensive mind, and he often had difficulty in separating the relevant mathematics neatly from its scientific and social implications and even from his personal experiences. The reader to whom he appears to be addressing himself seems to alternate in a random order between the layman, the undergraduate student of mathematics, the average mathematician, and Wiener himself.

Wiener wrote a most unusual autobiography. Although it conveys an extremely egocentric view of the world, I find it an agreeable story and not offensive, because it is naturally frank and there is no pose, least of all that of false modesty. All in all it is abundantly clear that he never had the slightest idea of how he appeared in the eyes of others. His account of the ill-starred trip to Europe in 1926–1927 is a particularly good example. Although he says almost nothing about the work of the mathematicians whom he met, he recalled after twenty-five years meeting J. B. S. Haldane and setting him straight over an error in his book *The Gold-Makers*: Haldane had used a Danish name for a character supposed to be an Icelander (*l Am a Mathematician*, 160). In his autobiography Wiener comes through as a fundamentally good-na-tured person, realistic about his human responsibilities and serious enough to be a good friend, a good citizen, and a good cosmopolite. Despite his broad erudition, the philosophical interludes are no more than common sense, if not downright flat. Unlike many autobiographers, he never usurps the role of a prophet who long ago predicted the course that things have taken. A good biography ought to be written of him, one that would counterbalance his autobiography and do him more justice than anyone can do in a book about himself.

According to his own account, Wiener's understanding of modern mathematics began in 1918, when he came across works on integration, functionals, and differential equations among the books of a young Harvard student who had died. At that time he met I. A. Barnett, who by suggesting that he work on integration in function spaces, put Wiener on the track that would lead him to his greatest achievements, the first of which was differential space. It was already characteristic of Wiener's openness of mind that, rather than being satisfied with a general integration theory, he looked for physical embodiments to test the theory. The first he tried, turbulence, was a failure; but the next, Brownian motion (1921), studied earlier by Einstein, was a success. Wiener conceived a measure in the space of one-dimensional paths that leads to the application of probability concepts in that space (see *Selected Papers*, no. 2). The construction is surprisingly simple. Take the set of continuous functions $x(t)$ of $t \geqq 0$ with $X(0) = 0$ and require that the probability of x passing for t_i between α_i and $\beta(_i = 1,. . .,K$ is provided by the Einstein-Smoluchowski formula that gives for the probability density of a point at x staying at y after a lapse of time $t$the expression

In later work Wiener made this measure more explicit by a measure-preserving mapping of the real number line on function space. He also proved that almost all paths are nondifferentiable and that almost all of them satisfy a Lipschitz condition of any degree <1/2, although almost none does so with such a condition of degree >1/2. “Differential space” is a strange term for this function space with a measure, promising a measure defined not by finite but by differential methods. Although vaguely operative on the background, this idea was never made explicit by Wiener when he resumed use of the term “differential space” in later work.

In 1923–1925 Wiener published papers that greatly influenced potential theory: Dirichlet’s problem, in its full generality (see *Selected Papers*, no. 3). The exterior problem of a compact set K in 3-space led him to the capacitory potential of a measure with support K as a basic tool.

From Brownian motion Wiener turned to the study of more general stochastic processes, and the mathematical needs of MIT’s engineering department set him on the new track of harmonic analysis. His work during the next five years culminated in a long paper (1930) on generalized harmonic analysis (see *Selected Papers*, no. 4), which as a result of J. D. Tamarkin’s collaboration is very well written. Rather than on the class L^2, Wiener focused on that of measurable functions f with

existing for all x, which is even broader than that of almost periodic functions. He borrowed the function π from physics as a key to harmonic analysis and connected it later to communication theory. Writing π as a Fourier transform,

he obtained what is now called the spectral distribution S. The most difficult step was to connect S to the integrated Fourier transform g of f by an analogue of the classical formula $d\lambda$. A brilliant example is: If $f(x) = \pm 1$ for $x_n \leq x < x_{n+1}$, where the signs are fixed by spinning a coin, then the spectral distribution of f is almost certainly continuous.

A key formula in this field was placed by Wiener on the cover of the second part of his autobiography:

When Wiener attempted to prove this, A. E. Ingham led him to what Hardy and Littlewood had called Tauberian theorems; but Wiener did more than adapt their results to his own needs. He gave a marvelous example of the unifying force of mathematical abstraction by recasting the Tauberian question as follows (see *Selected Papers*, no. 5): To prove the validity of

by which kind of more tractable kernel K_1, K may be replaced. The answer is (for K and K_1 □ L_1): If the Fourier transform of K_1 vanishes nowhere, the validity with K_1 implies that with K. Tauberian theorems have lost much of their interest today, but the argument by which Wiener proved his theorem is still vigorous. Wiener showed that in L_1 the linear span of the translates of a function is dense if its Fourier transform vanishes nowhere. This, again, rests on the remark that the Fourier transform class L_1 is closed with respect to division (as far as possible). Wiener’s work in this area became the historical source of the theory of Banach algebras. The “Wiener problem,” that is, the problem of deciding whether it is true that in L_1 a function $f_{,1}$ belongs to the closure of the span of the translates of f_2 if and only if the Fourier transform of f_1 always vanishes together with that of f_2, greatly influenced modern harmonic analysis; it was proved to be wrong by Paul Malliavin in 1959.

Fourier transforms and Tauberian theorems were also the subject of Wiener and R. E. A. C. Paley’s collaboration, which led to *Fourier Transforms* (1934). Another cooperative achievement was the study of the Wiener-Hopf equation (see *Selected Papers*, no. 6),

generalizing Eberhard Hopf’s investigation on radiation equilibrium. In *I Am a Mathematician* (p. 177), Wiener remarked that although originally accounting for the discontinuity of two physical media at x=0, it can even better serve to embody the discontinuity of knowledge at the boundary of future and past. The previous work on the Wiener Hopf equation became influential in Wiener’s prediction theory.

Until the late 1930’s stochastic processes, as exemplified by Brownian motion, and harmonic analysis were loose ends in the fabric of Wiener’s thought. To be sure, they were not isolated from each other: the spectrally analyzed function f was thought of as a single stochastic happening, and the earlier cited example shows that such a happening could even be conceived as embedded in a stochastic process. Work of others in the 1930’s shows the dawning of the idea of spectral treatment of stationary stochastic processes; at the end of the decade it became clear that the “Hilbert space trick” of ergodic theory could serve this aim also. Initially Wiener had neglected ergodic theory; in 1938–1939 he fully caught up (see *Selected Papers*, nos. 7–8), although in later work he did not avail himself of these methods as much as he might have done.

Communication theory, which for a long time had been Wiener’s background thought, became more prominent in his achievements after 1940. From antiaircraft fire control and noise filtration in radar to control and communication in biological settings, it was technical problems that stimulated his research Although linear prediction was investigated independently by A. N. Kolmogorov, Wiener's approach had the merit of dealing with prediction and filtering under one heading. If on the strength of ergodicity of the stationary stochastic process $f(f_t \varepsilon L_2)$, the covariances $\phi(t) = (f_t, f_0)$ are supposed to be provided by the data of the past, linear predicting means estimating the future of f by its projection on the linear span of the past f_t On the other hand, linear filtering means separating the summands "message" and "noise" in f = where again the autocovariances and cross covariances $\Phi(t) = (f_t, f_0)$ and are supposed to be known and the message is estimated by its projection on the linear span of the past signals f_t Both tasks lead to Wiener-Hopf equations for a weighting distribution w,

respectively.

The implications of these fundamental concepts were elaborated in a wartime report that was belatedly published in 1949; it is still difficult to read, although its contents have become basic knowledge in communication theory. Nonlinear filtering was the subject of Wiener's unpublished memorandum (1949) that led to combined research at MIT, as reported by his close collaborator Y. W. Lee (see *Selected Papers*, pp. 17–33). A series of lectures on this subject was published in 1958. One of its main subjects is the use of an orthogonal development of nonlinear (polynomial) Volterra functionals by R. H. Cameron and W. T. Martin (1947) in a spectral theory and in the analysis and synthesis of nonlinear filters, which, rather than with trigonometric inputs, are probed with white Gaussian inputs.

After this brief exposition of Wiener's mathematics of communication, it remains to inspect the broad field that Wiener himself vaguely indicated as cybernetics; he tells how he coined this term, although it had not been unusual in the nineteenth century to indicate government theory. Whilestudying antiaircraft fire control, Wiener may have conceived the idea of considering the operator as part of the steering mechanism and of applying to him such notions as feedback and stability, which had been devised for mechanical systems and electrical circuits. No doubt this kind of analogy had been operative in Wiener's mathematical work from the beginning and sometimes had even been productive. As time passed, such flashes of insight were more consciously put to use in a sort of biological research for which Wiener consulted all kinds of people, except mathematicians, whether or not they had anything to do with it. *Cybernetics, or the Control and Communication in the Animal and the Machine* (1948) is a rather eloquent report of these abortive attempts, in the sense that it shows there is not much to be reported. The value and influence of *Cybernetics,* and other publications of this kind, should not, however, be belittled. It has contributed to popularizing a way of thinking in communication theory terms, such as feedback, information, control, input, output, stability, homeostasis, prediction, and filtering . On the other hand, it also has contributed to spreading mistaken ideas of what mathematics really means *Cybernetics* suggests that it means embellishing a nonmathematical text with terms and formulas from highbrow mathematics. This is a style that is too often imitated by those who have no idea of the meaning of the mathematical words they use. Almost all so-called applications of information theory are of this kind.

Even measured by Wiener's standards, *Cybernetics* is a badly organized work–a collection of misprints, wrong mathematical statements, mistaken formulas, splendid but unrelated ideas, and logical absurdities. It is sad that this work earned Wiener the greater part of his publicrenown, but this is an afterthought. At that time mathematical readers were more fascinated by the richness of its ideas than by its shortcomings. Few, if any, reviewers voiced serious criticism.

Wiener published more writings of this kind. The last was a booklet entitled *God and Golem, Inc*. It would have been more appropriate as the swan song of a lesser mathematician than Wiener.

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