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(b. Elea, Lucania, *ca*. 490 B.C.; d. Elea, *ca*. 425 B. C.),

philosophy, mathematics.

Zeno became a friend and disciple of Parmenides, with whom, according to Plato's dialogue *Parmenides*, he visited Athens in the middle of the fifth century B. C. SomeSInce it is difficultl ater Greek authors, however, considered this visit an invention of Plato's According to a widespread legend with many greatly differing versions, Zeno was tortured and killed by a tyrant of Elea or of Syracuse, against whom he had conspired.

Zeno's fame and importance both for philosophy and for the mathematical theory of the continuum rest on his famous paradoxes. There is, however, a tradition, preserved by Diogenes Laörtius (IX,5,29), that he also developed a cosmology, according to which there existed several "worlds" κόσμοι composed of "warm" and "cold," "dry" and "wet" , but no empty space. Since it is difficult to find any direct connection between this cosmology and Zeno's paradoxes, that tradition has been questioned in recent times. The cosmology has some affinity, however, to certain medical doctrines of the fifth century B.C. There is at least the possibility that it was part of a theory of the phenomenal world analogous to Parmenides' theory of the world of belief (δόξα);and there is no other known Greek philosopher who held exactly these beliefs. Thus there is no compelling reason to reject the tradition.

According to Plato *(Parmenides,* 127 ff.), as a young man Zeno elaborated his paradoxes in defense of the philosophy of Parmenides but did not, like Parmenides, try to prove positively that there is nothing but the One and that plurality, change, and motion are mere illusions. He simply tried to show that if one assumes the existence of plurality and motion, no less strange consequences follow than if one denies their existence. In his commentary on Plato's *Parmenides* (127D), Proclus affirms that Zeno elaborated forty different paradoxes following from the assumption of plurality and motion, all of them apparently based on the difficulties deriving from an analysis of the continuum.

The best-known of the paradoxes is that of Achilles and the tortoise, according to which Achilles cannot overtake the tortoise. Though he always runs a hundred times faster than the tortoise, the latter will, before Achilles has reached the tortoise's starting point, have moved 1/100 of the original distance; and while Achilles traverses this second distance, the tortoise will have traversed 1/100 of the latter; and so on ad infinitum-so that Achilles can never catch the tortoise. Aristotle tried to refute this argument by pointing out that not only space but also time is infinitely divisible, so that the time particles, the sum of which is finite, correspond to the particles to be traversed and no difficulty arises for Achilles. At first sight this argument appears all the more convincing because in what seems to be the original formulation of the paradox, Achilles apparently finds it easy to traverse the distance between his starting point and that of the tortoise and experiences difficulty only after the distance between him and the tortoise has become smaller. There is, however, a much more subtle form of the argument, according to which Achilles cannot even begin: Before he can traverse the distance between his starting point and that of the tortoise, he has first to traverse half of that distance, and before that one-quarter, before that one-eighth, and so on ad infinitum. Thus he never gets going.

That this difficulty cannot be overcome quite so easily by referring to the infinite divisibility of time is shown by the second famous paradox of Zeno, that of the flying arrow that cannot move. In its simplest form this paradox says that the arrow can move neither in the place where it is nor in a place where it is not. In a more elaborate form it says that at any given instant, the arrow occupies a space equal to its size. It can neither occupy a larger space nor be in two different places at the same time. Since there is nothing between one instant and the next, and since the arrow cannot move in an instant, it phenomenal world analogous to Parmenidesgument by pointing out that the "now" νῦν although it divides time into past and future, is not a part of time, since time is extensive and since any part of that which has extension must in its turn have extension. In other words, time is not composed of "nows" or instants.

In recent times interest in the problem has been so great that hardly a year has passed without the publication of one or more attempts at its solution. G. Vlastos very cleverly pointed out that if we use the mathematical formula for velocity $v = s/t$ and apply it to the instant, which is supposed to be extensionless, we obtain the value $v = 0/0$, which is no fixed value at all and certainly not the fixed value of 0, thus indicating that the arrow has no velocity but remains at rest. In order to obtain the result 0, t must have a positive value: $v = 0/t = 0$. This is quite true. But Vlastos' further observation that the problem is similar to the problem of how a circle can be curved although it is supposed to be composed of points—and points are not curved—shows that what the mathematical formula reveals so clearly is still not quite realizable by human imagination.

The most thorough study of the problem from this latter point of view has been made by A. Grunbaum. He begins by distinguishing between two different concepts of time, one "mind-dependent" and one "mind-independent." The former is characterized by the experience of the fleeting "now" , which—as in Aristotle's theory of time— divides time into a constantly approaching future and into a constantly receding past into which what until "now" had been future sinks back, and in virtue of which things come into being and pass away. This kind of time, according to Grunbaum, can exist only in a mind consciously experiencing time. From it he distinguishes what he calls the "mind-independent" time of the physicist, within which one can clearly distinguish between an "earlier" and a "later" but within which, strictly speaking, there is no future, present, or past. In analogy to mind-dependent time, however, mind-independent time can be divided by "points of simultaneity," which can be assumed in any point of the time coordinate. These points of simultaneity can more strictly be considered as (extensionless) mathematical points than can the "now points" of mind-dependent time, which permits a kind of quasi-instantaneous awareness of succession—as, for instance, in the perception of the unity of a melody consisting of a succession of sounds.

By this distinction between two sorts of time, Grünbaum tries to eliminate from the discussion the result of observations made by [William James](#) and A. N. Whitehead, who tried to show that time cannot strictly be considered a continuum in the sense of [Georg Cantor](#)'s theory of aggregates, since, as careful self-observation shows, human time-consciousness is not continuous, but discrete ("Pulsating," not "Punctual"). This according to Grünbaum, is true of mind-dependent, but not of mind-independent, time. The latter is strictly a continuum with an absolute density of mathematical points. Within the context of this theory an attempt is then made to solve the Zenonian problem on the basis of the assumption that any extended magnitude or interval contains (in the sense of Cantor's theory of aggregates) an indenumerable or superdenumerable infinity of extensionless (or, as Grunbaum expresses it, degenerative) elements, which has extension although its elements do not. In other words, the theory, in contrast with that of Aristotle, who admitted only the "potential infinity" of unending division, postulates that any line or curve actually represents (or is composed of) a nondenumerable infinity of nonextended points that nevertheless has extension.

In a way, then, the paradox that the arrow cannot move in any given instant, but appears to move in a succession of instants, is "solved" by replacing it with the paradox that an infinite aggregate of nonextended points nevertheless has extension: that is, the paradox becomes connected with the intricacies of the modern theory of the continuum, which is still hotly debated. Grünbaum unquestionably, however, drew attention to the psychological aspects of the problem—or, rather, to those aspects where it touches upon the theory of knowledge. As H. Fränkel pointed out, the paradoxes discussed really derive from the fact that

The human mind, when trying to give itself an accurate account of motion, finds itself confronted with two aspects of the phenomenon. Both are inevitable but at the same time they are mutually exclusive. Either we look at the continuous flow of motion; then it will be impossible for us to think of the object in any particular position. Or we think of the object as occupying any of the positions through which its course is leading it; and while fixing our thought on that particular position we cannot help fixing the object itself and putting it at rest for one short instant ["Zeno of Elea's Attacks on Plurality," pp. 8-9].

The two main arguments against motion discussed so far have not survived in their original wording but appear in the ancient tradition in more or less refined formulations. Zeno's arguments against plurality, however, are-at least in part-quoted literally by Simplicius. These quotations are much less clearly and precisely formulated than the paradoxes of Achilles and the tortoise and the flying arrow. They show with what difficulties Zeno had to struggle when trying to express his thoughts, and they therefore require a good deal more interpretation. The difficulty is increased by the fact that Simplicius quotes only the second part of the argument literally and summarizes the first part. In his introduction he says that Zeno had first tried to prove that that which has no "magnitude" (extension[?]; μέγεθος) nor thickness (πάχος) nor body (ογκος) cannot exist. "For if it is added to something else, it will not make it bigger, and if it is subtracted, it will not make it smaller. But if it does not make a thing bigger when added to it nor smaller when subtracted from it, then it appears obvious that what was added or subtracted was nothing." The literal quotation then continues:

If this is [so?], then it is necessary that the one must have a certain distance (ἀπέχειν) from the other. and this is also so with that which protrudes (περί τοῦ προέχοντος) For this also will have "magnitude" [extension] and something of it will protrude. And to say this once and to say it again and again is the same thing. For nothing of it in this way will be the outermost (τό ἔαχατον) and never will any of them be without proportion to the others. Therefore, if there are many things it is necessary that they are small and big: small to the extent of having no size [extension] at all and big to the extent of infinite extension [H. Diels and W. Kranz, *Die Fragmente der Vorsokratiker,* 7th ed., I, no. 29, p. 266].

The use of the word ἀπροέχεω which usually means "to be away from" or "to be at a distance from," has induced some commentators to interpret Zeno's argument in the following way: If there are many things, they must be distinguished from one another. If they are distinguished, they must be separate. If they are separate, there must be something between them. Since Zeno (according to his cosmology) denied the existence of empty space, what separates things must itself be a thing, which in its turn must be separated by another thing that separates it from the things that it separates from one another, and so on ad infinitum. But this interpretation is hardly reconcilable with what precedes and what follows in Simplicius' account. The argument as a whole is understandable only on the assumption that ἀπροέχεω is used as a synonym of προέχεω, the term that is used in the remainder of the argument. The gist of the argument then appears to have been: What has size is divisible. What is divisible is not a real One, since it has parts. But any part that "lies beyond" or protrudes from a given part of something that has size, has size in its turn; hence it has parts, and so on ad infinitum, so that it becomes both small and big beyond all measure, If this is the meaning of the argument, we are back with the problem of the continuum.

Vlastos has claimed that in the last sentence of the fragment quoted, Zeno committed "a logical gaffe" by assuming that through infinitely continued division, one finally ends with particles "of no size!" But on closer inspection it seems clear that Zeno, at least in the first half of his last argument, committed no such logical error. Far from assuming that by infinitely continued division one finally comes to particles "of no size," his argument is based on the very opposite assumption: however far the division may have proceeded, what remains still always has size, hence is further divisible, hence has parts, hence is not really One. Therefore, in order to be really One (indivisible)- and this conclusion, if one starts from Zeno's assumption, is perfectly sound-it must be without size. But-and here the preliminary argument reported by Simplicius is brought in-what has no size does not make a thing to which it is added bigger, nor a thing from which it is subtracted smaller, and hence appears to be nothing.

This interpretation and analysis of the argument is also in perfect agreement with a statement elsewhere attributed to Zeno, to the effect that if someone could really explain to him what the One is, then he would also be in a position to explain plurality. At the same time it shows that Plato was right when he reported that Zeno did not try to give direct support to Parmenides' doctrine that only the One exists, but merely tried to show that from the assumption of a plurality of things, no less strange conclusions could be drawn than from the assumption that there is nothing but the One.

Granting this, one may still contend that Zeno committed a "logical gaffe" in the second part of his last argument, where he speaks of an infinite number of parts that would make the size of the object composed of them grow beyond all measure: this statement appears to be at variance with one of the most elementary applications of the theory of convergent series: that the sum of the infinite series $1/2 + 1/4 + 1/8 + 1/16 + \ldots = 1$. But this mathematical formula is a convenient symbol for the fact that infinitely continued bisection of a unit cannot exceed the unit, a fact of which Zeno, as other fragments clearly show, was perfectly aware. What he obviously did try to point out is that it is not possible for the human mind to build up the sum of such an infinite series starting, so to speak, from the other end, the end with the "degenerative elements," as Grunbaum calls them. When building up a sum, one has always to start with elements that have size. The difficulty is essentially that which H. Fränkel so lucidly described in regard to motion.

The other paradoxes of Zeno that have been preserved by ancient tradition are not so profound and can be resolved completely. One of them is that of the falling millet: If a falling bushel of millet makes a noise, so must an individual grain; if the latter makes no noise, neither can the bushel of grain, for the size of the grain has a definite ratio to that of the heap. The same must then be true of the noises. The resolution here lies in the limitation of perception, which also plays a role in the modern discussion of the perception of time. Interestingly, Zeno argues on the basis of the mathematical argument that there must be a definite proportion.

Another is the paradox of the moving blocks: If four blocks *BBBB* of equal size move along four blocks *AAAA* of the same size which are at rest, and at the same time four blocks *CCCC,* again of the same size and the first two of which have arrived below the last two of the row *AAAA* move with the same speed as *BBBB* in the opposite direction from BBBB, then BBBB will pass two blocks of AAAA in the same time in which they pass four blocks of CCCC .But since their speed remains the same, and yet time is measured by the distance traveled at the same time, half the time is equal to the double time.

Alexander of Aphrodisias made the following diagram to illustrate Zeno's moving blocks argument :

It is interesting that this argument contains the first glimpse in ancient literature of an awareness of the relativity of motion. It is the only Zenonian paradox preserved that has nothing to do with the problem of the continuum, although there have been some attempts in modern times by Paul Tannery and R. E. Siegel to show that there is a connection.

Concerning Zeno's importance for the development of ancient Greek mathematics, the most various views have been held and are still held by modern historians of science. Tannery was the first to suggest that Zeno's relation to the philosophy of Parmenides may have been less close than ancient tradition affirms and that Zeno was much more deeply influenced by problems arising from the discovery of incommensurability by the Pythago-reans. On the basis of the same assumption, H. Hasse and H. Scholz tried to show that Zeno was the "man of destiny" of ancient mathematics. They attempted to prove that the Pythagoreans, after having discovered the incommensurability of the diameter of a square with its side, had tried to overcome the resultant difficulties by assuming the existence of infinitely small elementary lines (*lineae indivisibile s*).It was against this inaccurate handling of the infinitesimal that Zeno protested, thus forcing the next generation of Pythagorean mathematicians to give the theory a better and more accurate foundation.

Other scholars (W. Burkert, A. Szabó, J. A. Philip) contend that since, according to ancient tradition, the Pythagoreans engaged in a rather abstruse number mysticism such a profound mathematical discovery as that of incommensurability cannot have been made by them, but must have been made by "practical mathematicians" influenced by Zeno's paradoxes. But there is no direct road leading from Zeno's paradoxes to the proof of incommensurability in specific cases, whereas some of the speculations supporting the Pythagoreans (when carried through with the consistency characteristic of the philosophers of the first century) must almost inevitably have led to the discovery, although we do not know exactly how it was first made ; and there is no tradition concerning an effect of Zeno's speculations on the development of mathematics in the second half of the fifth century B.C. B. L. van der Waerden has shown that what we know of mathematical theories of the second half of the fifth century B.C.-when the discovery of incommensurability undoubtedly was made-is rather at variance with the assumption that Zeno had any considerable influence on the development of mathematics at that time.

This, however, does not necessarily mean that Zeno's name has to be stricken from the history of ancient Greek mathematics. In all likelihood he received the first impulse toward the invention of his paradoxes not from mathematics but, as attested by Plato, from the speculations of Parmenides, and did not immediately have a strong influence on the development of Greek mathematics. But it is hardly by chance that Plato wrote his dialogue Parmenides, in which he refers to Zeno's paradoxes, around the time that Eudoxus of Cnidus, who revised the theory of proportions in such a way as to enable him to handle the infinitesimal with an accuracy that has remained unsurpassed, spent some years at Athens and was a member of Plato's academy. Zeno's paradoxes can hardly have failed to have been thoroughly discussed then, and so Zeno may still have had some influence on Greek mathematics at that decisive point in its development.

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II. Secondary Literature. See Guido Calogero, *Studi sull'Eleatisnw* (Rome. 1932): and *Storia delta logica antica,* I (Bari, 1967), 171 - 208 ; H. Fränkel, "Zeno of Elea's Attacks on Plurality," in American Journal of Philology, **63** (1942), 1-25, 193-206 : Adolf Grünbaum, "A Consistent Conception of the Extended Linear Continuum as an Aggregate of Unextended Elements," in *Philosophy of Science*, **19** (1952). 288-305: "The Nature of Time," in *Frontiers of Science and Philosophy,* **1** (1962), 149- 184 : and *Modern Science and Zeno's Paradoxes*(Middletown. Conn. 1967): H. Hasse and H. Scholz. *Die Grundlagenkri,vis der griechi.schen Mathe-matik* (Berlin -Charlottenburg, 1928) : J. A. Philip. *Pythagoras and Early Pythagoreanism* , (Toronto, 1966), 206–207; R. E. Siegel, "The Paradoxes of Zeno," in Janus, **48** (1959), 42 ff.; A. Szabó, *Anfänge der grie-chischen Marthermatik* (Munich-Vienna, 1939), 333 ff.; P. Tannery, "Le concept scientifique du continu. Zénon d'élée et Georg Cantor," in *Revue philosophique de la France et de l'étranger*, **20** (1885), 385–410, esp. 393–394; and *Pour l'histoire de la science hellène.* 2nd ed. (Paris, 1930), 248 ff.; B. L. van der Waerden, "Zen-on und die Grundlagenkrise der griechischen Mathema-tik," in *Mathematische Annalen*, **117** , no. **2** (1940), 141–161, esp. 151 ff,; and G. Vlastos, "A Note on Zeno's Arrow," in *Phronesis*, **11** (1966), 3–18; and "Zeno's Race Course," in *Journal of the History of Philosophy*, **4** (1966), 95–108.

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